

# MATHEMATICS OF GRAVITATION

Part II

GRAVITATIONAL WAVE DETECTION

*Editor of the Volume*

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## PREFACE

This volume, together with its companion volume (Part I), constitute the Proceedings of the workshop “Mathematical Aspects of Theories of Gravitation” which was held in the Banach Center of the Institute of Mathematics of the Polish Academy of Sciences, Warsaw, between February 29 and March 30, 1996.

The works presented here constitute a representative sample of the subjects discussed during the workshop. The organizing committee of the workshop consisted of P. Chruściel (University of Tours, France and Institute of Mathematics, Polish Academy of Sciences), W. Kondracki (Institute of Mathematics, Polish Academy of Sciences), A. Królak (Institute of Mathematics, Polish Academy of Sciences), and G. Schäfer (Max-Planck-Society, Germany). The members of the Scientific Committee were Prof. Marek Demiański (Copernicus Astronomical Center, Polish Academy of Sciences), Prof. Jerzy Kijowski (Center for Theoretical Physics, Polish Academy of Sciences), Dr Alan Rendall (Institut des Hautes Etudes Scientifiques, France, and Max-Planck-Society, Germany), Prof. Bernard F. Schutz (Max-Planck-Society, Germany), Prof. Kip S. Thorne (California Institute of Technology, USA), Prof. Andrzej Trautman (Institute of Theoretical Physics, University of Warsaw), together with the members of the organizing committee.

The first part of the workshop, organized by P. Chruściel, was entitled *Global problems of Lorentzian geometry and of the Einstein equations*. Emphasis was put on a) the methods of differential geometry and of differential topology; and b) the methods of the theory of partial differential equations. More precisely, that part of the workshop was concerned with various global properties of space-times. In recent years a number of global results concerning solutions of the Einstein equations have been proved using methods from the theory of nonlinear hyperbolic partial differential equations, and one of our aims was to promote the interaction between the hyperbolic PDE community and the general relativity community. For this reason we had two series of lectures by J. Shatah and C. Sogge on the theory of hyperbolic PDE's, presenting the latest developments of the theory to the participants of the workshop. We had lecture series by J. K. Beem, H. Friedrich, G. Neugebauer, and A. Rendall on various state-of-the-art results in GR, presenting various open PDE problems that arise in GR to the PDE audience. These lectures were the core of the first part of the workshop. In addition to that we had PDE and GR lectures on various topics related to the subject of the workshop, as well as lectures concerning alternative methods to study global properties of space-times.

The second part of the workshop, organized by A. Królak, was entitled *Mathematical problems of observation theory in general relativity*. The main concerns were a) the mathematical foundations of the theory of post-Newtonian and post-Minkowskian approximations; and b) the mathematical methods of signal analysis, including wavelet methods. More precisely, the main interest was in the development of optimal methods of statistics and stochastic processes for the analysis of data from the detectors of gravitational waves. Such detectors will be built on Earth in a few years' time as a result of LIGO (USA), VIRGO (France/Italy), GEO600 (Germany), and TAMA300 (Japan) projects that are already funded. Moreover there are plans to construct such detectors in space: LISA (European Space Agency). The purpose of this part of the workshop was to bring together specialists working on the analysis of gravitational-wave solutions of the Einstein equations, specialists working on the detection and estimation of the parameters of such signals in the noise of the detectors, together with experts on statistics and stochastic processes. Optimal data analysis may be essential for the success of the projects and will help to reduce their enormous cost. There were lecture series on two alternative post-Newtonian approximation methods by Blanchet and Schäfer. There were introductory lecture series on statistics by Zieliński and on stochastic processes by Stetner, as well as on signal analysis by Usowicz. There was a series of lectures on principles of gravitational wave detection by Rüdiger. J. Gil organized a special session on pulsars, in which area there are similar data analysis problems. Some of the members of the detector projects, responsible for designing data analysis schemes, took part in the workshop and presented the current status of the construction of the detectors.

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We wish to thank all the lecturers for their contributions, and the participants for the enjoyable atmosphere of the workshop.

Warszawa, October 4th, 1996

*Piotr Chruściel, Andrzej Królak*

# CONTENTS

List of participants . . . . .	9–10
<b>I. Sources of gravitational waves</b>	
B. F. SCHUTZ, Gravitational radiation from accreting neutron stars . . . . .	11–17
T. A. APOSTOLATOS, Gravitational waveforms from spinning objects . . . . .	19–22
M. DEMIAŃSKI, Cosmological background of gravitational waves . . . . .	23–30
K. D. KOKKOTAS, Stellar pulsations and gravitational-waves . . . . .	31–41
<b>II. Post-Newtonian approximations</b>	
G. SCHÄFER, Post-Newtonian approximations and equations of motion of general relativity . . . . .	43–53
P. JARANOWSKI, Technicalities in the calculation of the 3rd post-Newtonian dynamics . . . . .	55–63
R. A. CAPON, Progress towards a local expression for radiation reaction . . . . .	65–70
R. RIETH, On the validity of Wilson’s approach to general relativity . . . . .	71–74
M. SASAKI, Post-Newtonian approximation in the test particle limit . . . . .	75–83
H. ASADA, Post-Newtonian hydrodynamic equations using the $(3 + 1)$ formalism in general relativity . . . . .	85–93
<b>III. Gravitational wave detectors</b>	
J. K. BLACKBURN, The laser interferometer gravitational wave observatory project: LIGO . . . . .	95–135
R. FLAMINIO <i>et al.</i> , VIRGO: a wide band gravitational wave detector . . . . .	137–143
M. TINTO, Theory of spacecraft Doppler tracking . . . . .	145–154
—, A xylophone detector in space . . . . .	155–162
J. A. LOBO, Spherical detectors of gravitational waves . . . . .	163–178
<b>IV. Data analysis methods</b>	
J.-M. INNOCENT and B. TORRÉSANI, Wavelet transform and binary coalescence detection . . . . .	179–208
R. ZIELIŃSKI, Theory of parameter estimation . . . . .	209–220
S. D. MOHANTY and S. V. DHURANDHAR, Gravitational waves from coalescing binaries: a hierarchical signal detection strategy . . . . .	221–233
P. JARANOWSKI, Inverse problem for networks of laser interferometers . . . . .	235–237

**V. Related topics**

J. GIL, A. KRAWCZYK and G. MELIKIDZE, Pulsar radiation . . . . .	239–255
A. STARUSZKIEWICZ, On the quantal nature of the Coulomb field . . . . .	257–262
R. W. TUCKER and C. WANG, Non-Riemannian gravitational interactions . . . . .	263–271
S. L. BAŻAŃSKI, Is the geodesic hypothesis in general relativity falsifiable? . . . . .	273–285

**CONTRIBUTIONS TO PART I**

J. K. BEEM, Lorentzian geometry in the large.
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A. D. RENDALL, An introduction to the Einstein–Vlasov system.
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C. D. SOGGE, Fourier integral operators and nonlinear wave equations.
R. BEIG, TT-tensors and conformally flat structures on 3-manifolds.
Y. CHOQUET-BRUHAT and J. W. YORK, Well posed reduced systems for the Einstein equations.
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M. HELLER and W. SASIN, The closed Friedman world model with the initial and final singularities as a non-commutative space.
P. S. JOSHI and A. KRÓLAK, Nature of the central singularity in Szekeres models.
M. KRIELE, A stable class of spacetimes with naked singularities.
G. REIN, Selfgravitating systems in Newtonian theory—the Vlasov–Poisson system.
O. RICHTER and C. KLEIN, Algebro-geometric approach to the Ernst equation. I. Mathematical preliminaries.
L. B. SZABADOS, Quasi-local energy-momentum and the Sen geometry of two-surfaces.
N. M. J. WOODHOUSE, Integrability and Einstein’s equations.
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