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Application of Markov chains to quality evaluation of information entering by a computer system user

by

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Abstract: Problems of human-computer interaction modelling are considered in this paper. The aim was the quality evaluation of the user activity. The need of performing such evaluations occurs in the case of computer systems design, destined for control of complex processes, as well as devices being commonly in use. The simplest models are designed to calculate basic characteristics of the user activity, i.e. the command entering time and the command correct execution probability. The theory of Markov chains with rewards is the basis for constructing these models. The use of the elaborated model is shown on an example of simple command entering.

Keywords: manual entering of computer commands, user activity modelling, Markov chains with rewards.

1. Introduction

Both in daily life and in work we deal with diverse information systems. Information service systems are those in which a user operates a computer system. These are: the commonly used client service systems offered in the Internet, seat reservation systems, information searching systems (from timetables to bibliographic and scientific information), etc. We also come across the so-called "open systems", e.g. in the case of mobile telecommunications and banks. Cash machine is a typical example of interaction between an unqualified operator and a specialised computer. In the case of such a system the user interface should ensure fast access to a demanded function.

The second group of systems consists of specialised systems that are used to control actions (e.g., air traffic control systems, installations control systems, rescue-intervention systems, military systems, etc.). The characteristic features of systems that are used to control actions are (among other things): human participation in system tasks, incomplete information about the controlled process, high dynamics of system state changes, limited time and high requirements for the correctness of task execution by the system. The usage of system's executive elements is determined by an information-decision subsystem (Bubnicki, 1993; Paszkowski, 2002). Computer system tasks are reduced to processing of collected data and their presentation for a user (an operator). The user tasks, performed by the use of a complex interface, reside in: observation of displayed information, obtaining of required information, complementary information entering, evaluation of situation, decision making and entering of the decisions to the computer system. Thus, operations performed by the user are reduced to entering commands to a computer.

On the stage of computer system design the problem of system quality evaluation occurs. Particularly, the problem of quality evaluation of execution of tasks performed by a man occurs in the process of human-computer interaction design (Cacciabue, 1998; Hopkin, 1989). On the basis of such an evaluation a designer can change the project in such a way that the computer system will execute tasks with an assumed quality. For the system designer the quality characteristics of tasks executed by a user are essential, when the elementary actions of the tasks are established. On the basis of knowledge of such characteristics the designer can evaluate: quality characteristics of tasks executed by the system as an integrated whole, usefulness of changes in the ways, in which information is entered, sensitivity to changes in algorithms of task execution by the user, etc. For the sake effectiveness of the system's functionality the user activity is limited in time. It is also important that the user enter information to the system faultlessly (Cacciabue, 1998; Dhillon, 1986; Hopkin, 1999).

In order to aid the designer in solving the problems mentioned above some models are indispensable. On the basis of these models it will be possible to evaluate the quality of action of the user entering information through the designed interface.

The quality evaluation of task execution by a user is difficult because of the following factors (Donigiewicz, 2002):

- random character of the user's work, or, more precisely, random execution time of elementary operations;
- random choice of partial tasks, whose execution leads to completion of the whole task;
- changeability in time of the quality characteristics of partial task execution by the user;
- disturbances in the user's work, e.g. stress caused by limited time.

In this paper the user action with respect to a computer system is limited to information entering (i.e. command entering) into the system. Content-related problems of decisions making are not accounted for. An technical side of the user's activity is considered. The analysis is based on the concept of an elementary action or operation, i.e. the basic unit of user action. It is supposed that the user performs elementary actions that are parts of the task of entering information. An elementary user action consists e.g. in pressing a key or clicking an object. Tasks executed by the operator are presented in the form of a net of elementary actions.

2. Command entering process description

In the functioning of a computer system a user interacts with the computer in task execution. The user receives necessary information from the system and enters supplementary information into it. Both the receiving and entering processes are executed by entering an appropriate command into the system (see Fig. 1).

NAME Par_1 Par_2 ... Par_i ... Par_n ACCEPTANCE

Figure 1. A typical command form (where: NAME - identifier of the command kind, Par_i - the i-th command parameter, ACCEPTANCE - the end of the command entering)

A simplified command example may be a command without parameters, consisting of a single sign that is both the command name and the acceptance, and is entered by pressing a key by the user.

The algorithm of entering one sign by the user is shown in Fig. 2, and the appropriate transition graph is shown in Fig. 3. The transition between the states (see Fig. 3) should be interpreted adequately as the process of entering or withdrawing a sign.

The states of the command entering process can be described in the following way:

a_1 - entering a sign;

a_2 - withdrawing a sign in a situation when the sign was correctly entered and the user made a wrong decision, the state may be called "false alarm state";

a_3 - withdrawing a sign in a situation when the sign was incorrectly entered and the user made a good decision, the state may be called "correct alarm state";

a_4 - the end in a situation when the sign was correctly entered and the user made a good decision, the state may be called "correct entering state";

a_5 - the end in a situation when the sign was incorrectly entered and the user made a wrong decision, the state may be called "false quiet state".

Upon taking the reliability characteristics of the user activity into consideration the following quality characteristics of the command entering process can be determined (Donigiewicz, 2002):

- the probability that the command entering process achieved the state "entered correctly" after k steps,
- the reward (i.e. time) obtained by the command entering process in transition from the initial state.

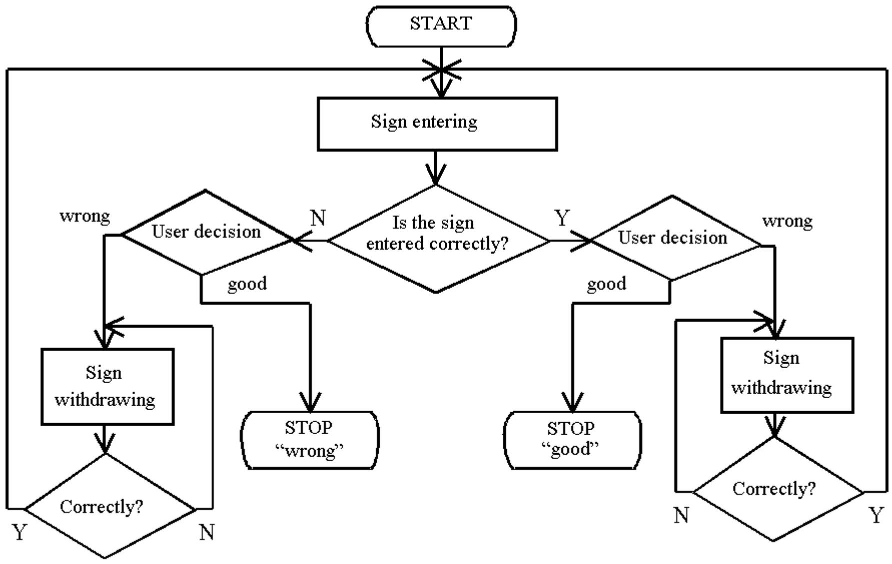

Figure 2. Algorithm of entering one sign

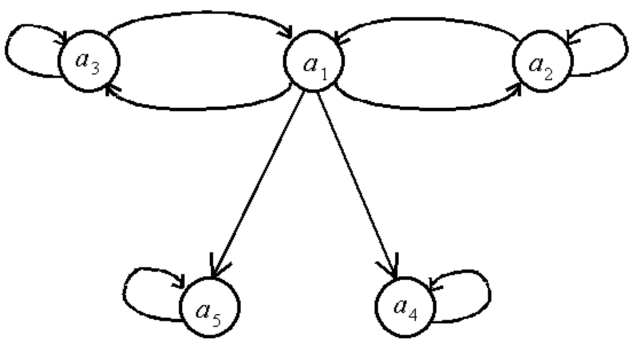

Figure 3. Graph of the process of entering one sign

3. The Markov model of command entering

The process of command entering by a user will be described with the use of the Markov chain. The Markov chain can be determined in the following way (Howard, 1960, 1971):

- initial distribution:

$$P(0) = P\{X(\tau_0) = a_i\}, \quad a_i \in A, \tag{1}$$

where: $P\{X(\tau_0) = a_i\}$ - probability of the event that the process is in state a_i at moment τ_0 ; A - finite set of states $A = \{a_1, \ldots, a_i, \ldots, a_m\}$;

- transition probability matrix of the Markov chain:

$$\mathbf{\Pi} = [p_{ij}] \quad i, j = 1, \ldots, m, \tag{2}$$

where: $p_{ij} = P\{X(\tau_{n+1}) = a_j \mid X(\tau_n) = a_i\}$ - probability of the event that the process is in state a_j at moment τ_{n+1}, on condition that the process was in state a_i at moment τ_n.

For the sake of notation simplicity the events $X(t) = a_i$ will be represented as $X(t) = i,\ i \in \mathbf{M} = \{1, \ldots, m\}$, and the set of time moments is assumed to be discrete.

3.1. Probabilities of state attainment

Let us determine the following vector of state probabilities:

$$\mathbf{P}(k) = [P_1(k), P_2(k), \ldots, P_i(k), \ldots, P_m(k)], \tag{3}$$

where: $P_i(k) = P\{X(k) = i\}$ - probability that the process assumes state i after k steps, $i \in \mathbf{M}$.

We are interested in the limiting values of the vector sequence $\mathbf{P}(k)$, i.e. the vector of limiting probabilities:

$$\mathbf{P}^\infty = \lim_{k \to \infty} \mathbf{P}(k). \tag{4}$$

The following Markov chain, described by the Chapman-Kolmogorov equation, is considered:

$$\mathbf{P}(k+1) = \mathbf{P}(k)\mathbf{\Pi}, \tag{5}$$

where $\mathbf{\Pi} = [p_{ij}]$, $i, j = 1, \ldots, m$ - is a stochastic matrix; $p_{ij} = P\{X(k+1) = j \mid X(k) = i\}$ - probability of the event that the process is in state j after $k+1$ steps, on condition that the process was in state i after k steps.

Let us assume that stochastic matrix $\mathbf{\Pi}$ is regular (i.e., the matrix is regular if eigenvalue 1 is a single root of characteristic equation). This is necessary and sufficient condition for the existence of the limit (Gantmakher, 1959):

$$\mathbf{\Pi}^\infty = \lim_{k \to \infty} \mathbf{\Pi}^k. \tag{6}$$

We will define the z-transform of function $\mathbf{\Pi}^k$ in the following way:

$$\mathbf{P}(z) = \sum_{k=0}^{\infty} \mathbf{\Pi}^k z^{-k}. \tag{7}$$

It is easy to calculate that

$$\mathbf{P}(z) = z(z\mathbf{I} - \mathbf{\Pi})^{-1}. \tag{8}$$

From the z-transform property it follows that

$$\mathbf{\Pi}^{\infty} = \lim_{k\to\infty} \mathbf{\Pi}^k = \lim_{z\to 1}(z-1)\mathbf{P}(z) = \lim_{z\to 1} z(z-1)(z\mathbf{I} - \mathbf{\Pi})^{-1}. \tag{9}$$

Since only linear elementary divisors correspond to eigenvalue 1 of stochastic matrix $\mathbf{\Pi}$ (Gantmakher, 1959), the following representation of matrix $(z\mathbf{I}-\mathbf{\Pi})^{-1}$ is possible:

$$(z\mathbf{I} - \mathbf{\Pi})^{-1} = \frac{1}{z-1}\mathbf{C} + \mathbf{T}(z), \tag{10}$$

where matrix $\mathbf{C}$ is independent of z, and matrix $\mathbf{T}(z)$ satisfies the condition:

$$\lim_{z\to 1}(z-1)\mathbf{T}(z) = 0. \tag{11}$$

Thus, with the assumption that matrix $\mathbf{\Pi}$ is regular, we have

$$\mathbf{\Pi}^{\infty} = \lim_{k\to\infty} \mathbf{\Pi}^k = \mathbf{C}. \tag{12}$$

If matrix $\mathbf{\Pi}$ is regular, then the following vector

$$\mathbf{P}^{\infty} = \lim_{k\to\infty} \mathbf{P}(k) = \lim_{k\to\infty} \mathbf{P}(0)\mathbf{\Pi}^k \tag{13}$$

does not depend on the initial value $\mathbf{P}(0)$ (Gantmakher, 1959). We will determine vector $\mathbf{P}(k)$ analytically, and, as a consequence, the vector $\mathbf{P}^{\infty}$, by the use of the discrete Laplace transform.

Using z-transform with respect to equation (5) we have $z\mathbf{P}(z) - z\mathbf{P}(0) = \mathbf{P}(z)\mathbf{\Pi}$, where $\mathbf{P}(z)$ is z-transformation of function $\mathbf{P}(k)$. Thus $\mathbf{P}(z)$ can be determined from the equation

$$\mathbf{P}(z) = z\mathbf{P}(0)(z\mathbf{I} - \mathbf{\Pi})^{-1}, \tag{14}$$

where $\mathbf{I}$ is the identity matrix.

By applying the inverse z-transform to equation (14) we obtain $\mathbf{P}(k)$. The value $P_4(k)$ is the desirable probability of the event that the process of entering a sign is in the state of correct sign entering after k steps.

3.2. The time of state attainment

We will combine the process of sign entering with the process of rewards generation.

Let us determine the function of reward R in the following way. $R(a_i, a_j) = r_{ij}$ is the reward for transition from state a_i to state a_j and the expected total reward of state a_i after k steps is as follows (Howard, 1960; White, 1993):

$$g_i(k) = \psi_i + \sum_j p_{ij} g_j(k-1), \tag{15}$$

where

$$\psi_i = g_i(1), \quad g_i(1) = \sum_j r_{ij} p_{ij} .$$

In the case of matrix record the equation of expected reward with the initial reward 0 becomes

$$\mathbf{G}(k+1) = \mathbf{\Psi} + \mathbf{\Pi G}(k), \tag{16}$$

where $\mathbf{G}(k) = [g_1(k), g_2(k), \ldots, g_m(k)]^T$ - reward vector for step k;
$\mathbf{\Psi} = [\psi_1, \psi_2, \ldots, \psi_m]^T$ - expected reward for one step (rewards vector);
$\mathbf{\Pi}$ - stochastic matrix.

We will make the assumption that for the initial moment the reward is

$$\mathbf{G}(0) = [0, 0, \ldots, 0]^T. \tag{17}$$

Let us define the z-transformation of function $\mathbf{G}(k)$ in the following way:

$$\mathbf{G}(z) = \sum_{k=0}^{\infty} \mathbf{G}(k) z^{-k}. \tag{18}$$

From equation (16) under condition (17) we have

$$\mathbf{G}(z) = \frac{z}{z-1}(z\mathbf{I} - \mathbf{\Pi})^{-1}\mathbf{\Psi}. \tag{19}$$

Using the assumption that matrix $\mathbf{\Pi}$ is regular, according to (10), we have

$$\mathbf{G}(z) = \frac{z}{(z-1)^2}\mathbf{C\Psi} + \frac{z}{z-1}\mathbf{T}(z)\mathbf{\Psi}. \tag{20}$$

Hence

$$\mathbf{G}(k) = \mathbf{C\Psi}k + \mathbf{R}(k), \tag{21}$$

where

$$\mathbf{R}(k) = Z^{-1}\left\{\frac{z}{z-1}\mathbf{T}(z)\mathbf{\Psi}\right\}$$

and Z^{-1} is the inverse z-transform. For matrix $\mathbf{\Pi}$, according to (11), we have

$$\lim_{k\to\infty} \mathbf{R}(k) = \lim_{z\to 1}(z-1)\frac{z}{z-1}\mathbf{T}(z)\mathbf{\Psi} = \mathbf{T}(1)\mathbf{\Psi}. \tag{22}$$

The limiting reward value per one step is as follows:

$$\lim_{k\to\infty} \frac{1}{k}\mathbf{G}(k) = \mathbf{C}\boldsymbol{\Psi}. \tag{23}$$

The following cases are distinguished as typical:

1^0 $\mathbf{C}\boldsymbol{\Psi} = 0$, then the limiting reward value exists and is equal

$$\mathbf{G}_\infty = \lim_{k\to\infty} \mathbf{G}(k) = \mathbf{T}(1)\boldsymbol{\Psi}. \tag{24}$$

2^0 $\mathbf{C}\boldsymbol{\Psi} \neq 0$, then value $\mathbf{R}(k)$ is unimportant for big k, and the limiting reward value per one step is determined by equation (23).

The value $g_1(k)$ may be calculated analytically by the use of inverse z-transform in relation to that determined by equation (19). Value $g_1(k)$ is the desirable reward (time) obtained by the entering process, after k steps in transition from the state a_1.

4. Examples of determination of quality characteristics of the sign entering

We will show the changes of values $P_4(k), P_5(k), P_2(k)$ and $g_1(k)$, for the illustrative data of the sign entering, while the selected values of the process of sign entering (see Figs. 2 and 3) are changing.

Let us establish the following parameters of a user:

p - probability that the user enters the sign incorrectly,
p_d - probability that the user's decision is wrong,
p_c - probability that the user withdraws the sign incorrectly,
t_b - time of sign entering by the user,
t_c - time of sign withdrawal by the user.

We will determine $\mathbf{P}(k)$ for the way of sign entering shown in Fig. 3. For the considered case the transition probability matrix $\boldsymbol{\Pi}$ is as follows:

$$\boldsymbol{\Pi} = \begin{bmatrix} 0 & p_{12} & p_{13} & p_{14} & p_{15} \\ p_{21} & p_{22} & 0 & 0 & 0 \\ p_{31} & 0 & p_{33} & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{25}$$

where: $p_{12} = (1-p)p_d$, $p_{13} = p(1-p_d)$, $p_{14} = (1-p)(1-p_d)$, $p_{15} = pp_d$, $p_{21} = 1-p_c$, $p_{22} = p_c$, $p_{31} = 1-p_c$, $p_{33} = p_c$.

In the case of initial moment the process considered is characterised by $\mathbf{P}(0) = [1, 0, 0, 0, 0]$. By the use of z-transform, according to equation (14), we have

$$\mathbf{P}(z) = \left[\frac{z(z-p_c)}{\gamma}, \frac{zp_{12}}{\gamma}, \frac{zp_{13}}{\gamma}, \frac{zp_{14}(z-p_c)}{(z-1)\gamma}, \frac{zp_{15}(z-p_c)}{(z-1)\gamma}\right], \tag{26}$$

where $\gamma = z(z - p_c) - (1 - p_c)(p_{12} - p_{13})$.

The limiting distribution of probabilities of being in particular states, for the considered process of sign entering, is as follows:

- for state a_4 we have

$$P_4^{\infty} = \lim_{k\to\infty} P_4(k) = \lim_{z\to 1}(z-1)P_4(z) = \frac{(1-p)(1-p_d)}{1-p+2pp_d-p_d}, \tag{27}$$

- for state a_5 we have

$$P_5^{\infty} = \lim_{k\to\infty} P_5(k) = \lim_{z\to 1}(z-1)P_5(z) = \frac{pp_d}{1-p+2pp_d-p_d}, \tag{28}$$

- for state a_2 we have

$$P_2^{\infty} = \lim_{k\to\infty} P_2(k) = \lim_{z\to 1}(z-1)P_2(z) = 0.$$

It is easy to notice that for $p = 0$ (i.e., when the user does not make mistakes when entering the sign) or $p_d = 0$ (i.e. when the user's decision is correct) we have $P_4^{\infty} = 1$ and $P_5^{\infty} = 1$.

In order to calculate $P_4(k)$, $P_5(k)$ and $P_2(k)$ we will use the inverse z-transform for the selected components of the vector $\mathbf{P}(z)$. By transforming $P_4(z)$ in the following way we have

$$P_4(z) = D_1\frac{z}{z-z_1} + B_1\frac{z}{z-z_2} + U_1\frac{z}{z-z_3}, \tag{29}$$

where:

$$D_1 = \frac{p_{14}(1-p_c)}{(z_2-1)(z_3-1)},$$

$$B_1 = \frac{p_{14}(z_2-p_c)}{(z_2-1)(z_2-z_3)},$$

$$U_1 = \frac{p_{14}(p_c-z_3)}{(z_3-1)(z_2-z_3)},$$

$$z_1 = 1, \quad z_2 = \frac{p_c+\sqrt{p_c^2+4\delta}}{2}, \quad z_3 = \frac{p_c-\sqrt{p_c^2+4\delta}}{2},$$

$$\delta = (1-p_c)(p_{12}+p_{13}),$$

p_{12}, p_{13}, p_{14} as in the case of equation (25).

Now, using the inverse z-transform on $P_4(z)$ we obtain

$$P_4(k) = D_1 + B_1 z_2^k + U_1 z_3^k. \tag{30}$$

By transforming $P_5(z)$ we also obtain

$$P_5(z) = D_2 \frac{z}{z - z_1} + B_2 \frac{z}{z - z_2} + U_2 \frac{z}{z - z_3}, \tag{31}$$

where:

$$D_2 = \frac{p_{15}(1 - p_c)}{(z_2 - 1)(z_3 - 1)},$$

$$B_2 = \frac{p_{15}(z_2 - p_c)}{(z_2 - 1)(z_2 - z_3)},$$

$$U_2 = \frac{p_{15}(p_c - z_3)}{(z_3 - 1)(z_2 - z_3)}.$$

Using the inverse z-transform on $P_5(z)$ we obtain

$$P_5(k) = D_2 + B_2 z_2^k + U_2 z_3^k. \tag{32}$$

Transforming $P_2(z)$ yields

$$P_2(z) = B_3 \frac{z}{z - z_2} + U_3 \frac{z}{z - z_3}, \tag{33}$$

where:

$$B_3 = \frac{p_{12}}{z_2 - z_3}, \quad U_3 = \frac{p_{12}}{z_3 - z_2}.$$

Using the inverse z-transform on $P_2(z)$ we obtain

$$P_2(k) = B_3 z_2^k + U_3 z_3^k. \tag{34}$$

We will calculate the expected total reward $g_1(k)$ for state a_1 after k steps. In the case of considered process of one sign entering the vector $\mathbf{\Psi}$ is as follows

$$\mathbf{\Psi} = [t_b, t_c, t_c, 0, 0]^T, \tag{35}$$

where: t_b - time of the sign entering by the user, t_c - time of the sign withdrawal by the user.
Transforming (19) yields

$$g_1(z) = \frac{z}{z - 1} \frac{(z - p_c)t_b + t_c(p_{12} + p_{13})}{\gamma}, \tag{36}$$

with γ, p_{12}, p_{13} as in equations (25-26). From the above we have

$$g_1^{\infty} = \lim_{k \to \infty} g_1(k) = \lim_{z \to 1} (z - 1) g_1(z) = \frac{t_b(1 - p_c) + t_c(p_d - 2pp_d + p)}{(1 - p_c)(1 - p_d + 2pp_d - p)}. \tag{37}$$

It is easy to notice that for $p = 0$ and $p_d = 0$ we obtain $g_1^{\infty} = t_b$. We will get $g_1(k)$ by the use of the inverse z-transform with respect to equation (36). Transforming $g_1(z)$ we obtain

$$g_1(z) = D\frac{z}{z - z_1} + B\frac{z}{z - z_2} + U\frac{z}{z - z_3}, \tag{38}$$

where:

$$D = \frac{t_b(1 - p_c) + t_c(p_{12} + p_{13})}{(z_2 - 1)(z_3 - 1)},$$

$$B = \frac{t_b(z_2 - p_c) + t_c(p_{12} + p_{13})}{(z_2 - 1)(z_2 - z_3)},$$

$$U = \frac{t_b(p_c - z_3) - t_c(p_{12} + p_{13})}{(z_3 - 1)(z_2 - z_3)},$$

$z_1, z_2, z_3, p_{12}, p_{13}$ as in equations (29).

Using the inverse z-transform on $g_1(z)$ we obtain

$$g_1(k) = D + Bz_2^k + Uz_3^k. \tag{39}$$

5. The effects of user parameters' influence on quality characteristics of sign entering

The following input data have been used in the study: p and p_c varying within the interval [0.005; 0.15]; p_d varying within the interval [0; 0.5] and $t_b = t_c$ =1s. Figs. 4 to 8 show the influence of probabilities p, p_c and p_d on probabilities $P_4(k)$, $P_5(k)$ and $P_2(k)$, in the case of one sign entering by a user.

The probability $P_4(k)$ of achieving the state of correct sign entering (see Fig. 4) changes considerably with the changes of probabilities p and p_c, but in a different way, depending on the number of steps k.

The differences in $P_4(k)$, occurring for various values of probability p_d (a wrong decision is taken), decrease along with the increase of number of steps k, $(k = 1, 3, 4, 5)$. A bigger number of steps means greater probability that the sign is entered correctly. In case of $k = 2$ the graph is omitted because state a_4 cannot be attaind after two steps. The influence of wrong decision probability p_d on probability $P_4(k)$ is quite considerable. The range of $P_4(k)$ variability along with the change of p_d is the same as in the case of $P_4(k)$ variability along with changes of p and p_c - this results from the probability of transition between states a_1 and a_4 (see equation 25).

Probability $P_5(k)$ of achievement of the false quiet state (see Fig. 6) increases along with p and p_c. The relative increase of $P_5(k)$ along with the change of p_d is considerable, but the differences connected with the various number of steps

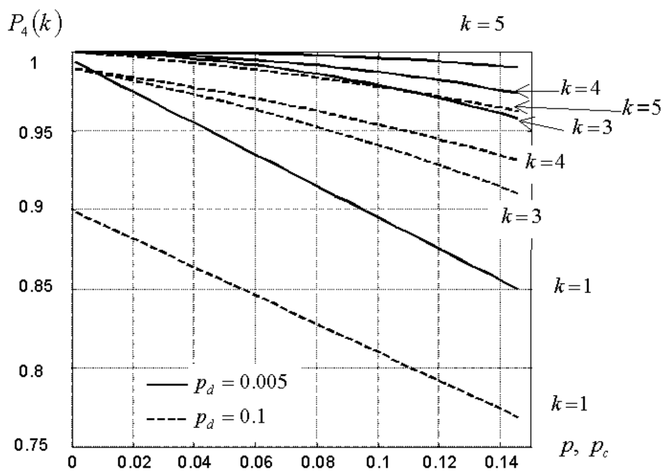

Figure 4. The influence of probabilities p and p_c (simultaneously changed) on probability $P_4(k)$, which describes attainment of the state a_4 when the sign is entered correctly

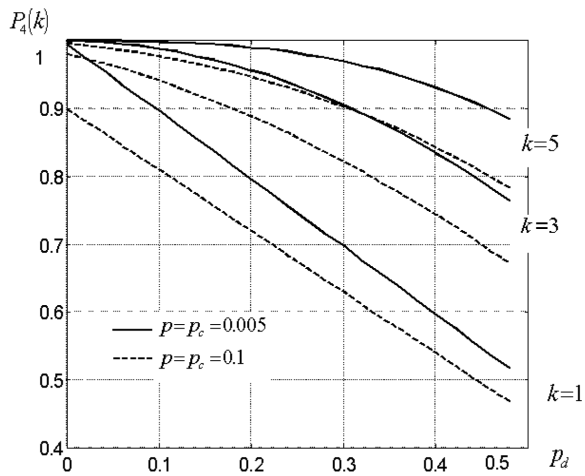

Figure 5. The influence of probabilities p and p_c (The influence of probability p_d on probability $P_4(k)$, which describes attainment of state a_4 for two different values p and p_c)

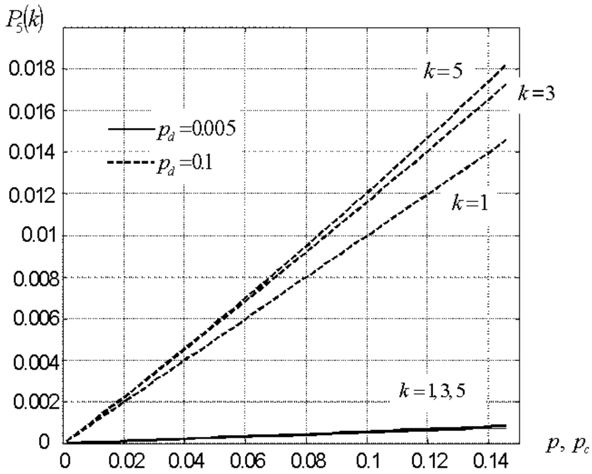

Figure 6. The influence of probabilities p and p_d on probability $P_5(k)$, which describes attainment of state a_5 for two different values of p_d

are not too big. For $k = 4$ the graph should be located between the curves for $k = 3$ and $k = 5$, and has been omitted for the sake of clarity.

The increase of probability p_d has influence on probability $P_5(k)$. For a big change of p_d (see Fig. 6) the increase of probabilities p and p_c causes considerable increase of false quiet probability $P_5(k)$. The achievement of state a_5 means that the user did not detect his mistake during entering information.

Figs. 7 and 8 show the influence of individual values on time $g_1(k)$ of sign entering. Deterioration of user characteristics (i.e., increase of p, p_c and p_d) causes significant increase of time $g_1(k)$. This increase, amounting to about 20%, also depends on the number of steps k. In Fig. 7 the graph for p_d=0.1 and k=1, is not visible because its position is the same as that of the graph for p_d=0.005 and k=1. Probability p_d has no influence on $g_1(k)$. The times t_b and t_c have an essential influence on $g_1(k)$, which is obvious.

6. Conclusions

The here presented quality evaluation model of information entering does not take into consideration all the conditions of user activity. Both the estimated probability that the user makes a mistake when entering information and the time of information entering are the basis of the system quality estimation. It is of particular significance in the case of real-time systems.

An application of the presented model to evaluation of quality of information entering (for a bigger number of operations) and extension of the model to tasks executed by the technical part of the system make possible better consideration

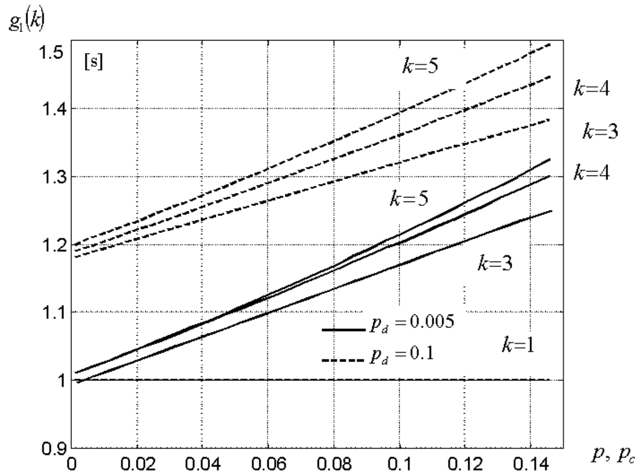

Figure 7. The influence of simultaneously changed probabilities p and p_c on time $g_1(k)$ for two different values p_d

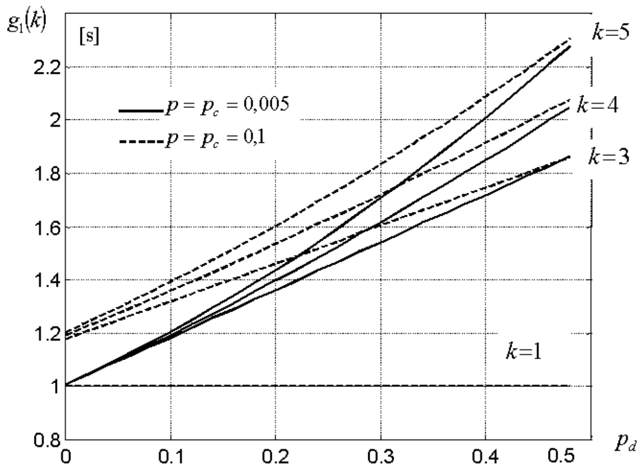

Figure 8. The influence of probability p_d on time $g_1(k)$ for k steps of sign entering

of real conditions of user's work. The only problem is the difficulty in analytical calculation of quality characteristics of the command entering process.

This concerns both the probability that the process attained the correct command entering state and the reward (time) obtained by the sign entering process after k steps. The difficulty lies in the calculation of the inverse z-transform for equations (14) and (19). The problem may be eliminated by the direct use of equations (5) and (16) supported by contemporary computing techniques.

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