SYMPLECTIC SINGULARITIES AND GEOMETRY OF GAUGE FIELDS

Editors of the Volume

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PREFACE

This volume contains papers written on the occasion of the Banach Center Symposium on Differential Geometry and Mathematical Physics in Spring 1995. The Symposium brought together researchers from several different areas of contemporary research in topics involving methods of symplectic geometry and topology, singularity theory and mathematical physics. The Symposium was coordinated by S. Janeczko and J. Kijowski. It was organized in the form of five active workshops:

1. Singularities and PDE's
   (organizers: S. Łojasiewicz, B. Ziemian)
2. Symplectic singularities
   (organizers: S. Janeczko, A. F. Künzle)
3. Hamiltonian and Lagrangian mechanics
   (organizers: J. Grabowski, G. Marmo)
4. Quantum and classical gauge theory
   (organizers: J. Kijowski, W. Kondracki, G. Rudolph)
5. Canonical and quantum gravity
   (organizers: P. Hájíček, J. Kijowski, J. Lewandowski)

The papers in the volume are contributions of the participants of the second and fourth workshops, whose papers, after refereeing, were considered suitable for publication in the volume. They range from expository to quite technical articles also in theoretical physics, and as a whole represent a good survey of contemporary work in symplectic singularities and mathematical physics.

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INTRODUCTION

It *is* possible to practise (cultivate) pure mathematics without any interest in its applications. Some people are even convinced that the latter might be dangerous for the purity in question.

It *is* possible to practise (cultivate) theoretical physics without any interest in the nature of mathematical tools used. If, travelling towards the Sanctuary of the Knowledge about the Microcosmos, you encounter a door, which cannot be unlocked with the keys you have in your pocket, don't worry: you may always ask a mechanic to try *his* sophisticated tools. Once the door is open, we continue our journey, forgetting as soon as possible the strange shape of the tools and the funny way the mechanic used them.

But there is also another, very noble, tradition in science. It goes back to Archimedes, Newton or Gauss. According to this tradition, mathematics is considered *the language of physics*. On the other hand, applications are *the soul of mathematics*. Even as we are obliged to specialize, because of an enormous amount of new results which we are not able to grasp, even as we are forced to divide our Universities into different Faculties and Departments, even as we work in buildings which are situated far away from each other, we never forget our common goal: to understand the Universe.

The above tradition is very strong in Poland. Stefan Banach, one of the fathers of functional analysis and the patron of the International Mathematical Center in Warsaw, wrote an excellent textbook in theoretical mechanics. Leopold Infeld, considered one of the founder-fathers by many Polish theoretical physicists, practised the job of a mathematician, when—in early 30-ties—he investigated abstract spinorial structures. He was called "an outstanding mathematician" by Canadian newspapers, when they reported his decision to settle back in Poland.

I was introduced to this tradition by my teacher, Krzysztof Maurin. Having it in mind, we proposed, together with prof. S. Janeczko, to organize in the International Stefan Banach Center in Warsaw a Semester devoted to mathematical physics and—especially—to its geometric structures. The Semester was composed of five workshops devoted to different subjects, especially those close to our personal research interests. This way the Spring '95 in Warsaw became an unceasing Feast of Mathematical Physics.

Being a physicist, I expect a lot from such encounters. With great pain I observe how far we are from the construction of a consistent theory of spacetime and matter on a subnuclear scale. The Quantum Field Theory—a natural candidate for such a theory since the late thirties—contains a lot of fundamental difficulties which terribly obscure the simple, heuristic idea of "interacting quantum fields". This idea, already proposed during the *golden age of quanta*, was never accomplished on the level of a mathematically complete and physically "handy" theory. Till now, quantum fields exist only in non-interacting (and, therefore, non-interesting), linear case. All calculations leading to precise values of physical quantities, which fit so miraculously some experimental data, are based on perturbative recipes. The notion of a quantum field enters here on a heuristic level only, as a kind of a philosophical "leading principle". In this context, any attempt to reconcile quanta with General Relativity and to construct the quantum theory of gravity may seem to be several orders of magnitude more difficult than to construct exactly e.g. Quantum Chromodynamics in a flat Minkowskian spacetime. It is hard to believe that these (still so remote!) goals may be achieved without an extremely deep, mathematical insight into the very notion of quantum field.

The Workshops organized during the Semester gave us a chance to meet in Warsaw many outstanding mathematicians and mathematical physicists. We heard a lot of interesting lectures and discussions. Unfortunately, only the organizers of the second and the fourth workshops were able to convince the lecturers to present their talks in a written form. The articles obtained this way provide an extremely interesting material, which we dare to present here as one of the many fruits of our Semester.

Jerzy Kijowski

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