Remark 2. If $Z(=S)$ = a group, and $V(u, t) \equiv I_E$, then the above statement is just the assertion of Naĭmark dilation theorem — see [4], [5], [6].

Remark 3. Using standard arguments of dilation theory — see [3], [5], one can prove suitable theorems, which explain how some continuity properties of B and $V(\cdot, \cdot)$ imply such properties of $\pi(\cdot)$.

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ON JORDAN MODELS OF C_0-CONTRACTIONS

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The lecture was devoted to the results of paper [4].

In [1] the following theorem was proved:

THEOREM. *Let T be a contraction of the class C_0 on a separable Hilbert space Then there exists a unique (up to constant factors of modulus* 1) *sequence* $\{m_i\}_{i=1}^{\infty}$ *of inner scalar functions such that*:

(1) m_{i+1} *divides* m_i *for each* i,

(2) T *is quasisimilar to* $S(m_1)\oplus S(m_2)\oplus \ldots$ *(the "Jordan model" of* T*).*

In [3] and [5] it was proved that if T has finite defect indices $\delta_T = \delta_{T*} = n$ then, for $i = 1, 2, \ldots, n$, m_i is equal to the $(n-i+1)$-th invariant factor of the characteristic function of T.

In [7] the problem was raised what is the relation of the functions m_i to the characteristic function of T in the general case. An answer to this question was given independently in [4] and [2]. The main result of [4] is the following theorem:

THEOREM. *Let T be an operator of class C_0 acting on a separable Hilbert space, Θ its characteristic function and let Ω be a contractive analytic function such that $\Theta\Omega = \Omega\Theta = \psi\cdot I_n$ where $\psi \in H^\infty$ is inner and n is the defect index of T (such an Ω exists by* [6], VI.3.1). *Let $S(m_1)\oplus S(m_2)\oplus \ldots$ be the Jordan model of T. Then $m_r = \psi/\varepsilon_r(\Omega)$ for every natural number $r \leqslant n$, where $\varepsilon_r(\Omega)$ denotes the r-th invariant factor of Ω (if n is finite then in this notation $m_i = 1$ for $i > n$).*

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THE SZEGÖ AND BEURLING THEOREMS AND OPERATOR FACTORIZATIONS

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In 1949 Beurling gave the now famous factorization of analytic functions on the unit disk as the product of an inner function and an outer function. Years earlier Szegö had solved the somewhat similar problem of describing the positive functions on the unit circle that can be written as the modulus of an analytic function. (The best known contemporary reference for these theorems is Hoffman's book [3]). Within the past few decades many theorems in operator theory have exhibited some resemblance to the above results. Not always has the relationship between the operator theory and the classical function theory been clear. The purpose here is to describe one direction in which such research has moved, to tie together some theorems that have appeared in the past few years, and to answer a few previously unanswered questions concerning these theorems. Most of the ideas contained herein arose in conversations with Ralph Gellar. In particular, the multiplicity theorem (Theorem 4) was his conjecture.

First we briefly review the necessary language of Hardy spaces. For each of the L^p spaces of functions on the unit circle, H^p is the subspace consisting of those functions whose Fourier coefficients of negative index are all zero. In H^2, an *outer function* is a function f having the property that fH^∞ is dense in H^2, and an *inner function* is a function that is of modulus one almost everywhere on the unit circle. The *bilateral shift* is the operator $U: L^2 \to L^2$ defined by $U: f(z) \to zf(z)$. The *unilateral shift* S on H^2 is the operator $S = U|H^2$.

Recall also that there is an easy description of the operators that commute with U or S. (We shall assume throughout that "operator" means "bounded operator." Also, for simplicity in the multiplicity discussion surrounding Theorem 4 we shall assume that the underlying Hilbert space is separable.) The commutant of U is L^∞, i.e. each operator B that commutes with U is of the form $A = A_\varphi$: $f \to \varphi f$ for all $f \in L^2$, where $\varphi \in L^\infty$. In the same sense, H^∞ is the commutant of S.

The original factorization theorems of Beurling and of Szegö are as follows:

THEOREM 1. *If $f \in H^2$, then $f = gh$ where g is inner and h is outer.*