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Book review:

Mathematical Finance and Probability. A Discrete Introduction

by

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The title of this book precisely reflects its contents - the volume deals with relations between arbitrage pricing of derivative instruments and probability theory, and both subjects are treated as equally important and not separated by chapters. The discrete-time approach was chosen, what reduces mathematical sophistication at the price of lower level of practical use. This is surprising, since both authors are affiliated with commercial companies. In my own personal view - discrete time was a good choice - at the level of the chosen mathematical rigour, the continuous-time approach would not be readable.

The structure of the book is as follows: the authors start with a brief description of the basic notions of both arbitrage pricing and probability: price, market, risk, return, hedging, replication, random variable, probability, etc. Then the very fundamental Law of One Price is introduced - the whole book is based on that approach. In the subsequent chapters authors develop a more formal probability theory from the measure-theoretical point of view. The choose the Hilbert space (L2) approach. After the necessary mathematical introduction, one-period pricing models are studied in details in both complete and incomplete market settings. Both fundamental asset pricing theorems for one-period models are formulated and proven via the method of change of the numeraire. Then, in Chapter 7 the relation between randomness and information is studied - a topic without very direct link to arbitrage pricing but very helpful for understanding of the probability tools made use of. In Chapter 8 the authors investigate the notion of independence. All chapters up to this point may be considered to be an introduction into the main part of the book: chapters 9-12 on the multi-period models. Chapter 9 is devoted to definitions of all objects under investigation: contingent claims, securities, hedging, attainability and replication. Chapter 10 is the most mathematically sophisticated one - it deals with conditional expectations and martingales, in particular, the Doob decomposition is proven in a nicely and compact manner. The fundamental asset pricing theorems for fully general multi-period models are formulated and proven via the method of change of numeraire in Chapter 11. Chapter 12 is devoted to the celebrated discrete-time option pricing model of Cox, Ingersoll and Ross. This chapter closes with the option pricing formulae for put and call stock options in the Cox, Ingersoll and Ross model.

A book on option pricing without the Black-Merton-Scholes formula cannot be called complete. The authors use the Central Limit Theorem, which is formulated and proven in Chapter 13. Then in Chapter 14 the Black-Merton-Scholes formula is derived as a limit of the option prices from the Cox, Ingersoll and Ross model.

The probably most important applications of discrete-time models are American and Bermudan options. The very last chapters of the book are devoted to that problem. First, authors formulate the problem in the abstract optimal stopping settings in Chapter 15. General optimal stopping theorems are formulated and proven via the method of Snell envelopes. Then the general results are applied to American claims. The book is complemented by two appendices: one on linear spaces and another on the Theorem of de Moivre-Laplace.

This is probably the best written book on discrete-time models of mathematical finance. It is self consistent, all notions used in it are carefully defined. That is a mathematical book - by mathematicians and for mathematicians, which also means that its practical applications are restricted. The bibliography is complete. I strongly recommend that title as an introduction to mathematical finance.

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P.K. Medina, S. Merino *Mathematical Finance and Probability. A Discrete Introduction.* Birkhäuser Verlag, Basel-Berlin-Boston, 340 pages, 2002. ISBN 3-7643-6921-3. Price: EUR 45.00 (softcover).