In our definition of the Griss algebra, we can replace condition (8) by the following equation:

$$(x \vee y)*z = (x*z) \vee (y*z).$$

PROBLEM 8. *Discuss the relationship between a Griss algebra and an algebra satisfying* (7), (9), (10) *and* $(x \vee y)*(x \vee z) \leqslant y \vee z$.

Many results on BCK-algebras and Griss type algebras are found in papers inserted into Mathematics Seminar Notes, Kobe University.

Added in proof. Problem 1 has been solved by Professor A. Wronski. He proved that the class of BCK-algebras is not a variety.

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ON AN ALGEBRAIC AND KRIPKE SEMANTICS FOR INTERMEDIATE LOGICS

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The topic of this paper is to show that Kripke semantics and algebraic semantics for intermediate predicate logics are incomplete.

Then we prove a weaker version of the converse theorem to the following one:

Let $\boldsymbol{K}$ *be a class of intermediate logics such that for every* $L \in \boldsymbol{K}$ *the formula of the form* $\forall x\big(\alpha(x) \cup \beta\big) \Rightarrow \big(\forall x\, \alpha(x) \cup \beta\big)$, *where* x *does not appear in* β *is provable in* L.

If $L \in \boldsymbol{K}$ *and* L *has a characteristic Kripke model, then* L *has a characteristic set of algebraic models.*

First of all, we describe a language $\mathscr{L}$ in order to define intermediate predicate logics. Let $\mathscr{L}$ consist of a countable infinite set V of individual variables $x, y, z, \ldots$; a countable infinite set of n-ary predicate variables $p^{(n)}, q^{(n)}, r^{(n)}, \ldots$, and a countable infinite set of constants. 0-ary predicate variables are identified with propositional variables. The logical symbols of $\mathscr{L}$ are $\cap, \cup, \Rightarrow, \neg, \forall, \exists$. The set of formulas F is defined in the usual way and elements of F we denote by $\alpha, \beta, \gamma, \ldots$

We will identify a logic with the set of formulas provable in it. Thus, by LK we mean the *set of all formulas provable in the classical predicate calculus* and by LI we mean the *set of all formulas provable in the intuitionistic predicate calculus*. Let α be a formula provable in LK. Then by $\mathrm{LI}+\alpha$ we denote an *intermediate predicate logic* obtained by adding an axiom scheme α to LI.

DEFINITION 1. A set of formulas L is said to be an *intermediate predicate logic* if it satisfies the following conditions:

(1) $\mathrm{LI} \subset L \subset \mathrm{LK}$,

(2) L is closed with respect to the same rules of inference as LI.

Let D, E, K, $Q_m(x)$, N_m be defined as follows:

$$D = \big(\forall x(\alpha(x)\cup\beta) \Rightarrow (\forall x\,\alpha(x)\cup\beta)\big),$$ where x does not appear in β,

$$E = (\neg\neg\exists x\,\alpha(x) \Rightarrow \exists x\,\neg\neg\alpha(x)),$$

$$K = (\alpha\cup\neg\alpha),$$

$$Q_1(x) = \alpha_1(x),$$

$$Q_{m+1}(x) = \Big(\big(\bigwedge_{i=1}^{m}\neg\alpha_i(x)\big)\wedge\alpha_{m+1}(x)\Big) \quad (m \geqslant 0),$$

$$N_m = \Big(\bigwedge_{i=1}^{m}\exists x Q_i(x) \Rightarrow \forall x\big(\bigvee_{i=1}^{m}Q_i(x)\big)\Big),$$

where α, β, $\alpha(x)$, $\alpha_i(x)$ are formulas of $\mathscr{L}$.

The logics LI$+D$, LI$+E$, LI$+K\vee N_m$ $(m>0)$ are examples of intermediate logics. In the sequel LI$+D$ and IJ$+E$ we will denote by LD and LE.

DEFINITION 2. A pair (T, D_t) is said to be a *Kripke structure* if

(i) *T is a nonempty ordered set (with the order relation $\leqslant$),*

(ii) *for every $t\in T$, D_t is a nonempty countable set such that if $t\leqslant s$, then $D_t\subseteq D_s$.*

Let $\vDash$ be a subset of $T\times(\bigcup_{t\in T}D_t)^V\times F$ such that (to simplify the notation we shall write $t\vDash_v\alpha$ instead of $\langle t, v, \alpha\rangle\in\vDash$ for $t\in T$, $v\in(\bigcup_{t\in T}D_t)^V$, $\alpha\in F$):

(1) for any propositional variable p, $t\vDash_v p$ iff $v(p)=1$, where 1 is the element of the set of truth values $\{0,1\}$ and if $t\leqslant s$ and $t\vDash_v p$, then $s\vDash_v p$,

(2) for every n-ary $(n>0)$ predicate p
$t\vDash_v pa_1\ldots a_n$ iff $p(a_1,\ldots,a_n)\subset D_t^n$, $v(x_i) = a_i\in D_t$,

(3) $t\vDash_v(\alpha\cup\beta)$ iff $t\vDash_v\alpha$ or $t\vDash_v\beta$,

(4) $t\vDash_v(\alpha\cap\beta)$ iff $t\vDash_v\alpha$ and $t\vDash_v\beta$,

(5) $t\vDash_v(\alpha\Rightarrow\beta)$ iff $\bigwedge_{s\in T}(t\leqslant s$, if $s\vDash_v\alpha$ then $s\vDash_v\beta)$,

(6) $t\vDash_v\neg\alpha$ iff $\bigwedge_{s\in T}(t\leqslant s$, non $s\vDash_v\alpha)$,

(7) $t\vDash_v\exists x\,\alpha(x)$ iff $t\vDash_{v'}\alpha(a)$, for some $a\in D_t$,

(8) $t\vDash_v\forall x\,\alpha(x)$ iff $\bigwedge_{s\in T}\big(t\leqslant s, \bigwedge_{a\in D_s}s\vDash_{v'}\alpha(a)\big)$,

where

$$v'(y) = a \quad\text{if}\quad y = x,$$
$$v'(y) = v(y) \quad\text{if}\quad y\neq x.$$

In the sequel if $\mathfrak{M}$ is a Kripke structure and $\vDash$ is a relation such that conditions (1)–(8) are fulfilled, then we call $\vDash$ a *forcing relation* in $\mathfrak{M}$.

DEFINITION 3. A formula α of $\mathscr{L}$ is said to be *valid in* $\mathfrak{M}$ if for every forcing relation $\vDash$ in $\mathfrak{M}$ and for any $t\in T$ $t\vDash_v\alpha$, where $v\in(\bigcup_{t\in T}D_t)^V$.

Denote by $L(\mathfrak{M})$ the set of all formulas valid in $\mathfrak{M}$.

DEFINITION 4. We say that an intermediate logic L has a *characteristic Kripke model* $\mathfrak{M}$ if $L = L(\mathfrak{M})$.

We mention that LJ, LK, LD have characteristic Kripke models. The following theorem is well known.

THEOREM 1. *If $\mathfrak{M} = (T, D_t)$ is a Kripke structure with constant domains, i.e. $D_t = D_s$, for every $t, s\in T$, then $D\in L(\mathfrak{M})$.*

LEMMA 1 [1]. *Let $\mathfrak{M}$ be a Kripke structure. Then there exists a family of Kripke structures $\{\mathfrak{M}_i\}_{i\in I}$, $\mathfrak{M}_i = (T^i, D_t^i)$, such that*

(i) $L(\mathfrak{M}) = \bigcap_{i\in I}L(\mathfrak{M}_i)$,

(ii) *every $\mathfrak{M}_i$ has the base point, i.e. T^i has the least element.*

LEMMA 2. *If $\mathfrak{M}$ has the base point and a formula of the form E is valid in $\mathfrak{M}$, then $\mathfrak{M}$ has constant domains.*

Proof. Let $\mathfrak{M}$ have the base point (denoted by $\mathbf{0}$) and let a formula of the form E be valid in $\mathfrak{M}$. Thus

$$(*)\qquad \mathbf{0}\vDash_v(\neg\neg\exists x\,\alpha(x)\Rightarrow\exists x\,\neg\neg\alpha(x))$$

for every forcing relation in $\mathfrak{M}$ and every valuation v.

By the definition of $\vDash$, $(*)$ is equivalent to

$$(**)\qquad \bigwedge_{t\in T}\Big(\bigvee_{t\leqslant s}\bigwedge_{s\leqslant u}\bigwedge_{a\in D_u}\text{non } u\vDash_v\alpha(a) \text{ or } \bigvee_{b\in D_t}\bigwedge_{t\leqslant s}\bigvee_{s\leqslant w}w\vDash_v\alpha(b)\Big).$$

Suppose that $\mathfrak{M}$ has no constant domains, i.e. for some $r_0\in T$, $D_r - D_0\neq\emptyset$. Define a forcing relation $\vDash$ in the following way:

For any $r\in T$ and a formula β

$$r\vDash_v\beta \quad\text{if}\quad v(\beta)\in D_r - D_0\neq\emptyset,$$
$$\text{non } r\vDash_v\beta \quad\text{if}\quad v(\beta)\in D_0.$$

Then it is easy to observe that $(**)$ does not hold, a contradiction which proves Lemma 2.

THEOREM 2. LE *has no characteristic Kripke model.*

Proof. Suppose that LE has a characteristic Kripke model $\mathfrak{M}$. Then LE $= L(\mathfrak{M})$. On account of Lemma 1 there exists a family $\{\mathfrak{M}_i\}_{i\in I}$ of Kripke structures $\mathfrak{M}_i$ such that

$$L(\mathfrak{M}) = \bigcap_{i\in I} L(\mathfrak{M}_i)$$

and each $\mathfrak{M}_i$ has the base point. By Lemma 2, each $\mathfrak{M}_i$ has constant domains. So by Theorem 1 we infer that, for every $i\in I$, $D\in L(\mathfrak{M}_i)$, i.e. the $D\in$ LE. But Umezawa proved in [4] that D is not provable in LE. So we have a contradiction which completes the proof of Theorem 2.

DEFINITION 5. Let A be a pseudo-Boolean algebra and let J be a non-empty set. Let R be a realization of the language $\mathscr{L}$ in J and A. For any formula α we define the mapping $\alpha_R\colon J^V\to A$ as follows (cf. [2]): for any valuation $v\in J^V$,

(1) $(px_1\ldots x_n)_R(v) = p_R\big(v(x_1), \ldots, v(x_n)\big)(v)$,

(2) $(\beta\cup\gamma)_R(v) = \beta_R(v)\cup\gamma_R(v)$,

(3) $(\beta\cap\gamma)_R(v) = \beta_R(v)\cap\gamma_R(v)$,

(4) $(\beta\Rightarrow\gamma)_R(v) = \beta_R(v)\Rightarrow\gamma_R(v)$,

(5) $(\neg\beta)_R(v) = \neg\beta_R(v)$,

(6) $\big(\exists x\, a(x)\big)_R(v) = \bigcup_{j\in J} a_R(v_j)$,

(7) $\big(\forall x\, a(x)\big)_R(v) = \bigcap_{j\in J} a_R(v_j)$,

where v_j is the valuation such that

$$v_j(y) = \begin{cases} j & \text{if} \quad y = x, \\ v(y) & \text{if} \quad y\neq x. \end{cases}$$

In the sequel we call the pair $\mathfrak{A} = (A, J)$ an *algebraic structure* and R a *realization in* $\mathfrak{A}$.

DEFINITION 6. Let $\mathfrak{A}$ be an algebraic structure and R a realization in $\mathfrak{A}$. A formula α of $\mathscr{L}$ is said to be *valid in* R if for every valuation v, $\alpha_R(v) = \bigvee$, where $\bigvee$ is the greatest element of A. Denote by $L(\mathfrak{A}_R)$ the set of all formulas valid in R. A formula α is *valid in algebraic structure* $\mathfrak{A}$ if for every R in $\mathfrak{A}$ $\alpha_R(v) = \bigvee$. In this case we shall say that $\mathfrak{A} = (A, J)$ is an *algebraic model for* α or α is *valid in* $\mathfrak{A}$.

Denote by $L(\mathfrak{A})$ the set of all formulas valid in $\mathfrak{A}$.

The following lemma shows a difference between algebraic semantics and Kripke one.

LEMMA 3. *The set of formulas provable in* $\mathrm{LI}+(a\cup\neg a\cup N_1)$ *is equal to the set* $L(\mathfrak{A}_0)\cap L(\mathfrak{A}_1)$, *but it is not equal to* $L(\mathfrak{A})$ *for any algebraic structure* $\mathfrak{A}$, *where* $\mathfrak{A}_0 = (A_0, J_0)$ *is an algebraic structure such that* A_0 *is a Boolean algebra,* $|J_0| < \aleph_0$, $\mathfrak{A}_1 = (A_1, J_1)$ *is the algebraic structure such that* $\mathfrak{A}_1$ *is the Lindenbaum algebra of* LI, $J_1 = \{a\}$.

This lemma suggests the following

DEFINITION 7. A set of algebraic structures $\{\mathfrak{A}_i\}_{i\in I}$ is said to be a *characteristic set* of algebraic models for an intermediate predicate logic L if $L = \bigcap_{i\in I} L(\mathfrak{A}_i)$.

THEOREM 3. $\mathrm{LI}+K\cup N_m$ $(m>1)$ *has no characteristic set of algebraic models.*

Proof. Suppose that $\{(A_i, J_i)\colon i\in I\}$ is a characteristic set of algebraic models for $\mathrm{LI}+K\cup N_m$. Since $K\cup N_m\in L(A_i, J_i)$ for every $i\in I$ we can show that either $|J_i|\leqslant m$ or A_i is Boolean algebra. In either case D is valid in (A_i, J_i) for any $i\in I$. Thus D must be provable in $\mathrm{LI}+K\cup N_m$. Let (T, D_t^m) be the Kripke structure such that

(1) T is the set $\{0, 1\}$ with the order relation defined as follows: $0\leqslant 0$, $0\leqslant 1$, and $1\leqslant 1$.

(2) $D_0^m = \{0\}$ and $D_1^m = \{i\colon 0\leqslant i< m\}$.

Then it is easy to verify that $K\cup N_m$ is valid in (T, D_t^m) but D is not valid in (T, D_t^m). This is a contradiction which proves Theorem 3.

Theorem 3 can be easily extended to any intermediate predicate logic of the form $\mathrm{LI}+\{F_i\colon i\in I\}+C\cup N_m$ $(m>1)$, where each F_i is a formula of propositional logics which is valid in the 3-valued linear pseudo-Boolean algebra and C is any formula provable in LK such that D is provable in $\mathrm{LI}+C$. Thus there exist $2^{\aleph_0}$ intermediate predicate logics having no characteristic sets of algebraic models. By Theorem 2 and Theorem 3 we know that for intermediate predicate logics the power of Kripke semantics and the power of algebraic semantics are incomparable.

THEOREM 4. *Let* $D\in L$. *If* L *has a characteristic Kripke model, then* L *has a characteristic set of algebraic models.*

Proof. Let L have a characteristic Kripke model $\mathfrak{M} = (T, D_t)$ and let $D\in L$. Then $\mathfrak{M}$ has constant domains.

Now we will construct an algebraic structure (A, J) in the following way. Put $J = D_t$. We take for a pseudo-Boolean algebra A the algebra of all open subsets of T ($a\subset T$ is open if $t\in a$, then for every s such that $t\leqslant s$, s also belongs to a). It is well known that the operations $\Rightarrow$ and $\neg$ in A are given by the formulas

$$a\Rightarrow b = \{t \in T \;\bigwedge_{s\in T} t\leqslant s,\ \text{if}\ s\in a\ \text{then}\ a\in b\},$$

$$\neg a = \{t \in T: \bigwedge_{s \in T} t \leqslant s,\ s \notin a\},$$

where a and b are open subsets of T.

Now let R be any realization in the algebraic structure $\mathfrak{A} = (A, J)$ defined above. Then for $v \in J^V$

$$a_R(v) = \{t \in T: t \models_v a\}, \quad \text{where } \models \text{ is any forcing relation in } \mathfrak{M}.$$

It is easy to observe that $L(\mathfrak{A}) = L(\mathfrak{M}) = L$.

THEOREM 5. *Let L be an intermediate predicate logic such that the formula of the form D is provable in L. Let $\mathfrak{A} = (A, J)$ be an algebraic structure and R a realization in $\mathfrak{A}$ such that $L = L(\mathfrak{A}_R)$. Then there exist a Kripke structure $\mathfrak{M} = (T, D_t)$ and a forcing relation $\models$ in $\mathfrak{M}$ such that for any $a \in \mathscr{L}$*

$$a \in L(\mathfrak{A}_R) \textit{ if and only if for every } t \in T,\ t \models_v a,$$

where $v \in (\bigcup_{t \in T} D_t)^V$.

Proof. Suppose that $\mathfrak{A} = (A, J)$ is an algebraic structure and R is a realization in $\mathfrak{A}$ such that $L = L(\mathfrak{A}_R)$. By our assumption $D \in L$. Thus in the pseudo-Boolean algebra A both the infinite distributive laws are satisfied.

To construct the required Kripke structure $\mathfrak{M} = (T, D_t)$ we use the following

LEMMA 4 [3]. *Let A be a D-pseudo-Boolean algebra, i.e. in A both the infinite distributive laws are fulfilled. Let (Q) be the set of infinite joins and meets in A:*

$$(Q) \qquad a_{2n} = \bigcup_{a \in A_{2n}} a, \qquad b_{2n+1} = \bigcap_{b \in B_{2n+1}} b,$$

where $A_{2n}, B_{2n+1} \subset A$ for $n \in \omega$.

Let x, y be the elements of A such that the relation $x \leqslant y$ does not hold. Then there exists a prime filter ∇ such that ∇ preserves joins and meets in (Q) and $x \in \nabla$ and $y \notin \nabla$.

By this lemma the set of all prime filters in A preserving all joins and meets in Q, i.e. the set of all Q-filters in A, is non-empty. We take for T the collection of all Q-filters in A.

Now we observe that by Theorem 1 the required Kripke model must have constant domains.

Let us put for every $\nabla \in T$ $D_\nabla = J$.

So $\mathfrak{M} = (T, D_\nabla)$, where T and D_∇ are defined above, is the required Kripke structure.

We define a forcing relation $\models$ in $\mathfrak{M}$ as follows: for any atomic formula $px_1 \ldots x_n$

$$\nabla \models_v px_1 \ldots x_n \quad \text{if and only if} \quad p_R(a_1, \ldots, a_n) \in \nabla,$$

where $a_i = v(x_i)$, $i = 1, \ldots, n$, $v \in J^V$.

We show that for any formula a

$$\nabla \models_v a \quad \text{if and only if} \quad a_R(v) \in \nabla.$$

We prove this fact by the induction on the length of a. We check only the case where a is of the form $(\beta \Rightarrow \gamma)$. The remaining cases are left to the reader.

Assume $\beta_R(v) = b$ and $\gamma_R(v) = c$. By Definition 2 (5) it is sufficient to show:

$$b \Rightarrow c \in \nabla \quad \text{if and only if} \bigwedge_{\nabla' \in T} \nabla \subset \nabla',\ \ b \notin \nabla' \text{ or } c \in \nabla'.$$

Suppose that $b \Rightarrow c \in \nabla$. Let for any $\nabla' \in T$ such that $\nabla \subset \nabla'$, $b \in \nabla'$. Then $b \Rightarrow c \in \nabla'$ and it is obvious that $c \in \nabla'$.

Suppose that $\nabla \in T$ and $b \Rightarrow c \notin \nabla$. Thus in the quotient algebra A/∇ the inequality $|b| \leqslant |c|$ does not hold. It is obvious that A/∇ is a D-pseudo-Boolean algebra. Let h be the natural homomorphism from A onto A/∇. For simplicity we denote the set $h(Q)$ by Q. By the lemma there exists a Q-filter $\tilde{\nabla}$ in A/∇ such that $|b| \in \tilde{\nabla}$ and $|c| \notin \tilde{\nabla}$.

Let $\nabla' = \{a \in A: |a| \in \tilde{\nabla}\}$. ∇' is not empty as $|b| \in \nabla'$ and it is easy to check that $\nabla' \in T$. Moreover, $c \notin \nabla'$ and $\nabla \subset \nabla'$. Indeed, let $a \in \nabla$; then $|a| = \bigvee_{A/\nabla}$. Thus $|a| \in \tilde{\nabla}$, so $a \in \nabla'$ which proves the existence of $\nabla' \in T$ such that $\nabla \subset \nabla'$, $b \in \nabla'$ and $c \notin \nabla'$. This proves that condition (5) is satisfied.

Now, let $a \in L(\mathfrak{A}_R)$. Then $a_R(v) = \bigvee$, i.e., for every $\nabla \in T$ $a_R(v) \in \nabla$ which proves that for every $\nabla \in T$, $\nabla \models_v a$.

Let $\nabla \models_v a$. Then $a_R(v) \in \nabla$ and $|a_R(v)| = \bigvee_{A/\nabla}$. Let h be the natural homomorphism from A/∇ onto A. Then $a_R(v) = h(|a_R(v)|) = \bigvee$ which completes the proof of Theorem 5.

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SOME OPEN PROBLEMS AND DIRECTIONS FOR FURTHER RESEARCH IN SEMIGROUPS OF CONTINUOUS SELFMAPS

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This paper contains a few new results but, as the title suggests, it is primarily a discussion of various possibilities for further research in the theory of semigroups of continuous selfmaps. Most of the problems discussed here are natural developments from three lectures I gave at the Stefan Banach International Mathematical Center and I would like to take this opportunity to thank everyone there for their kind hospitality.

The problems fall within four different topics: dense subsemigroups of $S(X)$, congruences on $S(X)$, homomorphisms on $S(X)$ and Green's relations for $S(X)$.

1. Dense subsemigroups of $S(X)$

Throughout this paper, $S(X)$ will denote the semigroup, under composition, of all continuous selfmaps of the topological space X. For general information about $S(X)$, one may consult [4] and [8].

It has been known for quite a long time that if a Hausdorff space X is locally compact, then $S(X)$ is a topological semigroup when given the compact-open topology. The converse is not true. That is, there exist Hausdorff spaces X for which $S(X)$ is a topological semigroup under the compact-open topology and yet X is not locally compact. In fact, they are as far from being locally compact as they can be. J. de Groot [6] proved the existence of 2^{c} 1-dimensional connected locally connected subspaces of the Euclidean plane with the property that the only continuous maps from one space into another are the constant maps and for any such space X, $S(X)$ consists entirely of the constant maps together with the identity map. Thus, $S(X)$ is a left zero semigroup with identity and is therefore a topological semigroup for any topology whatsoever on