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J. Ławrynowicz (ed.)

**Generalizations  
of Complex Analysis  
and their Applications  
in Physics**

WARSZAWA 1996

## PREFACE

The Symposium, held May 30 – July 1, 1994, was attended by 141 mathematicians, mathematical physicists and physicists from 29 countries. It embraced 147 lectures and numerous informal discussions. The number of foreign participants was 79, including 11 research students. The number of Polish research students was 20.

The participants discussed a broad range of topics.

A. *Workshop on Generalizations of Complex Analysis* (May 30 – June 12) covered: 1) generalized complex structures, 2) Clifford-type structures, 3) Hurwitz pairs and triples. The workshop was organized by K. Habetha (Aachen), J. Ławrynowicz (Łódź) – Chairman, J. Rembieliński (Łódź), O. Suzuki (Tokyo), and P. M. Tamrazov (Kiev). It took place in the Banach Center.

B. *XI Conference on Analytic Functions* (June 13 – June 19) covered: 1) extremal problems related to one complex variable, 2) potential theory in  $\mathbb{R}^n$ , 3) real and complex analytic geometry, 4) complex mappings and related structures. The conference was organized by Z. Charzyński (Łódź), S. Dimiev (Sofia), P. Dolbeault (Paris), H. Grauert (Göttingen), Z. Jakubowski (Łódź), J. Krzyż (Lublin), J. Ławrynowicz (Łódź) – Chairman, O. Martio (Helsinki), V. V. Napalkov (Ufa), and J. Siciak (Kraków). It took place in the Banach Center and the Military Holiday Center at Rynia, ca. 25 kms to the north of Warsaw, in the picturesque forest area at the Zegrzyńskie Lake. The preceding conferences were organized in:

1. Łódź	1954,	6. Kraków	1974,
2. Lublin	1958,	7. Kozubnik	1979,
3. Kraków	1962,	8. Błazejewko	1982,
4. Łódź	1966,	9. Lublin	1986,
5. Lublin	1970,	10. Szczyrk	1990.

The present conference gave an opportunity for the exchange of new ideas and stimulated cooperation leading to joint papers. It is hoped that this series of conferences will continue to give valuable scientific contributions.

C. *Seminar on Deformations of Mathematical Structures Applied in Physics* (June 20 – July 1) covered: 1) complex and related structures of physical interest and applicable in physics, 2) Clifford- and Hurwitz-type structures applicable in physics, 3) direct physical

applications. The seminar was organized by J. Lawrynowicz (Łódź) – Chairman, J. Rembieliński (Łódź), W. Rodrigues (Campinas), and C. Surry (Saint-Étienne). It took place in the Banach Center.

During the Symposium, its distinguished guest, Professor Friedrich Hirzebruch (Bonn), President of the Scientific Council of the Banach Center, delivered a plenary lecture entitled *The Riemann-Roch theorem and holomorphic symplectic manifolds*.

The present volume of selected papers submitted for publication contains 32 papers on extremal problems related to one complex variable, real and complex analytic geometry, complex mappings and related structures, generalized complex structures, Clifford-type structures, Hurwitz pairs and triples, complex and related structures applicable in physics, Clifford- and Hurwitz-type structures applicable in physics, and direct physical applications. Two other volumes, containing together 20 papers, have recently appeared in *Bull. Soc. Sci. Lettres Łódź* **45 Sér. Rech. Déform.** **19** (1995), 138 pp., and **20** (1995), 176 pp. The latter includes complete lists of organizers, lecturers and participants, amounting to 156 scientists from 30 countries. As in the case of the other volumes, each paper has been sent to two referees.

The Editors would like to thank warmly the authors of the papers submitted for their contributions. Special thanks are due to Mrs. Elżbieta Gałuszka and Miss Agnieszka Wieczorek, M.A., who took care of the technical preparation of this volume, and to the staff of the Publications Department of the Institute of Mathematics of the Polish Academy of Sciences.

Łódź, May 1996

*The Editors*

Banach Center — a group of participants (see the opposite page)

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