

MATHEMATICS OF GRAVITATION

Part I

LORENTZIAN GEOMETRY AND
EINSTEIN EQUATIONS

Editor of the Volume

PIOTR T. CHRUSCIEL

WARSZAWA 1997

PREFACE

This volume, together with its companion volume (Part II), constitute the Proceedings of the workshop “Mathematical Aspects of Theories of Gravitation” which was held in the Banach Center of the Institute of Mathematics of the Polish Academy of Sciences, Warsaw, between February 29 and March 30, 1996.

The works presented here constitute a representative sample of the subjects discussed during the workshop. The organizing committee of the workshop consisted of P. Chruściel (University of Tours, France and Institute of Mathematics, Polish Academy of Sciences), W. Kondracki (Institute of Mathematics, Polish Academy of Sciences), A. Królak (Institute of Mathematics, Polish Academy of Sciences), and G. Schäfer (Max-Planck-Society, Germany). The members of the Scientific Committee were Prof. Marek Demiański (Copernicus Astronomical Center, Polish Academy of Sciences), Prof. Jerzy Kijowski (Center for Theoretical Physics, Polish Academy of Sciences), Dr Alan Rendall (Institut des Hautes Etudes Scientifiques, France, and Max-Planck-Society, Germany), Prof. Bernard F. Schutz (Max-Planck-Society, Germany), Prof. Kip S. Thorne (California Institute of Technology, USA), Prof. Andrzej Trautman (Institute of Theoretical Physics, University of Warsaw), together with the members of the organizing committee.

The first part of the workshop, organized by P. Chruściel, was entitled *Global problems of Lorentzian geometry and of the Einstein equations*. Emphasis was put on a) the methods of differential geometry and of differential topology; and b) the methods of the theory of partial differential equations. More precisely, that part of the workshop was concerned with various global properties of space-times. In recent years a number of global results concerning solutions of the Einstein equations have been proved using methods from the theory of nonlinear hyperbolic partial differential equations, and one of our aims was to promote the interaction between the hyperbolic PDE community and the general relativity community. For this reason we had two series of lectures by J. Shatah and C. Sogge on the theory of hyperbolic PDE's, presenting the latest developments of the theory to the participants of the workshop. We had lecture series by J. K. Beem, H. Friedrich, G. Neugebauer, and A. Rendall on various state-of-the-art results in GR, presenting various open PDE problems that arise in GR to the PDE audience. These lectures were the core of the first part of the workshop. In addition to that we had PDE and GR lectures on various topics related to the subject of the workshop, as well as lectures concerning alternative methods to study global properties of space-times.

The second part of the workshop, organized by A. Królak, was entitled *Mathematical problems of observation theory in general relativity*. The main concerns were a) the mathematical foundations of the theory of post-Newtonian and post-Minkowskian approximations; and b) the mathematical methods of signal analysis, including wavelet methods. More precisely, the main interest was in the development of optimal methods of statistics and stochastic processes for the analysis of data from the detectors of gravitational waves. Such detectors will be built on Earth in a few years' time as a result of LIGO (USA), VIRGO (France/Italy), GEO600 (Germany), and TAMA300 (Japan) projects that are already funded. Moreover there are plans to construct such detectors in space: LISA (European Space Agency). The purpose of this part of the workshop was to bring together specialists working on the analysis of gravitational-wave solutions of the Einstein equations, specialists working on the detection and estimation of the parameters of such signals in the noise of the detectors, together with experts on statistics and stochastic processes. Optimal data analysis may be essential for the success of the projects and will help to reduce their enormous cost. There were lecture series on two alternative post-Newtonian approximation methods by Blanchet and Schäfer. There were introductory lecture series on statistics by Zieliński and on stochastic processes by Stetner, as well as on signal analysis by Usowicz. There was a series of lectures on principles of gravitational wave detection by Rüdiger. J. Gil organized a special session on pulsars, in which area there are similar data analysis problems. Some of the members of the detector projects, responsible for designing data analysis schemes, took part in the workshop and presented the current status of the construction of the detectors.

The funding of the workshop was ensured by the Polish Committee for Scientific Research KBN through a grant to the Banach Center. The semester and the publication of these proceedings were, moreover, supported by the Foundation for the Polish-German Cooperation (Warsaw). The Stefan Batory Foundation (Warsaw) provided some financial support for participants from Eastern Europe and from the former Soviet Union. Some participants were supported by the exchange programs between Poland and France, Hungary, India and Spain. We express our gratitude to all the above institutions.

We wish to thank all the lecturers for their contributions, and the participants for the enjoyable atmosphere of the workshop.

Warszawa, October 4th, 1996

Piotr Chruściel, Andrzej Królak

CONTENTS

List of participants	9–10
I. Lecture series	
J. K. BEEM, Lorentzian geometry in the large	11–20
G. GALLOWAY, Some rigidity results for spatially closed spacetimes	21–34
A. D. RENDALL, An introduction to the Einstein–Vlasov system	35–68
J. SHATAH, Regularity results for semilinear and geometric wave equations	69–90
C. D. SOGGE, Fourier integral operators and nonlinear wave equations	91–108
II. Lectures	
R. BEIG, TT-tensors and conformally flat structures on 3-manifolds	109–118
Y. CHOQUET-BRUHAT and J. W. YORK, Well posed reduced systems for the Einstein equations	119–131
M. DAHL, The positive mass theorem for ALE manifolds	133–142
C. GUNDLACH, Critical phenomena in gravitational collapse	143–152
M. HELLER and W. SASIN, The closed Friedman world model with the initial and final singularities as a non-commutative space	153–161
P. S. JOSHI and A. KRÓLAK, Nature of the central singularity in Szekeres models . .	163–168
M. KRIELE, A stable class of spacetimes with naked singularities	169–178
G. REIN, Selfgravitating systems in Newtonian theory—the Vlasov–Poisson system .	179–194
O. RICHTER and C. KLEIN, Algebro-geometric approach to the Ernst equation. I. Mathematical preliminaries	195–204
L. B. SZABADOS, Quasi-local energy-momentum and the Sen geometry of two-surfaces	205–219
N. M. J. WOODHOUSE, Integrability and Einstein’s equations	221–232
E. WOOLGAR, Fastest curves and toroidal black holes	233–242

CONTENTS OF PART II

I. Sources of gravitational waves

B. F. SCHUTZ, Gravitational radiation from accreting neutron stars	11–17
T. A. APOSTOLATOS, Gravitational waveforms from spinning objects	19–22
M. DEMIAŃSKI, Cosmological background of gravitational waves	23–30
K. D. KOKKOTAS, Stellar pulsations and gravitational-waves	31–41

II. Post-Newtonian approximations

G. SCHÄFER, Post-Newtonian approximations and equations of motion of general relativity	43–53
P. JARANOWSKI, Technicalities in the calculation of the 3rd post-Newtonian dynamics	55–63
R. A. CAPON, Progress towards a local expression for radiation reaction	65–70
R. RIETH, On the validity of Wilson’s approach to general relativity	71–74
M. SASAKI, Post-Newtonian approximation in the test particle limit	75–83
H. ASADA, Post-Newtonian hydrodynamic equations using the $(3 + 1)$ formalism in general relativity	85–93

III. Gravitational wave detectors

J. K. BLACKBURN, The laser interferometer gravitational wave observatory project: LIGO	95–135
R. FLAMINIO <i>et al.</i> , VIRGO: a wide band gravitational wave detector	137–143
M. TINTO, Theory of spacecraft Doppler tracking	145–154
—, A xylophone detector in space	155–162
J. A. LOBO, Spherical detectors of gravitational waves	163–178

IV. Data analysis methods

J.-M. INNOCENT and B. TORRÉSANI, Wavelet transform and binary coalescence detection	179–208
R. ZIELIŃSKI, Theory of parameter estimation	209–220
S. D. MOHANTY and S. V. DHURANDHAR, Gravitational waves from coalescing binaries: a hierarchical signal detection strategy	221–233
P. JARANOWSKI, Inverse problem for networks of laser interferometers	235–237

V. Related topics

J. GIL, A. KRAWCZYK and G. MELIKIDZE, Pulsar radiation	239–255
A. STARUSZKIEWICZ, On the quantal nature of the Coulomb field	257–262
R. W. TUCKER and C. WANG, Non-Riemannian gravitational interactions	263–271
S. L. BAŻAŃSKI, Is the geodesic hypothesis in general relativity falsifiable?	273–285

LIST OF PARTICIPANTS

P. Aichelburg (Austria)
G. Allen (Germany)
G. Alekseev (Russia)
L. Andersson (Sweden)
M. Ansorg (Germany)
T. A. Apostolatos (Germany)
H. Asada (Japan)
A. B. Balakin (Russia)
H. Baum (Germany)
S. Bażański (Poland)
J. K. Beem (USA)
R. Beig (Austria)
P. Bizoń (Poland)
J. K. Blackburn (USA)
L. Blanchet (France)
W. Borgiel (Poland)
J.-P. Bourguignon (France)
G. A. Burnett (USA)
R. A. Capon (Germany)
Y. Choquet-Bruhat (France)
P. Chruściel (France)
B. Cimochoowski (Poland)
M. Dahl (Sweden)
M. Demiański (Poland)
E. E. Donets (Russia)
M. Dunajski (UK)
S. V. Dhurandhar (India)
J. Ehlers (Germany)
R. T. Faizulin (Russia)
F. de Felice (Italy)
A. Fischer (USA)
R. Flaminio (France)
H. Friedrich (Germany)
G. Galloway (USA)
J. Gil (Poland)
O. Grosshans (Germany)
C. Gundlach (Spain)
M. Heller (Poland)
M. Herzlich (France)
J.-M. Innocent (France)
J. Isenberg (USA)
T. Jacobson (The Netherlands)
P. Jaranowski (Poland)
J. Kannar (Germany)
W. Kaszlewicz (Poland)
L. Kidder (USA)
J. Kijowski (Poland)
C. Klein (Germany)
K. D. Kokkotas (Greece)
M. Kolonko (Poland)
S. Krasnikov (Rosja)
K. V. Krasnov (Ukraine)
A. Krawczyk (Poland)
M. Kriele (Germany)
A. Królak (Poland)
T. Ledvinka (Czech R.)
W. Lewandowski (Poland)
J. A. Lobo (Spain)
A. G. Lyne (UK)
R. Meinel (Germany)
E. Malec (Poland)
F. Markopoulou (UK)
E. Melas (UK)
M. Montero (Spain)
Z. G. Murzakhonov (Russia)
G. Neugebauer (Germany)
I. Novikov (Denmark)
Yu. N. Obukhov (Germany)
K. Osetrin (Russia)
V. Perlick (Germany)
I. Racz (Hungary)
G. Rcheulishvili (Russia)

M. Redmerska (Poland)
G. Rein (Germany)
A. Rendall (Germany)
O. Richter (Germany)
R. Rieth (Germany)
W. Rudnicki (Poland)
A. Rüdiger (Germany)
R. Kh. Saibatalov (Russia)
M. Sasaki (Japan)
G. Schäfer (Germany)
B. Schmidt (Germany)
B. F. Schutz (Germany)
M. A. Serrano-Moral (Spain)
A. Shadi Tahvildar-Zadeh (USA)
J. Shatah (USA)
W. Sieber (Germany)
W. Simon (Austria)
A. F. Skochilov (Russia)
L. Söderholm (Sweden)
C. D. Sogge (USA)
V. O. Soloviev (Russia)
A. Staruszkiewicz (Poland)
L. Stettner (Poland)
M. Syndyk (Poland)
L. B. Szabados (Hungary)
A. Szczęśniak (Poland)
J. Tafel (Poland)
M. Tinto (USA)
B. Torrèsani (France)
A. Trautman (Poland)
P. Trzaskoma (Poland)
R. Tucker (UK)
J. Usowicz (Poland)
L. V. Verozub (Ukraine)
R. M. Wald (USA)
G. Weinstein (USA)
J. Wojtkiewicz (Poland)
N. Woodhouse (UK)
E. Woolgar (Canada)
R. Zieliński (Poland)