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ON THE COUPLED SYSTEM OF NONLINEAR WAVE EQUATIONS WITH DIFFERENT PROPAGATION SPEEDS

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Dedicated to Professor Rentaro Agemi on the occasion of his 60th birthday

1. Introduction and Theorem. In the present paper we consider the time local well-posedness in minimal regularity of the Cauchy problem for the coupled system of nonlinear wave equations with different paropagation speeds in three space dimensions:

$$\partial_t^2 u - \Delta u = f(u, \partial u, v, \partial v), \quad t \in [-T, T], \quad x \in \mathbf{R}^3, \tag{1.1}$$

$$\partial_t^2 v - c^2 \Delta v = g(u, \partial u, v, \partial v), \quad t \in [-T, T], \quad x \in \mathbf{R}^3, \tag{1.2}$$

$$\begin{aligned} & u(0,x) = u_0(x), \quad \partial_t u(0,x) = u_1(x), \\ & v(0,x) = v_0(x), \quad \partial_t v(0,x) = v_1(x), \end{aligned} \tag{1.3}$$

where $\partial_t = \partial/\partial t$, c is a propagation speed of equation (1.2) with $0 < c < 1$, T is the existence time of local solutions with $T > 0$ and $\partial = (\partial_t, \nabla_x)$. We assume that the nonlinear functions f and g are quadratic with respect to $(u, \partial u, v, \partial v)$. In the present paper, we study the problem about what the least regularity of initial data is for the time local well-posedness of (1.1)-(1.3).

Let $D = \mathcal{F}^{-1}|\xi|\mathcal{F}$, where $\mathcal{F}$ and $\mathcal{F}^{-1}$ denote the Fourier transform and the inverse Fourier transform, respectively. In order to make the setting of the above problem simple, we consider the following three cases.

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(Case 0) Assume that f and g are any of the following functions f_{0j} and g_{0j}, $j = 1, 2$, respectively.

$$f_{01} = uv, \quad f_{02} = v^2,$$
$$g_{01} = uv, \quad g_{02} = u^2.$$

(Case 1) Assume that f and g are any of the following functions f_{1j} and g_{1j}, $j = 1, 2, 3$, respectively.

$$f_{11} = uDv, \quad f_{12} = vDu, \quad f_{13} = vDv,$$
$$g_{11} = uDv, \quad g_{12} = vDu, \quad g_{13} = uDu.$$

(Case 2) Assume that f and g are any of the following functions f_{2j} and g_{2j}, $j = 1, 2$, respectively.

$$f_{21} = (Du)(Dv), \quad f_{22} = (Dv)^2,$$
$$g_{21} = (Du)(Dv), \quad g_{22} = (Du)^2.$$

Here, we give an example of the coupled system of nonlinear wave equations with different propagation speeds. The following system is called the Klein-Gordon-Zakharov equations, which appear in the plasma physics (see [7] and [27]).

$$\partial_t^2 u - \Delta u + u = -nu, \quad x \in \mathbf{R}^3,$$
$$\partial_t^2 n - c^2 \Delta n = \Delta |u|^2, \quad x \in \mathbf{R}^3,$$

where $0 < c < 1$. If we put $v = D^{-1}n$, then the above system is transformed into (1.1)-(1.2) with $f = -u(Dv) - u$ and $g = -D(|u|^2)$, whose local solvability can essentially be reduced to that of (1.1)-(1.2) with $f = f_{11}$ and $g = g_{13}$. The above system was studied in [21].

Remark 1.1. (i) In Cases 1 and 2, we can replace the nonlocal operator D by the usual derivatives ∂_t and ∂_{x_j}. It does not matter in our argument below at all.

(ii) We exclude the case that the nonlinear functions f and g are the terms consisting only of (u, Du) and (v, Dv), respectively. Because such nonlinear terms have the same property as in the case of $c = 1$.

Before we proceed to our problem, we briefly recall the known results for the case of $c = 1$. Because those suggest what happens to our problem. For simplicity, we take the following single equation:

$$\partial_t^2 u - \Delta u = f(u, Du), \quad t \in [-T, T], \quad x \in \mathbf{R}^3, \tag{1.4}$$
$$(u(0), \partial_t u(0)) = (u_0, u_1) \in H^s \oplus H^{s-1}. \tag{1.5}$$

In [23], Ponce and Sideris proved that if $f = u^2$, the Cauchy problem (1.4)-(1.5) is time locally well-posed for $s > 0$, that if $f = uDu$, the Cauchy problem (1.4)-(1.5) is time locally well-posed for $s > 1$, and that if $f = (Du)^2$, the Cauchy problem (1.4)-(1.5) is time locally well-posed for $s > 2$. Their proof in [23] is based on the Strichartz estimate and the standard energy estimate (for the Strichartz estimate, see, e.g., [24], [22], [20] and [9]). On the other hand, in [18] and [19], Lindblad proved that if $f = u^2$, the Cauchy problem (1.4)-(1.5) is ill-posed for $s \leq 0$, that if $f = u(\partial_t - \partial_{x_1})u$, the Cauchy problem

(1.4)-(1.5) is ill-posed for $s \leq 1$, and that if $f = ((\partial_t - \partial_{x_1})u)^2$, the Cauchy problem (1.4)-(1.5) is ill-posed for $s \leq 2$. So it may safely be said that $s_j = j$, $0 \leq j \leq 2$ are critical for $f = u^2$, $f = uDu$ and $f = (Du)^2$, respectively, when we consider the time local well-posedness of (1.4)-(1.5).

REMARK 1.2. (i) As is suggested by the proof of Ponce and Sideris [23], the breakdown of the Strichartz estimate for the limiting case causes the ill-posedness in a low regularity space (see also the introduction in [14]).

(ii) For the solution u of (1.4)-(1.5), we take the following scaling:

$$u_\eta = \eta^\alpha u(\eta t, \eta x), \quad \eta > 0.$$

If $f = u^2$ and $\alpha = 2$, u_η also satisfies equation (1.4) and $\|u_\eta(0)\|_{\dot{H}^{-1/2}} + \|\partial_t u_\eta(0)\|_{\dot{H}^{-3/2}}$ is invariant for any $\eta > 0$. If $f = uDu$ and $\alpha = 1$, u_η also satisfies equation (1.4) and $\|u_\eta(0)\|_{\dot{H}^{1/2}} + \|\partial_t u_\eta(0)\|_{\dot{H}^{-1/2}}$ is invariant for any $\eta > 0$. If $f = (Du)^2$ and $\alpha = 0$, u_η also satisfies equation (1.4) and $\|u_\eta(0)\|_{\dot{H}^{3/2}} + \|\partial_t u_\eta(0)\|_{\dot{H}^{1/2}}$ is invariant for any $\eta > 0$. Here, $\dot{H}^s$ denotes the homogeneous Sobolev space of order s (for the precise definition of this space, see, e.g., [3]). Accordingly, the scaling suggests that $s_c = -1/2, 1/2, 3/2$ are critical for $f = u^2$, $f = uDu$ and $f = (Du)^2$, respectively. However, s_c are not really critical in the case of nonlinear wave equations, as described above.

(iii) In a series of their papers [14]-[16], Klainerman and Machedon show that if the nonlinearity satisfies the null condition, the time local well-posedness of (1.1)-(1.3) holds even for $s_c < s \leq s_j$. Because the null condition compensates the breakdown of the Strichartz estimate for the limiting case. Their results suggest that a special structure of nonlinearity could recover the Strichartz estimate of the limiting case, which would lead to the time local well-posedness in a low regularity space.

So the following question naturally arises: Can the discrepancy of propagation speeds compensate the breakdown of the Strichartz estimate for the critical regularity $s = s_j$? Regarding this question, we have the following theorem.

THEOREM 1. *Assume that*

$$(u_0, u_1), \quad (v_0, v_1) \in H^s \oplus H^{s-1}.$$

(i) (Case 0) If $g \neq g_{02}$, then the Cauchy problem (1.1)-(1.3) is time locally well-posed for $s = 0$.

(ii) (Case 1) If $f \neq f_{12}$, then the Cauchy problem (1.1)-(1.3) is time locally well-posed for $s = 1$.

(iii) (Case 2) If $f \neq f_{21}$, then the Cauchy problem (1.1)-(1.3) is time locally well-posed for $s = 2$.

Theorem 1 (ii) is proved in [26], [21] and [25]. In section 2, we state the bilinear estimates needed for the proof of Theorem 1 and we also show that the bilinear estimates corresponding to the cases excluded in Theorem 1 are false. These results show that the discrepancy of propagation speeds is helpful for the proof of the time local well-posedness in most nonlinearity, but that it is not helpful in certain nonlinearity.

2. Sketch of proof of Theorem 1. For $b, s \in \mathbf{R}$ and $\lambda > 0$, we define the spaces $X_{b,s}^{\lambda,\pm}$ as follows:

$$X_{b,s}^{\lambda,\pm} = \{f \in \mathcal{S}'(\mathbf{R}^4); \|f\|_{X_{b,s}^{\lambda,\pm}} < \infty\},$$

where

$$\|f\|_{X_{b,s}^{\lambda,\pm}} = \left(\int_{\mathbf{R}^4} (1+|\tau \pm \lambda|\xi||)^{2b}(1+|\xi|)^{2s}|\hat{f}(\tau,\xi)|^2 \, d\tau d\xi\right)^{1/2}.$$

We put

$$\langle f, g\rangle = \int_{\mathbf{R}^4} f(t,x) \, \overline{g(t,x)} \, dtdx.$$

The spaces $X_{b,s}^{\lambda,\pm}$ are introduced by Bourgain [4] and [5] to study the nonlinear Schrödinger equation and the KdV equation. The Fourier restriction norm method developed by Bourgain was simplified and improved for the one dimensional case by Kenig, Ponce and Vega [11] and [12]. Recently, this method has been applied to various nonlinear dispersive wave equations (see, e.g., [1], [2], [6], [10], [11] and [13]). The related method was developed by Klainerman and Machedon [14]-[17] for the nonlinear wave equations.

The crucial part of proof of Theorem 1 is essentially redeuced to Proposition 2 below. In fact, once we have Proposition 2, we can prove Theorem 1 by the contraction argument (for the scheme of the Fourier restriction norm method, see, e.g., Bourgain [5, 6], Kenig, Ponce and Vega [11, 12] and Klainerman and Machedon [15]).

PROPOSITION 2. *(i) Assume that* $0 < a < 1/2 < b < 1$ *,and* $\lambda > 1$ *or* $0 < \lambda < 1$*. Let* a *and* b *be close enough to* $1/2$*. Then the following inequalities hold.*

$$|\langle w, vu\rangle| \le C\|w\|_{X_{a,0}^{\lambda,j}}\|v\|_{X_{b,0}^{1,k}}\|u\|_{X_{b,1}^{\lambda,l}}, \tag{2.1}$$

$$|\langle w, vu\rangle| \le C\|w\|_{X_{a,0}^{1,j}}\|v\|_{X_{b,0}^{\lambda,k}}\|u\|_{X_{b,1}^{\lambda,l}}, \tag{2.2}$$

where j*,* k *and* l *denote either of* $+$ *or* $-$ *sign. Furthermore, if* $\lambda < 1$*, we have the following inequalities.*

$$|\langle w, vu\rangle| \le C\|w\|_{X_{a,0}^{\lambda,j}}\|v\|_{X_{b,0}^{\lambda,k}}\|u\|_{X_{b,1}^{1,l}}, \tag{2.3}$$

where j*,* k *and* l *denote either of* $+$ *or* $-$ *sign.*

(ii) Assume that $\lambda > 1$ *and* $s \le 1$*. Let* a *and* b *be arbitrary real numbers. Then, the following inequalities are false.*

$$|\langle w, vu\rangle| \le C\|w\|_{X_{a,1-s}^{\lambda,j}}\|v\|_{X_{b,s-1}^{\lambda,k}}\|u\|_{X_{b,s}^{1,l}}, \tag{2.4}$$

$$|\langle w, vu\rangle| \le C\|w\|_{X_{a,1-s}^{1,j}}\|v\|_{X_{b,s-1}^{1,k}}\|u\|_{X_{b,s}^{1,l}}, \tag{2.5}$$

where $j = k = l = +$ *or* $-$*.*

REMARK 2.1. (i) Proposition 2 (i) gives the estimates needed for the proof of Theorem 1 (ii), that is, for Case 1. For Case 2, if we differentiate equations (1.1) and (1.2), the estimate in L^2 of nonlinear terms of the resulting equations can be redeuced to the estimate of Case 1. This gives the estimate in H^1 of the original nonlinear terms and so this shows the estimate in H^2 of solution for Case 2. Therefore, Proposition 2 (i) also implies the estimates needed for the proof of Theorem (iii). After a slight modification of Proposition 2 (i), we have Theorem (i) by the duality argument.

(ii) The breakdown of (2.4) suggests that the discrepancy between propagation speeds is not always helpful for the proof of the well-posedness. The breakdown of (2.5) corresponds to the counterexamples by Lindblad [18] and [19] for the case of the single equation.

Proof of Proposition 2. The proof of part (i) can be found in [21] and [25] and so we omit it. We consider part (ii). We first prove the failure of estimate (2.4) for the case of $j = k = l = -$. The proof for the $+$ sign case is the same as in the $-$ sign case.

Let N be a natural number to be chosen large enough later. Let C_j, $1 \le j \le 4$ be four sufficiently large positive numbers. We put $\theta_\lambda = \cos^{-1}(1/\lambda)$, $0 < \theta_\lambda < \pi/2$. For $\xi = (\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3) \in \mathbf{R}^3$, let θ be an angle between ξ and the $\tilde{\xi}_3$ axis. Let $\hat{u}$ denote the Fourier transform in both time and spatial variables of u. We define $\hat{v}(\tau,\xi)$, $\hat{u}(\tau,\xi)$ and $\hat{w}(\tau,\xi)$ as follows.

$$\hat{v}(\tau,\xi) = \begin{cases} |\xi|^{-2}, & 2^{N/2} \le |\xi| \le 2^N,\ \theta_\lambda - |\xi|^{-1} \le \theta \le \theta_\lambda + |\xi|^{-1}, \\ & \qquad |\tau - |\xi|| \le 1, \\ 0, & \text{otherwise}, \end{cases}$$

$$\hat{u}(\tau,\xi) = \begin{cases} 1, & 4^N - C_1 2^N \le |\xi| \le 4^N + C_2 2^N,\ 0 \le \theta \le C_3 2^{-N}, \\ & \qquad |\tau - \lambda|\xi|| \le C_4, \\ 0, & \text{otherwise}, \end{cases}$$

$$\hat{w}(\tau,\xi) = \begin{cases} 1, & \xi = (\tilde{\xi}_1, \tilde{\xi}_2, \tilde{\xi}_3),\ 4^N - 2^N \le \tilde{\xi}_3 \le 4^N,\ \sqrt{\tilde{\xi}_1^2 + \tilde{\xi}_2^2} \le 2^N, \\ & \qquad |\tau - \lambda|\xi|| \le 1, \\ 0, & \text{otherwise}. \end{cases}$$

We also define the relation $f(\tau,\xi;N) \sim g(\tau,\xi;N)$ as follows: For some $C_0 > 0$ independent of τ, ξ and N,

$$C_0^{-1}|f(\tau,\xi;N)| \le |g(\tau,\xi;N)| \le C_0|f(\tau,\xi;N)|.$$

We now show that

$$\hat{u}(\tau - \tau_1, \xi - \xi_1) = 1, \quad (\tau,\xi) \in \operatorname{supp} \hat{w}, \quad (\tau_1,\xi_1) \in \operatorname{supp} \hat{v}. \tag{2.6}$$

In fact, we note that

$$\lambda|\xi| - \lambda|\xi - \xi_1| - |\xi_1| = \frac{-(2\lambda|\xi||\xi_1|(1 - \lambda\cos\tilde{\theta}) + (\lambda^2 - 1)|\xi|^2)}{\lambda|\xi| - |\xi_1| + \lambda|\xi - \xi_1|}, \tag{2.7}$$

where $\tilde{\theta}$ is an angle between ξ and ξ_1. We also note that $|\tilde{\theta} - \theta_\lambda| \sim |\xi_1|^{-1}$ for $(\tau,\xi) \in \operatorname{supp} \hat{w}$ and $(\tau_1,\xi_1) \in \operatorname{supp} \hat{v}$. Accordingly, we have by the definitions of $\hat{w}$ and $\hat{v}$

$$|\tau - \tau_1 - \lambda|\xi - \xi_1|| \le |\tau - \lambda|\xi|| + |\tau_1 - |\xi_1|| + |\lambda|\xi| - \lambda|\xi - \xi_1| - |\xi_1|| \le C$$

for $(\tau,\xi) \in \operatorname{supp} \hat{w}$ and $(\tau_1,\xi_1) \in \operatorname{supp} \hat{v}$. Furthermore, we note that $||\xi| - 4^N| \le C2^N$ for $(\tau,\xi) \in \operatorname{supp} \hat{w}$ and that $|\xi_1| \le C2^N$ for $(\tau_1,\xi_1) \in \operatorname{supp} \hat{v}$. Hence, we easily see that

$$4^N - C2^N \le |\xi - \xi_1| \le 4^N + C2^N, \quad 0 \le \theta \le C2^{-N}$$

for $(\tau,\xi) \in \operatorname{supp} \hat{w}$ and $(\tau_1,\xi_1) \in \operatorname{supp} \hat{v}$, where θ is the angle between the vector $\xi - \xi_1$ and the third axis. These show (2.6).

We put $I(\tau,\xi)=\hat{u}*\hat{v}$. Here and hereafter, $*$ denotes the convolution with respect to the time and the spatial variables. For $(\tau,\xi)\in \operatorname{supp}\hat{w}$, we have by (2.6)

$$\begin{aligned} I(\tau,\xi) &= \int_{\mathbf{R}^4} \hat{u}(\tau-\tau_1,\xi-\xi_1)\ \hat{v}(\tau_1,\xi_1)\ d\tau_1 d\xi_1 \\ &\sim \int_{2^{N/2}}^{2^N}\int_{\theta_\lambda-|\xi_1|^{-1}}^{\theta_\lambda+|\xi_1|^{-1}} |\xi_1|^{-2}\sin\theta\ d\theta\ |\xi_1|^2\ d|\xi_1| \\ &\sim \int_{2^{N/2}}^{2^N} |\xi_1|^{-1}\ d|\xi_1| = \frac{N}{2}\log 2 \sim N. \end{aligned}$$

Therefore, we obtain

$$\langle \hat{w}, \hat{u}*\hat{v}\rangle \sim (2^N)^2\times(2^N)\times N \sim N(2^N)^3. \tag{2.8}$$

On the other hand, simple calculations yield

$$\begin{aligned} &\|(1+|\tau-\lambda|\xi||)^b(1+|\xi|)^{1-s}\hat{w}\|_{L^2} \\ &\quad\sim [(4^N)^{2(1-s)}\times(2^N)^2\times(2^N)]^{1/2}\sim(2^N)^{3/2+2(1-s)}, \end{aligned}\tag{2.9}$$

$$\begin{aligned} &\|(1+|\tau-\lambda|\xi||)^a(1+|\xi|)^{s-1}\hat{u}\|_{L^2} \\ &\quad\sim [(4^N)^{2(s-1)}\times(4^N)^2\times(2^N)\times(2^{-N})^2]^{1/2}\sim(2^N)^{3/2+2(s-1)}. \end{aligned}\tag{2.10}$$

In addition, if $s=1$, we have

$$\begin{aligned} &\|(1+|\tau-\lambda|\xi||)^a(1+|\xi|)\hat{v}\|_{L^2} \\ &\sim \left[\int_{2^{N/2}}^{2^N}\int_{\theta_\lambda-|\xi_1|^{-1}}^{\theta_\lambda+|\xi_1|^{-1}} |\xi_1|^{-4}\sin\theta\ d\theta\ (1+|\xi_1|)^2|\xi_1|^2\ d|\xi_1|\right]^{1/2} \\ &\sim \left[\int_{2^{N/2}}^{2^N}|\xi_1|^{-1}\ d|\xi_1|\right]^{1/2} = [\frac{N}{2}\log 2]^{1/2}\sim N^{1/2}. \end{aligned}\tag{2.11}$$

If $s<1$, we have

$$\begin{aligned} &\|(1+|\tau-\lambda|\xi||)^a(1+|\xi|)^s\hat{v}\|_{L^2} \\ &\sim\left[\int_{2^{N/2}}^{2^N}\int_{\theta_\lambda-|\xi_1|^{-1}}^{\theta_\lambda+|\xi_1|^{-1}} |\xi_1|^{-4}\sin\theta\ d\theta\ (1+|\xi_1|)^{2s}|\xi_1|^2\ d|\xi_1|\right]^{1/2} \\ &\sim\left[\int_{2^{N/2}}^{2^N}|\xi_1|^{-1-2(1-s)}\ d|\xi_1|\right]^{1/2}\sim 2^{-N(1-s)/2}. \end{aligned}\tag{2.12}$$

Therefore, if (2.4) is true, we must have by the Plancherel theorem and (2.8)-(2.12)

$$(2^N)^3 N \le C(2^N)^3\times\begin{cases} N^{1/2}, & s=1, \\ 2^{-N(1-s)/2}, & s<1. \end{cases}$$

where C is a positive constant independent of N. But this inequality fails as $N\to\infty$, which is a contradiction to the validity of (2.4).

We can prove the failure of (2.5) similarly and so we briefly describe how to adjust the above proof to the case of (2.5). Let $\hat{u}$ and $\hat{w}$ be as in the above proof of failure of

(2.4) except for $\lambda = 1$. We take $\theta_\lambda = 0$ and we define $\hat{v}$ as follows.

$$\hat{v}(\tau,\xi) = \begin{cases} |\xi|^{-2}, & 2^{N/2} \le |\xi| \le 2^N,\ 0 \le \theta \le |\xi|^{-1/2}, \\ & |\tau - |\xi|| \le 1, \\ 0, & \text{otherwise}. \end{cases}$$

Then (2.7) is replaced by

$$|\xi| - |\xi - \xi_1| - |\xi_1| = \frac{-(2|\xi||\xi_1|(1-\cos\tilde{\theta}))}{|\xi| - |\xi_1| + |\xi - \xi_1|}, \tag{2.13}$$

where $\tilde{\theta}$ is an angle between ξ and ξ_1. Here we note that $|1 - \cos\tilde{\theta}| \sim |\xi_1|^{-1}$ for $(\tau,\xi) \in$ supp $\hat{w}$ and $(\tau_1,\xi_1) \in$ supp $\hat{v}$. Hence, the absolute value of the right hand side of (2.13) is bounded by a constant independent of τ, ξ, τ_1 and ξ_1 for $(\tau,\xi) \in$ supp $\hat{w}$ and $(\tau_1,\xi_1) \in$ supp $\hat{v}$. The rest of the proof of failure of (2.5) is the same as above. ∎

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