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GENERALIZED IDENTIFICATION EXPERIMENTS FOR FINITE DETERMINISTIC AUTOMATA

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In the theory of state identification experiments different kinds of experiments can be considered: diagnosing experiments for the determination of initial states, homing experiments for the determination of states at the end of the experiments, and, for instance, control experiments for bringing the automaton in a certain state. In this paper there are introduced generalized identification experiments which can solve all these tasks as special cases, and which can also solve additional tasks that come by connections of diagnose, homing and control. Using the method described in [2] an algorithm is given for the solution of the existence and construction problem.

In [1] were introduced relations of indistinguishability for output words such that the experimentalist can identify only by output words which are distinguishable. Here we consider generalized identification experiments with such indistinguishability relations and solve the existence and construction problem for regular relations.

Let $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$ be a finite deterministic synchronous automaton with input set X, output set Y, state set Z, next state function $\delta\colon Z\times X \to Z$, output function $\lambda\colon Z\times X\to Y$ and the set of initial states $Z_0 \subseteq Z$.

(1) Definition. An input word $p \in X^*$ is a *generalized identification experiment* (g.i.e.) for $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$ and a given set $\mathcal{M} \subseteq 2^Z\times 2^Z$ iff there is an identification function ζ from $\lambda(Z_0, p)$ in $\mathcal{M}$ such that for all $z\in Z_0$ we have $z \in \varphi_1(\lambda(z,p))$ and $\delta(z,p)\in\varphi_2(\lambda(z,p))$ (φ_1, φ_2 are the components of φ, $\varphi(q) = [\varphi_1(q), \varphi_2(q)]$).

(2) Corollary. *An input word p is a* g.i.e. *for* $\mathfrak{A}$ *and* $\mathcal{M}$ *iff for all* $q\in\lambda(Z_0, p)$ *exists* $[M, N]\in\mathcal{M}$ *with*

$$M_q \underset{\mathrm{Df}}{=} \{z|\ z\in Z_0 \wedge \lambda(z,p) = q\} \subseteq M,$$
$$\delta(M_q, p) \subseteq N.$$

A g.i.e. performs only diagnose in the usual sense if $\mathcal{M} = \{\{z\}|\ \ z\in Z_0\}\times\{Z\}$, it performs homing if $\mathcal{M} = \{Z_0\}\times\{\{z\}|\ \ z\in Z\}$ and control respectively if $\mathcal{M} = \{Z_0\}\times\{\{z\}\}$ for the wanted control state z. Additionally a g.i.e. can perform tasks as "if the initial state is z_1, then after the experiment the automaton is to be in state z_1', but if the initial state is z_2, then after the experiment the automaton may be in state z_2' or z_2'', and we want to know, which state we have arrived". This could be realized by a g.i.e. for

$$\mathcal{M} = \{[\{z_1\}, \{z_1'\}], [\{z_2\}, \{z_2'\}], [\{z_2\}, \{z_2''\}]\}.$$

So we can describe tasks of connecting diagnose, homing and control. We can also describe such tasks for sets of states if we need not know the state exactly but only a set which it belongs to.

Let

$$\mathcal{M}_1 \underset{\mathrm{Df}}{=} \{M \mid \exists N (N \subseteq Z \wedge [M, N] \in \mathcal{M})\},$$

$$\mathcal{M}_2 \underset{\mathrm{Df}}{=} \{N \mid \exists M (M \subseteq Z \wedge [M, N] \in \mathcal{M})\},$$

be such that $\mathcal{M}_1 \times \{Z\}$ and $\{Z_0\} \times \mathcal{M}_2$ describes the "diagnosing part" and the "homing/control part" of $\mathcal{M}$, respectively. If $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$ (both parts "independent" from each other), we have

$$L_{\mathfrak{A}, \mathcal{M}} = L_{\mathfrak{A}, \mathcal{M}_1 \times \{Z\}} \cap L_{\mathfrak{A}, \{Z_0\} \times \mathcal{M}_2},$$

where $L_{\mathfrak{A}, \mathcal{M}}$ denotes the set of all g.i.e. for $\mathfrak{A}$ and $\mathcal{M}$. But, in general, we have only

$$L_{\mathfrak{A}, \mathcal{M}} \subseteq L_{\mathfrak{A}, \mathcal{M}_1 \times \{Z\}} \cap L_{\mathfrak{A}, \{Z\} \times \mathcal{M}_2}.$$

For proving this we can define the identification functions

$$\zeta'(q) \underset{\mathrm{Df}}{=} [\zeta_1(q), Z] \quad \text{for} \quad \mathcal{M}_1 \times \{Z\}$$

and

$$\zeta''(q) \underset{\mathrm{Df}}{=} [Z_0, \zeta_2(q)] \quad \text{for} \quad \{Z_0\} \times \mathcal{M}_2$$

if $\zeta(q) = [\zeta_1(q), \zeta_2(q)]$ is an identification function for $\mathcal{M}$ for any g.i.e. $p \in L_{\mathfrak{A}, \mathcal{M}}$.

If $\mathcal{M} = \mathcal{M}_1 \times \mathcal{M}_2$, we can also define an identification function $\zeta(q) \underset{\mathrm{Df}}{=} [\zeta_1'(q), \zeta_2''(q)]$ for $\mathcal{M}$ using the identification functions

$$\zeta' \colon \lambda(Z_0, p) \to \mathcal{M}_1 \times \{Z\}$$

and

$$\zeta'' \colon \lambda(Z_0, p) \to \{Z_0\} \times \mathcal{M}_2.$$

Obviously we can add (or omit) arbitrary states $z \in Z \setminus Z_0$ to the sets M of $\mathcal{M}_1$ without changing the set $L_{\mathfrak{A}, \mathcal{M}}$. We also can go from $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$ and a set $\mathcal{M} = \mathcal{M}_1 \times \{Z\}$ ($\mathcal{M}_1 \subseteq 2^Z$, $\mathcal{M}$ describing a "generalized diagnosing task") to $\mathfrak{A}' = [X, Y, Z, \delta, \lambda, Z]$ and $\mathcal{M}' = \mathcal{M}_1' \times \{Z\}$ with $\mathcal{M}_1' \underset{\mathrm{Df}}{=} \{M \cup (Z \setminus Z_0) \mid M \in \mathcal{M}_1\}$ and thereby we have

$$L_{\mathfrak{A}, \mathcal{M}} = L_{\mathfrak{A}', \mathcal{M}'}.$$

Thus we need not have the specification of Z_0 for describing such tasks of generalized diagnosing. Note that a similar result does not hold in the case of control, and also not for homing and generalized identification. To give a very simple example let $Z_0 = \{z\}$, $\mathcal{M} = \{[Z, \{z\}]\}$ for an arbitrary $z \in Z$. Then we have the empty word e in $L_{\mathfrak{A}, \mathcal{M}}$, but not the words $p \in X^*$ with $\delta(z, p) \neq z$. But in the case of $Z_0 = Z$, for each control task given by some $\mathcal{M}'$ from $e \in L_{\mathfrak{A}, \mathcal{M}'}$, it follows that $[Z, Z] \in \mathcal{M}'$ and then $L_{\mathfrak{A}, \mathcal{M}'} = X^*$.

Now for the solution of the existence and construction problem we define for a given automaton $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$ a function F from the set X^* into the finite (!) set

$$W \underset{\mathrm{Df}}{=} \{\varphi \mid \varphi \colon Z_0 \to Z\} \times \{\alpha \mid \alpha \text{ equivalence relation over } Z_0\}$$

by

$$F(p) \underset{\mathrm{Df}}{=} [\varphi_p, \alpha_p],$$

$$\varphi_p(z) \underset{\mathrm{Df}}{=} \delta(z, p),$$

$$z_1 \alpha_p z_2 \underset{\mathrm{Df}}{\Leftrightarrow} \lambda(z_1, p) = \lambda(z_2, p) \qquad (z, z_1, z_2 \in Z, p \in X^*).$$

Defining the function $f\colon W \times X \to W$ by

$$f([\varphi, \alpha], x) \underset{\mathrm{Df}}{=} [\varphi', \alpha'],$$

$$\varphi'(z) \underset{\mathrm{Df}}{=} \delta(\varphi(z), x),$$

$$z_1 \alpha' z_2 \underset{\mathrm{Df}}{\Leftrightarrow} z_1 \alpha z_2 \wedge \lambda(\varphi(z_1), x) = \lambda(\varphi(z_2), x)$$

we can show:

(3) LEMMA. *$F(px) = f(F(p), x)$ for all $p \in X^*$, $x \in X$.*

Proof. For $F(px) = [\varphi_{px}, \alpha_{px}]$ we have by definition $\varphi_{px}(z) = \delta(z, px) = \delta(\delta(z, p), x) = \delta(\varphi_p(x), z)$ and

$$\begin{aligned} z_1 \alpha_{px} z_2 &\Leftrightarrow \lambda(z_1, px) = \lambda(z_2, px) \\ &\Leftrightarrow \lambda(z_1, p) = \lambda(z_2, p) \wedge \lambda(\delta(z_1, p), x) = \lambda(\delta(z_2, p), x) \\ &\Leftrightarrow z_1 \alpha_p z_2 \wedge \lambda(\varphi_p(z_1), x) = \lambda(\varphi_p(z_2), x). \end{aligned}$$

Thus, $F(px) = [\varphi_{px}, \alpha_{px}] = f([\varphi_p, \alpha_p], x) = f(F(p), x)$ is true.

At last we define for the given set $\mathcal{M} \subseteq 2^Z \times 2^Z$ (describing the identification task) the set $W_0 \subseteq W$ by

$$W_0 \underset{\mathrm{Df}}{=} \{[\varphi, \alpha] \mid \forall M' (M' \in Z_0/\alpha \to \exists [M, N] ([M, N] \in \mathcal{M} \wedge \wedge M' \subseteq M \wedge \{\varphi(z) \mid z \in M'\} \subseteq N))\}.$$

From Corollary (2) follows

(4) LEMMA. *An input word $p \in X^*$ is a g.i.e. for $\mathfrak{A}$ and $\mathcal{M}$ iff $F(p) \in W_0$.*

Because of (3) and (4),

$$\mathrm{Acc}(\mathfrak{A}, \mathcal{M}) \underset{\mathrm{Df}}{=} [X, W, f, F(e), W_0]$$

(with input set X, state set W, next state function $f\colon W \times X \to W$ as defined above, initial state $F(e)$ and accepting state set W_0) is a finite acceptor for the set $L_{\mathfrak{A},\mathscr{M}}$ of all g.i.e. for $\mathfrak{A}$ and $\mathscr{M}$. Now we can decide with help of Acc$(\mathfrak{A}, \mathscr{M})$ whether a g.i.e. for $\mathfrak{A}$ and $\mathscr{M}$ exists, and we can construct one, if exists. We also can use the following algorithm:

(5) ALGORITHM.

Step 0: $P_0 := \Delta P_0 := \{e\}$ (e = empty word)
Step i ($i = 1, 2, \ldots$):
$\Delta P_i := \emptyset$.
For all $px \in \Delta P_{i-1} \cdot X$ do successively:
If $F(px) \notin \{F(p')|\ p' \in P_{i-1} \cup \Delta P_i\}$
then $\Delta P_i := \Delta P_i \cup \{px\}$.
If after that $\Delta P_i = \emptyset$ stop,
else $P_i := P_{i-1} \cup \Delta P_i$ and goto step $i+1$.

Obviously, this algorithm stops after no more than $|W|$ steps. When it stops in a step i, we have constructed a set P_{i-1} with

$$\{F(p')|\ p' \in P_{i-1}\} = \{F(p)|\ p \in X^*\}.$$

To have a simple proof of this fact we assume that the algorithm goes by using lexicographical ordering (for the successive tests of the words $px \in \Delta P_{i-1} \cdot X$). If $p \in X^*$ exists with $F(p) \notin \{F(p')|\ p' \in P_{i-1}\}$, let p be the first in the lexicographical order. Then there must be $p_1, p_2 \in X^*$, $x \in X$ with $p = p_1 x p_2$, $p_1 \in P_{|p_1|}$, $p_1 x \notin P_{|p_1|+1}$. Because of $p_1 x \notin P_{|p_1|+1}$ a word r must exist with $F(p_1 x) = F(r)$ and r coming before $p_1 x$ in our ordering. From Lemma (3) it follows that $F(p_1 x p_2) = F(r p_2)$ and therefore we have a contradiction: $p = p_1 x p_2$ is not the first in the lexicographical order with $F(p)$ not in $\{F(p')|\ p' \in P_{i-1}\}$.

Hence, when the algorithm stops in step i we have

$$L_{\mathfrak{A},\mathscr{M}} = \emptyset \Leftrightarrow P_{i-1} \cap L_{\mathfrak{A},\mathscr{M}} = \emptyset,$$

i.e. if a g.i.e. exists, we find one in the set P_{i-1}. We have proved

(6) THEOREM. *The existence of* g.i.e. *is decidable for finite* $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$ *and* $\mathscr{M} \subseteq 2^Z \times 2^Z$. *In case of existence, a* g.i.e. *can be constructed with length smaller than* $|Z|^{|Z_0|} \cdot |Z_0|!$.

Since we have constructed the acceptor Acc$(\mathfrak{A}, \mathscr{M})$, the sets $L_{\mathfrak{A},\mathscr{M}}$ are regular. On the other hand, if Acc $= [X, Z, \delta, z_0, Z_f]$ is an acceptor for a regular language L (i.e. $L = \{p|\ \delta(z_0, p) \in Z_f\}$), then for $\mathfrak{A} = [X, Y, Z, \delta, \lambda, \{z_0\}]$ and $\mathscr{M} = \{[\{z_0\}, Z_f]\}$ we have $L = L_{\mathfrak{A},\mathscr{M}}$. Hence the family of all sets $L_{\mathfrak{A},\mathscr{M}}$ is the set of the regular languages.

If we consider only (generalized) diagnosing experiments ($\mathscr{M}$ of the form $\mathscr{M} = \mathscr{M}_1 \times \{Z\}$, $\mathscr{M}_1 \subseteq 2^Z$), then we have $L_{\mathfrak{A},\mathscr{M}} = L_{\mathfrak{A},\mathscr{M}} \cdot X^*$ (because if $p \in X^*$ is a generalized diagnosing experiment, then for every $r \in X^*$ the word pr is a generalized diagnosing experiment, too). On the other hand, if Acc $= [X, Z, \delta, z_0, Z_f]$ is an acceptor for a regular language with $L = L \cdot X^*$, then we can construct

$$\mathfrak{A} \underset{\mathrm{Df}}{=} [X, \{0, 1\}, Z \times \{0, 1\}, \delta', \lambda, \{[z_0, 0], [z_0, 1]\}]$$

with

$$\delta'([z, i], x) \underset{\mathrm{Df}}{=} [\delta(z, x), i],$$

$$\lambda([z, i], x) \underset{\mathrm{Df}}{=} \begin{cases} 1, & \text{if} \quad i = 1 \wedge \delta(z, x) \in Z_f, \\ 0, & \text{otherwise,} \end{cases}$$

for $z \in Z$, $x \in X$, $i \in \{0, 1\}$. Again we have $L = L_{\mathfrak{A},\mathscr{M}}$ if we define $\mathscr{M} = \{\{[z_0, 0]\}, \{[z_0, 1]\}\} \times \{Z\}$. Thus we have characterized the family of all sets of generalized diagnosing experiments for finite deterministic automata to be the family of all regular languages L with $L = L \cdot X^*$.

Now let $U \subseteq (Y \times Y)^*$ be a distinguishability relation as introduced in [2]. The experimentalist is able to distinguish only between some output words $q = y_1 \ldots y_n$ and $q' = y'_1 \ldots y'_n$ if $\langle q, q' \rangle \subseteq U$ (that means $[y_1, y'_1] \ldots [y_n, y'_n] \in U$). We assume that for all $q, q' \in Y^*$ we have $\langle q, q \rangle \notin U$ and $\langle q, q' \rangle \notin U \rightarrow \langle q', q \rangle \notin U$ (i.e. $\overline{U} \underset{\mathrm{Df}}{=} (Y \times Y)^* \setminus U$, the indistinguishability relation, is a compatibility relation). Since we are interested only in synchronous automata, we need not consider $\langle q, q' \rangle$ for $|q| \neq |q'|$. Then we have

(7) DEFINITION. An input word $p \in X^*$ is a g.i.e. for $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$, $\mathscr{M} \subseteq 2^Z \times 2^Z$ and $U \subseteq (Y \times Y)^*$ iff there is an identification function ζ: $\lambda(Z_0, p) \rightarrow \mathscr{M}$ with

$$\forall q \forall z (q \in \lambda(Z_0, p) \wedge z \in Z_0 \wedge \langle q, \lambda(z, p) \rangle \notin U \rightarrow z \in \zeta_1(q) \wedge \delta(z, p) \in \zeta_2(q))$$

(thereby, as in Definition (1), $\zeta(q) = [\zeta_1(q), \zeta_2(q)]$).

(8) COROLLARY. *An input word p is a* g.i.e. *for* $\mathfrak{A}$, $\mathscr{M}$ *and* U *iff for all* $z \in Z_0$ *there exists* $[M, N] \in \mathscr{M}$ *with*

$$M_z \underset{\mathrm{Df}}{=} \{z'|\ z' \in Z_0 \wedge \langle \lambda(z, p), \lambda(z', p) \rangle \notin U\} \subseteq M,$$

$$\delta(M_z, p) \subseteq N.$$

(*Or iff for all* $q \in \lambda(Z_0, p)$ *there exists* $[M, N] \in \mathscr{M}$ *with*

$$M_q \underset{\mathrm{Df}}{=} \{z'|\ z' \in Z_0 \wedge \langle q, \lambda(z', p) \rangle \notin U\} \subseteq M,$$

$$\delta(M_q, p) \subseteq N.)$$

The experimentalist recognizing the output cannot distinguish all output words. So he takes one word $q \in \lambda(Z_0, p)$ with $\langle q, \lambda(z, p) \rangle \in U$ when $\lambda(z, p)$ was the real output after start in z. Then by $\zeta(q)$ he determines a pair $[M, N] \in \mathscr{M}$ with $z \in M$, $\delta(z, p) \in N$. The specification of $\mathscr{M}$ guarantees this identification to be exact as needed.

We have to remark that the experimentalist is allowed only to take a word $q \in \lambda(Z_0, p)$ (therefore he must know the set $\lambda(Z_0, p)$ for his recognition). Since $\overline{U}$ need not be transitive, it could even happen that for a g.i.e. p there is a word $q \notin$

$\notin \lambda(Z_0, p)$ with $\langle \lambda(z,p), q\rangle \notin U$ for all $z \in Z_0$. Recognizing such q, the experimentalist could make no difference between the states of Z_0.

For the solution of the existence and construction problem in case of regular distinguishability relations we can take the same method as above.

Let $\mathcal{U} = [Y\times Y, T, g, t_0, T_f]$ be an acceptor for U (with input set $Y\times Y$, state set T, next state function g: $T\times(Y\times Y) \to T$, initial state t_0 and set $T_f \subseteq T$ of accepting states, such that $U = \{\langle q, q'\rangle |\ g(t_0, \langle q, q'\rangle) \in T_f\}$). Then we define

$$W \underset{\mathrm{Df}}{=} \{\varphi|\ \varphi\colon Z_0 \to Z\} \times \{\psi|\ \psi\colon Z_0\times Z_0 \to T\},$$

$$F(p) \underset{\mathrm{Df}}{=} [\varphi_p, \psi_p]$$

with

$$\varphi_p(z) \underset{\mathrm{Df}}{=} \delta(z, p),$$

$$\psi_p(z_1, z_2) \underset{\mathrm{Df}}{=} g\big(t_0, \langle \lambda(z_1, p), \lambda(z_2, p)\rangle\big)$$

$(p \in X^*, z, z_1, z_2 \in Z)$,

$$f([\varphi, \psi], x) \underset{\mathrm{Df}}{=} [\varphi', \psi']$$

with

$$\varphi'(z) \underset{\mathrm{Df}}{=} \delta\big(\varphi(z), x\big),$$

$$\psi'(z_1, z_2) \underset{\mathrm{Df}}{=} g\Big(\psi(z_1, z_2), \big[\lambda\big(\varphi(z_1), x\big), \lambda\big(\varphi(z_2), x\big)\big]\Big)$$

$([\varphi, \psi] \in W, x \in X, z, z_1, z_2 \in Z)$,

$$W_0 \underset{\mathrm{Df}}{=} \{[\varphi, \psi]|\ [\varphi, \psi] \in W \wedge \forall z_0(z_0 \in Z_0 \to \exists [M, N]\,([M, N] \in \mathcal{M} \wedge$$
$$\wedge \{z|\ z \in Z_0 \wedge \psi(z, z_0) \notin T_f\} \subseteq M \wedge$$
$$\wedge \{\varphi(z)|\ z \in Z_0 \wedge \psi(z, z_0) \notin T_f\} \subseteq N)\}.$$

Then from $\varphi_{px}(z) = \delta\big(\varphi_p(z), x\big)$ and

$$\begin{aligned}\psi_{px}(z_1, z_2) &= g\big(t_0, \langle\lambda(z_1, px), \lambda(z_2, px)\rangle\big)\\ &= g\Big(g\big(t_0, \langle\lambda(z_1, p), \lambda(z_2, p)\rangle\big), \big[\lambda\big(\delta(z_1, p), x\big), \lambda\big(\delta(z_2, p), x\big)\big]\Big)\\ &= g\Big(\psi_p(z_1, z_2), \big[\lambda\big(\varphi_p(z_1), x\big), \lambda\big(\varphi_p(z_2), x\big)\big]\Big)\end{aligned}$$

we can follow (cf. Lemma (3)):

(9) Lemma. *$F(px) = f\big(F(p), x\big)$ for all $p \in X^*$, $x \in X$.*

By Corollary (8) we have (cf. Lemma (4)):

(10) Lemma. *p is a* g.i.e. *for $\mathfrak{A}, \mathcal{M}, U$ iff $F(p) \in W_0$.*

Thus, $\mathrm{Acc}(\mathfrak{A}, \mathcal{M}, U) \underset{\mathrm{Df}}{=} [X, W, f, F(e), W_0]$ is a finite acceptor for the set $L_{\mathfrak{A}, \mathcal{M}, U}$ of all g.i.e. for $\mathfrak{A}, \mathcal{M}, U$. We can decide "$L_{\mathfrak{A}, \mathcal{M}, U} = \emptyset$?" with help of algorithm (5). The proof is exactly the same as for g.i.e. for $\mathfrak{A}$ and $\mathcal{M}$.

Using the same method we have proved the similar result:

(11) Theorem. *The existence of* g.i.e. *is decidable for finite $\mathfrak{A} = [X, Y, Z, \delta, \lambda, Z_0]$, $\mathcal{M} \subseteq 2^Z \times 2^Z$ and regular distinguishability relations $U \subseteq (Y\times Y)^*$.*

In case of existence a g.i.e. *can be constructed with length smaller than $|Z|^{|Z_0|} \times |T|^{|Z_0|^2}$ where $|T|$ denotes the cardinality of the state set T of an acceptor for U.*

In [3] it was shown that the family of all sets of diagnosing experiments in case of regular distinguishability relations is the set of all regular languages. On the other hand, by our construction of the acceptor $\mathrm{Acc}(\mathfrak{A}, \mathcal{M}, U)$, the set $L_{\mathfrak{A}, \mathcal{M}, U}$ must be regular if U is regular. Thus, the family of all sets $L_{\mathfrak{A}, \mathcal{M}, U}$ for regular distinguishability relations U is the set of the regular languages (even in case of only diagnosing tasks). Also in [3] it was shown that for non-regular distinguishability relations the sets of diagnosing (or homing) experiments need not be regular and the existence problem in general is not decidable, even in the case of only context free relations.

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