TOPICS IN COMPLEX ANALYSIS
BANACH CENTER PUBLICATIONS, VOLUME 31
INSTITUTE OF MATHEMATICS
POLISH ACADEMY OF SCIENCES
WARSZAWA 1995

AN APPLICATION OF A THEOREM OF HUBER IN HOLOMORPHIC FOLIATION THEORY

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Introduction. In the following we are concerned with 1-codimensional holomorphic foliations on a connected paracompact complex manifold X of dimension n.

Let U be an open subset of X and $f : U \to C$ a holomorphic submersion onto a 1-dimensional complex manifold C. $f : U \to C$ is called a *regular local holomorphic foliation of codimension one.* Two regular local holomorphic foliations $f_1 : U_1 \to C_1$, $f_2 : U_2 \to C_2$ are called *compatible*, if for every $x \in U_1 \cap U_2$ there exist an open neighborhood $W \subset U_1 \cap U_2$ of x and a biholomorphic mapping $g : f_1(W) \to f_2(W)$ such that $f_2 = g \circ f_1$ on W. A (*global*) *regular holomorphic foliation* $\mathcal{F}$ of codimension one on X is a system $\{f_j : U_j \to C_j : j \in J\}$ of compatible regular local holomorphic foliations of codimension one such that $\bigcup_{j\in J} U_j = X$.

We identify two regular foliations $\mathcal{F}_1, \mathcal{F}_2$ on X if every local foliation of $\mathcal{F}_1$ is compatible with every local foliation of $\mathcal{F}_2$. In the following we assume that every regular foliation $\mathcal{F}$ on X contains every local foliation which is compatible with those of $\mathcal{F}$. By a theorem of Frobenius there is a one to one correspondence between the system of regular holomorphic foliations $\mathcal{F}$ of codimension 1 on X and the system of subsheaves Ω' of the sheaf Ω^1 of holomorphic Pfaffian forms on X such that Ω^1/Ω' is a locally free $\mathcal{O}$-sheaf of rank $n-1$ and $\omega \wedge d\omega = 0$ for every $\omega \in \Omega'_x$, $x \in X$.

Let $\mathcal{F}$ be a regular holomorphic foliation on X of codimension 1. A subset L of X is called a *local leaf* or a *plaque* of $\mathcal{F}$, if there is a local holomorphic foliation $f : U \to C$ of $\mathcal{F}$ such that L is a connected component of a fiber of f. The relatively open subsets of the local leaves of $\mathcal{F}$ constitute a base of a topology $\mathcal{T}$ on X. $\mathcal{T}$ is called the *$\mathcal{F}$-topology.* $(X, \mathcal{T})$ is a complex manifold of dimension $n-1$. It is not connected. The connected components L of $(X, \mathcal{T})$ are called

1991 *Mathematics Subject Classification*: 32L30, 32H20.
The paper is in final form and no version of it will be published elsewhere.

leaves of $\mathcal{F}$. We denote by $X/\mathcal{F}$ the space of all leaves and by $\pi : X \to X/\mathcal{F}$ the natural projection We equip $X/\mathcal{F}$ with the quotient topology and the natural ringed structure. Then π is an open morphism. In [Ho 1] Holmann has proved the following leaf-space theorem.

THEOREM H. *$X/\mathcal{F}$ is a complex space if and only if it is Hausdorff.*

If $X/\mathcal{F}$ is a complex space then it is a Riemannian surface. Theorem H is also true under more general conditions, especially for foliations of higher codimension.

We are concerned with (*singular*) 1-*codimensional holomorphic foliations* on X. Those are pairs $(\mathcal{F}', A')$, in which $A' \subset X$ is an analytic subset of codimension ≥ 2 and $\mathcal{F}'$ is a regular holomorphic foliation of codimension 1 on $X \backslash A'$. We identify two singular foliations $(\mathcal{F}_1', A_1')$, $(\mathcal{F}_2', A_2')$ if $\mathcal{F}_1' = \mathcal{F}_2'$ on $X \backslash (A_1' \cup A_2')$. If A is the smallest possible exceptional analytic subset of a singular foliation $\mathcal{F} = (\mathcal{F}^*, A)$, we call $\operatorname{sing} \mathcal{F} = A$ the *singular locus* of $\mathcal{F}$. $\mathcal{F}^*$ is the maximal corresponding regular foliation of $\mathcal{F}$. It is a foliation on $X^* = X \backslash \operatorname{sing} \mathcal{F}$. There is a one to one correspondence between the system of holomorphic foliations $\mathcal{F}$ of codimension 1 on X and the system of coherent analytic subsheaves Ω' of Ω^1 such that Ω^1/Ω' is a $\mathcal{O}$-sheaf without torsion of rank $n-1$ and $\omega \wedge d\omega = 0$ for every $\omega \in \Omega_x'$, $x \in X$. We get $\operatorname{sing} \mathcal{F} = \operatorname{sing} \Omega^1/\Omega'$, where $\operatorname{sing} \Omega^1/\Omega'$ is the set of all points $x \in X$ such that Ω_x^1/Ω_x' is not free. For the general theory of singular holomorphic foliations compare [B/R].

Let $\mathcal{F}$ be a holomorphic foliation on X of codimension 1. Let U be an open subset of X and $f : U \to C$ an open holomorphic mapping onto a 1-dimensional complex manifold C. f is called an *integral* of $\mathcal{F}$ on U if f is locally constant on $U^* = U \cap X^*$ relating to the leaf topology of $\mathcal{F}^*$, i.e. if f defines a regular local foliation belonging to $\mathcal{F}^*$ on a dense open subset of U^*. If theorem H is true the projection $\pi : X \to X/\mathcal{F}$ is an integral of $\mathcal{F}$. The notion of an integral is a generalization of the notion of a local foliation. For the general theory of integrals compare [B/R] and [Rf 2]. In [M/M] Mattei and Moussu gave a topological description of integrability in codimension 1.

THEOREM M/M. *Let $a \in X$. The following statements are equivalent*:

(1) *There exist an open neighborhood U of a and an integral of $\mathcal{F}$ on U.*

(2) *There exists an open neighborhood V of a such that*

- (a) *every leaf of $\mathcal{F}|V^*$ is a closed subset of V^* and*
- (b) *a is cluster point of a countable number of leaves of $\mathcal{F}|V^*$ at the most.*

Obviously (2) is necessary for (1). Even more, from (1) we conclude a stronger version of (b), namely

(b*) *a is cluster point of a finite number of leaves of $\mathcal{F}|V^*$.*

Corresponding to the conditions (2) of theorem M/M we define

0.1. DEFINITION. $\mathcal{F}$ is called *geometrically simple* if the following conditions are satisfied:

(a) every leaf of $\mathcal{F}^*$ is a closed subset of X^* and

(b) every point $x \in \operatorname{sing} \mathcal{F}$ is a cluster point of a countable number of leaves of $\mathcal{F}^*$ at the most.

Because of theorem M/M we may replace (b) by a stronger condition like (b*).

In section 2 we prove the following theorem.

0.2. Theorem. *Let $\mathcal{F}$ be geometrically simple and X compact. Then there exists an integral of $\mathcal{F}$ on X.*

The theorem is a corollary of the following technical theorem.

0.3. Theorem. *Let Γ be a set of leaves of $\mathcal{F}^*$. Assume that*

(a) *every $L \in \Gamma$ is a closed subset of X^*,*

(b) *Γ is a locally finite family of subsets of X^*,*

(c) *for every $L \in \Gamma$ the space*

$$(X^*/\mathcal{F}^*)\backslash\{M \in \Gamma : M \neq L\}$$

is Hausdorff,

(d) *for every $x \in \operatorname{sing} \mathcal{F}$ there exists at least one leaf L of $\mathcal{F}^*$, $L \notin \Gamma$, such that $x \notin \overline{L}$.*

Then there exists an integral of $\mathcal{F}$ on X.

From condition (c) in 0.3 we get integrals by theorem H. In section 1 we will combine technics to glue them together. For that purpose we use methods of hyperbolic analysis.

We call $\mathcal{F}$ *locally integrable*, if for every $a \in X$ there exists an integral f of $\mathcal{F}$ on an open neighborhood of a. By the aid of theorem M/M we get:

$\mathcal{F}$ is locally integrable iff $\mathcal{F}$ is locally geometrically simple.

Let $\mathcal{F}$ be locally integrable then we can define *local leaves*, *$\mathcal{F}$-topology* and *global leaves* in a similar way as in the regular case (comp. [B/R], [Rf 2]). Therefore we can also define the *leaf-space* $X/\mathcal{F}$ and the projection $\pi : X \to X/\mathcal{F}$. $X/\mathcal{F}$ is a ringed space in a natural way and π a morphism.

Using the result of [Rf 1] one can show that π is open.

An integral $f : U \to C$ of $\mathcal{F}$ is called *simple* if all fibers of f are connected. We conclude:

Let $\mathcal{F}$ be locally integrable. Then the following statements are equivalent:

(1) *There exists a simple integral $f : X \to C$ of $\mathcal{F}$.*

(2) *$X/\mathcal{F}$ is a complex space.*

In this case we can identify $C = X/\mathcal{F}$, $f = \pi$ and, especially $X/\mathcal{F}$ is a Riemannian surface.

We show in the situation of theorem 0.2 that there exists a simple integral of $\mathcal{F}$ on X. Therefore we get:

0.4. Corollary. *Let $\mathcal{F}$ be geometrically simple and X compact. Then $X/\mathcal{F}$ is a Riemannian surface.*

By the way we also show in the situation of theorem 0.3 that there exists an integral of a special type.

Let $\mathcal{F}$ be geometrically simple. We denote by sh $\mathcal{F}$ the $\mathcal{F}$-saturated hull of sing $\mathcal{F}$, i.e. the union of all leaves of $\mathcal{F}$ cutting sing $\mathcal{F}$. The following corollary of 0.3 is a generalization of theorem H in codimension 1.

0.5. Corollary. *Assume that $\mathcal{F}$ is geometrically simple and sh $\mathcal{F}$ is an analytic subset of X. Then $X/\mathcal{F}$ is a Riemannian surface if and only if it is Hausdorff.*

In the last section we will make some remarks on the proof of theorem M/M and generalizations of theorem M/M, theorem 0.2 and corollary 0.4.

1. Some extension theorems. In this section let X, X_1, X_2 be arbitrary complex manifolds and C, C_1, C_2 Riemannian surfaces. By $\mathcal{O}(X_1, X_2)$ we denote the set of all holomorphic mappings $f : X_1 \to X_2$. Let be $D = \{t \in \mathbb{C} : |t| < 1\}$ and $D^* := D\backslash\{0\}$. By $\tilde{C}$ we denote the universal covering of C. It is well known that $\tilde{C}$ is isomorphic to D, $\mathbb{C}$ or $\mathbb{P}^1$ (comp. for example [Fo]). By removing at most three points we get a Riemannian surface C' from C such that $\tilde{C}' \cong D$.

It is well known that for a Riemannian surface C the modern notion of hyperbolicity coincides with the classical one, i.e. C is hyperbolic iff $\tilde{C} \cong D$ (comp. [Ko]).

In [Hu] Huber has proved the following extension theorem.

Huber Theorem. *Let C be hyperbolic and $f \in \mathcal{O}(D^*, C)$. If there exists a sequence z_ν in D^* such that $\lim z_\nu = 0$ and $\lim f(z_\nu)$ exists then f has an extension $\tilde{f} \in \mathcal{O}(D, C)$.*

Huber proved his theorem using the Kobayshi-Royden length of curves in hyperbolic Riemannian surfaces. In [Kw] Kwack generalized the theorem by replacing C by an arbitrary hyperbolic manifold X.

With the aid of Huber's theorem we get:

1.1. Lemma. *Let $c_j \in C_j$, $j = 1, 2$, and $\varphi \in \mathcal{O}(C_1\backslash\{c_1\}, C_2\backslash\{c_2\})$ injective. If there exists a sequence x_ν in $C_1\backslash\{c_1\}$ such that $\lim x_\nu = c_1$ and $\lim f(x_\nu) = c_2$ then φ has an extension $\tilde{\varphi} \in \mathcal{O}(C_1, C_2)$.*

Proof. We may assume that C_2 is hyperbolic. Otherwise we remove some points of $C_2\backslash\{c_2\}$. Now apply Huber's theorem.

We need also the following lemma:

1.2. Lemma. *Let A be an analytic subset of X of codimension ≥ 2 and $f \in \mathcal{O}(X\backslash A, C)$. Assume that for every $a \in A$ there exists a point $c \in C$ such that a is no cluster point of $f^{-1}(c)$. Then there exists a holomorphic extension $\tilde{f} \in \mathcal{O}(X, C)$ of f.*

P r o o f. We must extend f locally. Therefore we may assume the following:

$$X = D^n, \quad A = D^m, \quad m \le n-2,$$

there exists a point $c \in C \backslash f(X\backslash A)$.

We have $\tilde{C} \cong D$ or $\tilde{C} \cong \mathbb{C}$ or $C \cong \mathbb{P}^1$. If $C \cong \mathbb{P}^1$ we remove c. Therefore we may assume that $\tilde{C} \cong D$ or $\tilde{C} \cong \mathbb{C}$.

Because $X\backslash A$ is simply connected we get a mapping $g \in \mathcal{O}(X\backslash A, \tilde{C})$ such that $f = \pi o g$, π being the projection $\pi : \tilde{C} \to C$. We extend g by the classical Riemannian extension theorem and set $\tilde{f} = \tilde{g} o \pi$, $\tilde{g}$ being the extension of g.

2. The proofs of 0.2, 0.3 and 0.5. In this section let X be a connected paracompact complex manifold of dimension n and $\mathcal{F}$ a holomorphic foliation on X of codimension 1.

2.1. Definition. Let $\mathcal{F}$ be locally integrable. Two leaves L, L' of $\mathcal{F}$ are called *not separable*, $L \leftrightarrow L'$, if $U \cap U' \ne \emptyset$ for every neighborhood U resp. U' of L resp. L' in $X/\mathcal{F}$.

2.2. Remark and definition. Let $f : X \to C$ be an integral of $\mathcal{F}$. Then there exists a unique mapping $\tilde{f} : X/\mathcal{F} \to C$ such that $\tilde{f} o \pi = f$. $\tilde{f}$ is a surjective open morphism. We get: $L \leftrightarrow M \Rightarrow \tilde{f}(L) = \tilde{f}(M)$. f is called *maximally separating* (m.s.) if $\tilde{f}(L) = \tilde{f}(M) \Leftrightarrow L \leftrightarrow L'$.

Let $f : X \to C$ be a m.s. integral. Then we can identify C with the quotient $(\widetilde{X/\mathcal{F}})$ of $(X/\mathcal{F})$ by $\leftrightarrow$ and f with the projection $\tilde{\pi} : X \to (\widetilde{X/\mathcal{F}})$.

A simple integral is a m.s. integral.

P r o o f o f 0.3. In 0.3 we allow that $\Gamma = \emptyset$. Then we only have the condition, that $X^*/\mathcal{F}^*$ is Hausdorff, and condition (d).

First assume that sing $\mathcal{F} = \emptyset$, i.e. that $\mathcal{F}$ is a regular foliation. If $\Gamma = \emptyset$ then 0.3 follows by theorem H. Let $\Gamma \ne \emptyset$.

Let $L \in \Gamma$. Then L is a closed subset of X. By a theorem of Holman (comp. [Ho 1]) we get that L is an analytic subset of X. Because Γ is locally finite we conclude that $A = \bigcup_{L \in \Gamma} L$ is an analytic subset of X. The sets $X_0 = X\backslash A$ and $X_L = X_0 \cup L$, $L \in \Gamma$ are open subsets of X. By theorem H we can conclude that

$$C_0 = (X/\mathcal{F})\backslash\Gamma \quad \text{and} \quad C_L = C_0 \cup \{L\}, L \in \Gamma,$$

are Riemannian surfaces. The natural projections $f_0 = X_0 \to C_0$, $f_L : X_L \to C_L$ are simple and therefore m.s. integrals. Now we consider the system $\mathcal{I}$ of all m.s. integrals $f : U \to C$, in which U is an open $\mathcal{F}$-saturated subset of X containing X_0. $f_0 = X_0 \to C_0$ and $f_L : X_L \to C_L$ are elements of $\mathcal{I}$. If we norm the m.s. integrals as described following 2.2 then we can identify every element $f : U \to C$ of $\mathcal{I}$ with U. If $f : U \to C$, $f' : U' \to C'$ belong to $\mathcal{I}$, $U \subset U'$, then we get $C \subset C'$ in a natural way. Therefore the inclusion of the domains of definiton of the integrals gives an ordering $\le$ on $\mathcal{I}$. The condition of Zorn's lemma is satisfied. We consider a maximal element $f : U \to C$ of $\mathcal{I}$.

By an indirect argument we show that $U = X$. Assume that there exists an element $L \in \Gamma$, $L \not\subset U$. We have $C_0 = C \cap C_L$ in a natural way. Let $S = f(U \cap A)$. There exist two alternatives.

1st case: For every $\sigma \in S$ there exist neighborhoods W of σ in C and V of L in C_L such that $W \cap C_0 \cap V = \emptyset$. Then we consider the disjoint union $C \cup C_L$ and the Riemannian surface

$$\tilde{C} := C \cup C_L / \mathrm{id}_{C_0}.$$

Let $\tilde{f} : U \cup L \to \tilde{C}$ be induced by f and f_L. It belongs to $\mathcal{I}$; a contradiction.

2nd case: There exist a point $\sigma \in S$ and a sequence x_ν in C_0 such that $x_\nu \to \sigma$ in C and $x_\nu \to L$ in C_L. Applying 1.1 we get an element $\tilde{f} : U \cup L \to C$ of $\mathcal{I}$; a contradiction.

Theorem 0.3 is proven in the regular case. We got a m.s. integral.

Now let sing $\mathcal{F} \neq \emptyset$. There exists a m.s. integral $f^* : X^* \to C$ of $\mathcal{F}^*$ on X^*. Because of condition (d) we can apply 1.2 and get an integral $f : X \to C$ of $\mathcal{F}$. It is a m.s. integral.

Proof of 0.5. We set $\Lambda = X^*/\mathcal{F}^*$, $S = \operatorname{sing} \mathcal{F}$. Let $L \in \Lambda$. Then L is a closed subset of X^* and therefore an analytic subset of X^*. By Thullen's extension we conclude that the closure $\overline{L}$ of L is an analytic subset of X because of dim $S \leq n-2$. We call L singular if $\overline{L} \cap S \neq \emptyset$, otherwise we call it regular. The set of all singular leaves we denote by Γ.

If $L \in \Gamma$ then there exists an irreducible component S' of S such that $S' \subset \overline{L}$. If S' is an irreducible component of S then the number of leaves $L \in \Gamma$ such that $S' \subset \overline{L}$ is greater than zero, but finite.

We need these considerations for proving 0.2 and 0.5. Now we start with the proof of 0.5. Let $X/\mathcal{F}$ be Hausdorff. We show that Γ satisfies the conditions of 0.3. (a), (c) and (d) are trivial. Because the sets $\overline{L}$, $L \in \Gamma$, constitute the irreducible components of sh $\mathcal{F}$, also (b) is valid.

Let $f : X \to C$ be a m.s. integral of $\mathcal{F}$. Then we can identify $C = \widetilde{(X/\mathcal{F})} = X/\mathcal{F}$.

Proof of 0.2. We use the same procedure as in the proof of 0.5 and show that Γ satisfies the conditions of 0.3. (a) and (d) are trivial.

Because S is compact it only has a finite number of irreducible components. Therefore Γ is finite and (b) is true.

We prove (c). Let be $X_0 := \bigcup_{L \in \Lambda \setminus \Gamma} L$. X_0 is an open $\mathcal{F}^*$-saturated connected subset of X^* and $\mathcal{F}$ induces a 1-codimensional regular foliation on X_0 with compact leaves. Therefore every $L \in \Lambda \setminus \Gamma$ is stable, i.e. every leaf $L \in \Lambda \setminus \Gamma$ has a fundamental system of open $\mathcal{F}$-saturated neighborhoods (comp. [Ka], [Ho 2]). Now consider $L, M \in \Lambda$, $L \notin \Gamma$. If $M \notin \Gamma$ then there exist disjoint open neighborhoods of L resp. M in Λ. Let be $M \in \Gamma$ and assume that there are no disjoint open neighborhoods of L resp. M in Λ.

Consider a distance d on X defining the topology of X and consider the space $\mathcal{A}(X) := \{Y \subset X : Y \text{ non-empty and closed}\}$. We equip $\mathcal{A}(X)$ with the Hausdorff distance δ. Then $(\mathcal{A}(X), \delta)$ is a compact metric space.

By our assumption we get a sequence $L_\nu \in \Lambda\backslash\Gamma$ converging in $\mathcal{A}(X)$ such that $L \cup M \subset \lim L_\nu$. Because L is stable we conclude that $\lim L_\nu = L$, $L = M$; a contradiction.

The conditions of 0.3 are satisfied.

Let $f : X \to C$ be a m.s. integral of $\mathcal{F}$. We show that f is simple. f is constant on the leaves of $\mathcal{F}$. We show that f separates different leaves of $\mathcal{F}$. We argue indirectly. Let $L, M \in \Gamma$. By $\tilde{L}, \tilde{M}$ we denote the leaves of $\mathcal{F}$ defined by L resp. M. Assume that $\tilde{L} \neq \tilde{M}$, but $f(\tilde{L}) = f(\tilde{M})$. Because f is open there exists a converging sequence $L_\nu \in \Lambda\backslash\Gamma$ such that $L \cup M \subset \lim L_\nu$. Because $\lim L_\nu$ is connected and $\tilde{L} \neq \tilde{M}$ we can find a point $a \in X_0 \cap \lim L_\nu$. Let N be the leaf passing through a. Then we get $\lim L_\nu = N$, $L = M = N$; a contradiction.

3. Remarks. For the following we refer to [Rf 3]. A more official publication will be made by G. Bohnhorst.

Again we consider a connected paracompact complex manifold of dimension n and a holomorphic foliation $\mathcal{F}$ on X of codimension 1.

Using the convergence techniques of the proof for 0.2 and using a local stability theorem of Bohnhorst one can give a new geometrical proof for theorem M/M.

In a similar way and using an idea of Milnor ([Mi]) and techniques of semianalytic geometry ([Ło]) one can prove a semiglobal generalization of theorem M/M (theorem 0.2 in [Rf 3]):

3.1. Theorem. *Assume that $\mathcal{F}$ is geometrically simple and let K be a compact subset of a leaf of $\mathcal{F}$. Then there exist an open neighborhood U of K in X and an integral f of $\mathcal{F}$ on U.*

In 3.1 we can choose U connected and f simple. Then $U/(\mathcal{F}|U)$ is a Riemannian surface.

Modifying the proof of 0.2 a little bit one can show (corollary 0.5 in [Rf 3]):

3.2. Theorem. *Let $\mathcal{F}$ be geometrically simple and* $\operatorname{sh}\mathcal{F}$ *an analytic subset of X. Assume that every regular leaf of $\mathcal{F}^*$ is compact. Then $X/\mathcal{F}$ is a Riemannian surface.*

Applying theorem 3.1 and 3.2 we get the following result (corollary 0.6 in [Rf 3]):

3.3. Theorem. *Let $\mathcal{F}$ be geometrically simple. Assume that all leaves of $\mathcal{F}$ are compact. Then $X/\mathcal{F}$ is a Riemannian surface.*

This is a generalization of Satz 3 in [Ka] resp. proposition 6.2 in [Ho 2] and of course of our corollary 0.4.

References

[B/R] G. Bohnhorst und H.-J. Reiffen, *Holomorphe Blätterungen mit Singularitäten*, Math. Gottingensis, Schriftenreihe des Sonderforschungsbereichs Geometrie und Analysis, Heft 5, 1985.

[Fo] O. Forster, *Riemannsche Flächen*, Heidelberger Taschenbücher 184, Springer, Berlin, 1977.

[Ho 1] H. Holmann, *Holomorphe Blätterungen komplexer Räume*, Comment. Math. Helv. 47 (1972), 185–204.

[Ho 2] —, *On the stability of holomorphic foliations with all leaves compact*, in: Variétés analytiques compactes, Colloque Nice, 1977, Lecture Notes in Math. 683, Springer, 1978.

[Hu] H. Huber, *Über analytische Abbildungen Riemannscher Flächen in sich*, Comm. Math. Helv. 27 (1953), 1–73.

[Ka] B. Kaup, *Ein geometrisches Endlichkeitskriterium für Untergruppen von* $\mathrm{Aut}(\mathbb{C},0)$ *und holomorphe* 1*-codimensionale Blätterungen*, Comment. Math. Helv. 53 (1978), 295–299.

[Ko] Sh. Kobayashi, *Hyperbolic Manifolds and Holomorphic Mappings*, Pure and Appl. Math. 2, Marcel Dekker, New York, 1970.

[Kw] M. H. Kwack, *Generalization of the big Picard Theorem*, Ann. of Math. 90 (1969), 9–22.

[Ło] S. Łojasiewicz, *Ensembles semianalytiques*, Institut des Hautes Etudes Scientifiques, Bures-sur-Yvette, France, 1965.

[M/M] J. F. Mattei et R. Moussu, *Holonomie et intégrales premières*, Ann. Sci. École Norm. Sup. (4) 13 (1980), 469–523.

[Mi] J. Milnor, *Singular Points of Complex Hypersurfaces*, Ann. of Math. Stud. 61, Princeton University Press and the University of Tokyo Press, 1968.

[Rf 1] H.-J. Reiffen, *Einfache holomorphe Funktionen*, Math. Ann. 259 (1982), 99–106.

[Rf 2] —, *Leaf spaces and integrability*, in: Holomorphic Dynamics, Proceedings of the Second International Colloquium on Dynamical Systems, Mexico, 1986, Lecture Notes in Math. 1345, Springer, 1988.

[Rf 3] —, *Integrals for holomorphic foliations of codimension one*, Osnabrücker Schriften zur Mathematik, Reihe M, Heft 11, 1992.