Put $y = \Lambda_a(n)\gamma_n$. We introduce a parameter h with

$$\frac{c_1}{H_n} \leqslant h \leqslant c_2 \frac{\Lambda_a(n)}{H_n^2}.$$

Further, using the method proposed in [9], we obtain relations (2.6) or (2.1) of Theorem 5 or Theorem 1.

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Presented to the semester
MATHEMATICAL STATISTICS
September 15–December 18, 1976

MATHEMATICAL STATISTICS
BANACH CENTER PUBLICATIONS, VOLUME 6
PWN—POLISH SCIENTIFIC PUBLISHERS
WARSAW 1980

ROBUSTNESS: A QUANTITATIVE APPROACH

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According to Box and Anderson [1] who introduced the notion, a test is "robust" if it is "sensitive to change, of a magnitude likely to occur in practice, in extraneous factors". Furthermore, a test is said to be "powerful" if it is "sensitive to change in the specific factor tested". In the note a real valued function on the parameter space of a statistical problem is constructed which measures robustness of a test similarly as the power function measures its "sensitivity to change in the factor tested".

More precisely, given a statistical structure $M_0 = (\mathscr{X}, \mathscr{A}, \mathscr{P}_0)$, $\mathscr{P}_0 \subset \mathscr{P}$, $\mathscr{P}$ being the set of all probability measures on $\mathscr{A}$, we will use a larger structure $M_1 \supset M_0$ to express "changes, of a magnitude likely to occur in practice, in extraneous factors". Let $\pi\colon \mathscr{P}_0 \to 2^{\mathscr{P}}$ be a function such that $\pi(P) \ni P$ and define $M_1 = (\mathscr{X}, \mathscr{A}, \mathscr{P}_1)$ with $\mathscr{P}_1 = \bigcup_{P \in \mathscr{P}_0} \pi(P)$. Let t be a fixed statistic and ϱ a real valued function on $\mathscr{P}_1^t$, $\mathscr{P}_1^t = \{P^t(\cdot) = P\big((t^{-1}(\cdot)\big),\ P \in \mathscr{P}_1\}$. A function $r_t\colon \mathscr{P}_0 \to \boldsymbol{R}^1$ defined as

$$r_t(P) = \sup\{\varrho(Q^t)\colon Q \in \pi(P)\} - \inf\{\varrho(Q^t)\colon Q \in \pi(P)\}$$

is called *ϱ-robustness* of the statistic t in the extension M_1 of M_0.

EXAMPLE. Let d be a metric in the space $\mathscr{P}$ and for a given statistic t let d_t be a metric in $\mathscr{P}^t$. For a given statistical structure $M_0 = (\mathscr{X}, \mathscr{A}, \mathscr{P}_0)$ consider M_1 defined as ε-extension of M_0 constructed by the mapping $\pi(P) = \{Q \in \mathscr{P}\colon d(P, Q) < \varepsilon\}$. The distribution-robustness of the statistic t in ε-extension of M_0 is given by

$$r_{t,\varepsilon}(P) = \sup\{d_t(P^t, Q^t)\colon Q \in \pi(P)\},$$

A qualitative Hampel's [2] definition of robustness is: t is robust in a neighbourhood of P if for any $\delta > 0$ there exists $\varepsilon > 0$ such that $r_{t,\varepsilon}(P) < \delta$; t is robust in the structure M_0 if for any positive δ there exists $\varepsilon > 0$ such that $\sup_{\mathscr{P}_0} r_{t,\varepsilon}(P) < \delta$.

The full text containing some further discussion and examples (power-robustness of the two-sided Student test with respect to change of variance; a risk-robustness of sample mean and sample median in estimating expected value of a normal distribution; power-robustness of a test with respect to unequal probabilities in Bernoulli trials; size-robust tests) appeared in [3].

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Presented to the semester
MATHEMATICAL STATISTICS
September 15–December 18, 1976

MATHEMATICAL STATISTICS
BANACH CENTER PUBLICATIONS, VOLUME 6
PWN—POLISH SCIENTIFIC PUBLISHERS
WARSAW 1980

COMPLETENESS FOR A FAMILY OF NORMAL DISTRIBUTIONS

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1. Introduction

In this paper the problem of completeness of minimal sufficient statistics for a family of multivariate normal distributions $\mathscr{P} = \{N(\mu, \Sigma): \mu \in \mathscr{X}, \Sigma \in \mathscr{V}\}$ is considered. Here $\mathscr{X}$ is a subspace of n-dimensional Euclidean space R^n and $\mathscr{V}$ is a set of $n \times n$ positive definite matrices containing non-empty open set relative to the sp $\mathscr{V}$ (the smallest linear space containing $\mathscr{V}$).

For special cases this problem has been considered by Graybill and Hultquist [1] and Seely [3], [4].

2. Minimal sufficient statistics

Without loss of generality we assume that the identity operator I belongs to $\mathscr{V}$. Let $\mathscr{W} = \{\Sigma^{-1}: \Sigma \in \mathscr{V}\}$ and let $W_1, \ldots, W_k$ form a basis for sp $\mathscr{W}$. Then for $W \in \mathscr{W}$ we have

$$W = \sum_{i=1}^{k} w_i(\Sigma) W_i, \tag{1}$$

where $w(\Sigma) = (w_1(\Sigma), \ldots, w_k(\Sigma))'$ is the vector of linearly independent functions from $\mathscr{V}$ to R^1. Let $\mathscr{R}$ be the smallest subspace of R^n such that $\Sigma^{-1}\mathscr{X} \subset \mathscr{R}$ for all $\Sigma \in \mathscr{V}$. Note that $\mathscr{X} \subset \mathscr{R}$, because $I \in \mathscr{V}$. Let $x_1, \ldots, x_p$ and $x_1, \ldots, x_p, x_{p+1}, \ldots, x_r$ be a basis for $\mathscr{X}$ and $\mathscr{R}$, respectively. Moreover, let for $\mu \in \mathscr{X}$ and $\Sigma \in \mathscr{V}$, $f(y|\mu, \Sigma)$ denote the $N(\mu, \Sigma)$ density functions with respect to Lebesgue's measure.

LEMMA 1. *The functions* $T(y) = (x_1'y, \ldots, x_r'y)'$ *and* $W(y) = (y'W_1y, \ldots, y'W_ky)$ *are minimal sufficient statistics for* $\mathscr{P}$.

Proof. Let R denote the $n \times r$ matrix $[x_1 \ldots x_r]$. Since $R(R'R)^{-1}R'$ is the orthogonal projection on $\mathscr{R}$, we have

$$\Sigma^{-1}\mu = R(R'R)^{-1}R'\Sigma^{-1}\mu = R\alpha(\Sigma, \mu),$$