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# Convergence of holomorphic chains

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**Abstract.** We endow the module of analytic $p$-chains with the structure of a second-countable metrizable topological space.

**1. Introduction.** A *holomorphic $p$-chain* in an open subset $\Omega$ of $\mathbb{C}^n$ is a formal locally finite sum $Z = \sum_{j \in J} k_j Z_j$ where $Z_j$ are pairwise distinct irreducible analytic subsets of $\Omega$ of pure dimension $p$ and $k_j \in \mathbb{Z} \setminus \{0\}$ for $j \in J$. The set $\bigcup_{j \in J} Z_j$ is called the *support* of the chain Z and denoted by $|Z|$. Each $Z_j$ is called a *component* of $Z$ and the number $k_j$ is the *multiplicity* of $Z_j$.

A holomorphic $p$-chain Z is *positive* if the multiplicities of all its components are positive. $\mathcal{G}^p_+(\Omega)$ denotes the set of positive $p$-chains in $\Omega$. The set of holomorphic $p$-chains in $\Omega$ is endowed with the structure of a free $\mathbb{Z}$-module. We denote it by $\mathcal{G}^p(\Omega)$.

Given a 0-chain and an open relatively compact subset $U$ of $\Omega$ the *total multiplicity* of $Z$ in $U$ is defined as the sum of multiplicities of all its components contained in $U$. We denote the total multiplicity by $\deg_U Z$. When $J$ is finite we extend this definition putting $\deg Z = \sum_{j \in J} k_j$.

One can define convergence of chains as the classical weak convergence of the associated currents (for details see e.g. [Ch, §14.1-2]). An attempt to explain the geometrical meaning of this convergence is made in [Ch].

In [Ch, § 12.2] the author proves that proper intersection is sequentially continuous and also states that this operation is continuous [Ch, §12.4]. However, he neither defines a topology nor proves the equivalence of sequential continuity and continuity.

The main aim of this note is to define a topology on $\mathcal{G}^p(\Omega)$ and to study some properties of this topological space. We shall prove that the result of this construction is second-countable, metrizable, and convergence in it coincides with the one defined in [Ch, §12.2].

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The topology constructed here is useful in studying the intersections of analytic sets (see [Tw], [R]).

**2. Topology of $p$-chains.** Let $0 \le p < n$ be integers, $\Omega$ be an open subset of $\mathbb{C}^n$. We shall use the following notation:

- $E = \{z \in \mathbb{C} : |z| < 1\}$,
- $\Lambda(n,p) = \{\lambda : \{1,\ldots,p\} \to \{1,\ldots,n\} : \lambda(1) < \ldots < \lambda(p)\}$,
- $e_1,\ldots,e_n$—the canonical basis of $\mathbb{C}^n$,
- $\pi_\lambda : (z_1,\ldots,z_n) \to (z_{\lambda(1)},\ldots,z_{\lambda(p)})$, $\pi = \pi_{(1,\ldots,p)} \,|\, E^n$,
- $\mathcal{A}(\Omega) = \{f : \mathbb{C}^n \to \mathbb{C}^n : f$ an affine isomorphism, $f(\overline{E^n}) \subset \Omega\}$,
- $\mu(h)$—order of a finite branched holomorphic covering $h$,
- for $Z = \sum k_j Z_j$, $z \in Z_s \setminus \bigcup_{j \ne s} Z_j$, $m(z,Z) = k_s$.

Suppose that $\Omega_1$, $\Omega_2$ are open subsets of $\mathbb{C}^n$ and $\Omega \subset \Omega_2$. Given a biholomorphic mapping $f : \Omega_1 \to \Omega_2$ and $Z = \sum_{j\in J} k_j Z_j$ belonging to $\mathcal{G}^p(\Omega)$, a new $p$-cycle in $f^{-1}(\Omega)$ can be defined by $f^*(Z) = \sum_{j\in J} k_j f^{-1}(Z_j)$.

Definition 2.1. Let $V$ be an open subset of $\mathbb{C}^n$ containing $\overline{E^n}$, and $Z \in \mathcal{G}^p(V)$, $Z = \sum_{j\in J} k_j Z_j$, such that $|Z| \cap (\overline{E^p} \times \partial E^{n-p}) = \emptyset$. Define

$$\mu(Z) = \sum_{j\in J} k_j \mu(\pi \,|\, Z_j \cap E^n).$$

Definition 2.2. Let $f_j \in \mathcal{A}(\Omega)$, $c_j \in \mathbb{Z}$ for $j = 1,\ldots,m$ and let $K$ be a compact subset of $\Omega$. Define $U(\{(f_1,c_1),\ldots,(f_m,c_m)\},K)$ to be the set of all $p$-chains Z in $\Omega$ such that $|Z| \cap K = \emptyset$ and

$$|Z| \cap f_j(\overline{E^p} \times \partial E^{n-p}) = \emptyset, \quad \mu(f_j^*(Z)) = c_j \quad \text{for} \quad j = 1,\ldots,m.$$

It is easy to verify the following

Proposition 2.3. *If $\Omega$ is an open subset of $\mathbb{C}^n$, then in $\mathcal{G}^p(\Omega)$ the family $\mathcal{U}(\Omega) = \{U(A,K) : A$ is a finite subset of $\mathcal{A}(\Omega) \times \mathbb{Z}$, $K$ is compact in $\Omega\}$, is a base of a topology.*

Definition 2.4. The *topology of $p$-chains* in $\Omega$ is defined to be the topology generated by $\mathcal{U}(\Omega)$.

The next proposition is an immediate consequence of the last definition.

Proposition 2.5. *Let $Z, Z^\nu, \widetilde{Z}^\nu, \widetilde{Z} \in \mathcal{G}^p(\Omega)$.*

1. *If $Z^\nu \to Z$, $\widetilde{Z}^\nu \to \widetilde{Z}$, and $|Z + \widetilde{Z}| = |Z| \cup |\widetilde{Z}|$, then $Z^\nu + \widetilde{Z}^\nu \to \widetilde{Z} + Z$.*
2. *If $Z^\nu \to Z$, $a \in \mathbb{Z}$, then $a \cdot Z^\nu \to a \cdot Z$.*
3. *$\sum_{\nu=0}^{\infty} Z^\nu$ is convergent iff $Z^\nu \to 0$.*
4. *If $f$ is an affine isomorphism, then $\mathcal{G}^p(\Omega) \ni Z \mapsto f^*(Z) \in \mathcal{G}^p(f^{-1}(\Omega))$ is a homeomorphism.*

Example 2.6. $\Omega = \mathbb{C}^2$, $Z^\nu = \{1/\nu\} \times \mathbb{C}$, $Z = \{0\} \times \mathbb{C}$, $\widetilde{Z}^\nu = (-1) \cdot (\{-1/\nu\} \times \mathbb{C})$, $\widetilde{Z} = (-1) \cdot (\{0\} \times \mathbb{C})$. Then $Z^\nu \to Z$ and $\widetilde{Z}^\nu \to \widetilde{Z}$ but $Z^\nu + \widetilde{Z}^\nu$ does not converge to $\widetilde{Z} + Z$. Hence addition is not continuous on $\mathcal{G}^p(\Omega)$. Proposition 2.5.1 and Theorem 2.9 give its continuity on $\mathcal{G}^p_+(\Omega)$.

Given $Z^1, \ldots, Z^k$ belonging to $\mathcal{G}^{p_1}(\Omega), \ldots, \mathcal{G}^{p_k}(\Omega)$, respectively, and satisfying the conditions

1. the sum of the codimensions of $|Z^j|$ is equal to $n$,
2. $\bigcap_{j=1}^k |Z^j|$ is zero-dimensional,

a 0-chain is defined by

$$Z^1 \wedge \ldots \wedge Z^k = \sum_{a \in |Z^1| \cap \ldots \cap |Z^k|} i(Z^1 \wedge \ldots \wedge Z^k, a) \cdot \{a\}$$

where $i(Z^1 \wedge \ldots \wedge Z^k, a)$ denotes the intersection multiplicity defined in [Dr] (see also [Ch]). It is easy to prove that in Definition 2.1,

$$\mu(Z) = \deg_{E^n}((\{w\} \times E^{n-p}) \wedge Z) \quad \text{for } w \in E^p. \tag{1}$$

If $f : \Omega_1 \to \Omega_2 \supset \Omega$ is biholomorphic, then by [Ch, §12.3],

$$i(Z^1 \wedge \ldots \wedge Z^k, f(a)) = i(f^*(Z^1) \wedge \ldots \wedge f^*(Z^k), a). \tag{2}$$

Proposition 2.7. *Let $Z^\nu, Z \in \mathcal{G}^p(\Omega)$. If for each compact $K \subset \Omega \setminus |Z|$ we have $|Z^\nu| \cap K = \emptyset$ for almost all $\nu$, then the following conditions are equivalent*:

1. *For each point $a \in \operatorname{Reg}|Z|$, each $(n-p)$-dimensional plane transversal to $|Z|$ at $a$ and each open set $U$ relatively compact in $L$ such that $\overline{U} \cap |Z| = \{a\}$ there is an index $\nu_0$ such that $\dim(|Z^\nu| \cap U) = 0$, $\deg_U(Z^\nu \wedge L) = \deg_U(Z \wedge L)$ for all $\nu > \nu_0$.*
2. *For each point $a$ from a given dense subset $D \subset \operatorname{Reg}|Z|$, each $(n-p)$-dimensional plane transversal to $|Z|$ at $a$ and each open set $U$ relatively compact in $L$ such that $\overline{U} \cap |Z| = \{a\}$ there is an index $\nu_0$ such that $\dim(|Z^\nu| \cap U) = 0$, $\deg_U(Z^\nu \wedge L) = \deg_U(Z \wedge L)$ for all $\nu > \nu_0$.*
3. *$Z^\nu \to Z$ in the topology of $p$-chains.*

Proof. The proposition is obvious for $p = 0$ or $Z = 0$. Let $p > 0$, $Z \neq 0$.

1⇒2. Obvious.

2⇒3. Let $Z \in U(A, K)$, $A = \{(f_1, c_1), \ldots, (f_m, c_m)\}$. We check that $Z^\nu \in U(A, K)$ for sufficiently large $\nu$. Since $U(A, K) = \bigcap_{j=1}^m U(\{(f_j, c_j)\}, K)$ we can assume $m = 1$. By Proposition 2.5.4 it suffices to consider $f_1 = \mathrm{id}_{\mathbb{C}^n}$. Fix $w \in E^p$ such that $\{w\} \times E^{n-p}$ is transversal to $|Z|$ at each point of the set $(\{w\} \times E^{n-p}) \cap |Z| = \{z_1, \ldots, z_s\}$. There exist $\varepsilon > 0$ and open pairwise disjoint relatively compact subsets $U_1, \ldots, U_s$ of $E^{n-p}$ such that:

- $w + \varepsilon\overline{E^p} \subset E^p$,
- $|Z| \cap (\{w\} \times \overline{U}_j) = \{z_j\}$ for $j = 1, \ldots, s$,
- $|Z| \cap K_1 = \emptyset$ where $K_1 = (w + \varepsilon\overline{E^p}) \times (\overline{E^{n-p}} \setminus (U_1 \cup \ldots \cup U_s))$.

Choose $\widetilde{z_j} \in D \cap ((w + \varepsilon E^p) \times U_j)$ for $j = 1, \ldots, s$. Then

$$\mu(Z) = \sum_{j=1}^{s} \deg(\{w\} \times U_j) \wedge Z = \sum_{j=1}^{s} \deg(\{\pi(\widetilde{z_j})\} \times U_j) \wedge Z.$$

For sufficiently large $\nu$ we have $|Z^\nu| \subset \Omega \setminus (K \cup K_1)$, and so

$$\sum_{j=1}^{s} \deg(\{\pi(\widetilde{z_j})\} \times U_j) \wedge Z = \sum_{j=1}^{s} \deg(\{\pi(\widetilde{z_j})\} \times U_j) \wedge Z^\nu = \mu(Z^\nu).$$

Then $Z^\nu \in U(A, K)$ for sufficiently large $\nu$ and condition 3 follows.

3⇒1. Fix $a = (a_1, \ldots, a_n), L, U$ as in 1. By Proposition 2.5.4 and (2) we can assume that $a = 0$, $L = \mathbb{C}\{e_{p+1}, \ldots, e_n\}$ and $\overline{E^{n-p}} \subset U$.

There is $\varepsilon > 0$ such that $|Z| \cap (\varepsilon\overline{E^p} \times \partial E^{n-p}) = \emptyset$ and $\varepsilon\overline{E^p} \times \overline{E^{n-p}} \subset \Omega$. Moreover,

$$|Z^\nu| \cap ((\varepsilon\overline{E^p} \times \partial E^{n-p}) \cup (\{0\}^p \times (U \setminus E^{n-p}))) = \emptyset$$

and

$$\mu(f^*(Z^\nu)) = \mu(f^*(Z)),$$

where $f = (\varepsilon\,\mathrm{id}_{\mathbb{C}^p}, \mathrm{id}_{\mathbb{C}^{n-p}})$ and $\nu$ is large enough.

The set $|Z^\nu| \cap U$ is compact and non-empty, hence $\dim(|Z^\nu| \cap U) = 0$. By (1),

$$\deg_U(Z^\nu \wedge L) = \deg_U(Z \wedge L).$$

Remark. Condition 2 resembles the one given in [Ch, §12.2]. The following example shows the slight difference between them.

EXAMPLE 2.8. $\Omega = \mathbb{C}^2$, $Z^\nu = (\{1/\nu\} \times \mathbb{C}) + (\{1 - 1/\nu\} \times \mathbb{C})$, $Z = (\{0\} \times \mathbb{C}) + (\{1\} \times \mathbb{C})$, $\widetilde{Z} = (\{0\}\times\mathbb{C}) + 2(\{1\}\times\mathbb{C})$. One can see that $Z^\nu \to Z$ and $Z^\nu \to \widetilde{Z}$ in the sense of [Ch, §12.2]. The definition in [Ch, §12.2] seems to be erroneous, for $[Z^\nu]$ does not converge to $[\widetilde{Z}]$ as a sequence of currents. Neither does it converge to $\widetilde{Z}$ in the topology of $p$-chains.

Let us define:

- $\mathcal{A}_{\mathbb{Q}}(\Omega) = \{f \in \mathcal{A}(\Omega) : f(0), f(e_1), \ldots, f(e_n) \in (\mathbb{Q} + i\mathbb{Q})^n\}$,
- $\widetilde{\mathcal{K}} = \{[q_1, q_2] \times \ldots \times [q_{4n-1}, q_{4n}] : q_1, \ldots, q_{4n} \in \mathbb{Q}\}$,
- $\mathcal{K} = \{\bigcup \mathcal{B} : \mathcal{B} \subset \widetilde{\mathcal{K}}, \mathcal{B} \text{ is finite}\}$,
- $\mathcal{U}_{\mathbb{Q}}(\Omega) = \{U(A, K) : A \subset \mathcal{A}_{\mathbb{Q}}(\Omega) \times \mathbb{Z},\ A \text{ is finite},\ K \in \widetilde{\mathcal{K}},\ K \subset \Omega\}$,
- $E(r_1, r_2) = r_1 E^p \times r_2 E^{n-p}$ for $r_1, r_2 > 0$.

THEOREM 2.9. *$\mathcal{U}_{\mathbb{Q}}(\Omega)$ is a base for the topology of $p$-chains in $\Omega$.*

Proof. The assertion is obvious for $p = 0$. Suppose that $p > 0$ and let $Z = \sum_{j\in J} k_j Z_j \in U(A, K)$. We can assume $A = \{(f_1, c_1)\}$ (see the proof of Proposition 2.7). Then there are $\widetilde{K} \in \mathcal{K}$ and $\varepsilon > 0$ satisfying

$$\widetilde{K} \cap |Z| = \emptyset, \quad K \subset \widetilde{K} \subset \Omega, \quad f_1(E(1+\varepsilon, 1+\varepsilon)) \subset \Omega, \tag{3}$$

$$f_1(E(1+\varepsilon, 1+\varepsilon) \setminus \overline{E(1+\varepsilon, 1-\varepsilon)}) \subset \widetilde{K}. \tag{4}$$

Fix $0 < r < 1$. By a simple computation there is a neighborhood $U \subset \mathcal{A}(\Omega)$ of $f_1$ in the Banach space of affine mappings $\mathbb{C}^n \to \mathbb{C}^n$ such that each $f \in U$ satisfies the following conditions:

$$\begin{aligned} f(E(1+\varepsilon/2, 1+\varepsilon/2) &\setminus \overline{E(1+\varepsilon/2, 1-\varepsilon/2)}) \\ &\subset f_1(E(1+\varepsilon, 1+\varepsilon) \setminus \overline{E(1+\varepsilon, 1-\varepsilon)}), \end{aligned} \tag{5}$$

$$f_1(\{0\}^p \times E^{n-p}) \subset f(E(r/2, 1+\varepsilon/2)) \subset f_1(E(r, 1+\varepsilon)), \tag{6}$$

$$\begin{aligned} (f_1^{-1} \circ f)(\{0\}^p \times E^{n-p}) &\cap (E^n \setminus f_1^{-1}(\widetilde{K})) \\ &= (f_1^{-1} \circ f)(\{0\}^p \times \mathbb{C}^{n-p}) \cap (E^n \setminus f_1^{-1}(\widetilde{K})), \end{aligned} \tag{7}$$

$$(f_1^{-1} \circ f)(\{0\}^p \times \mathbb{C}^{n-p}) \cap E^n \subset E(r, 1), \tag{8}$$

$$\pi_{(p+1,\ldots,n)}|(f_1^{-1} \circ f)(\{0\}^p \times \mathbb{C}^{n-p}) \text{ is a bijection.} \tag{9}$$

Let $f \in U$ and $W = \sum l_j W_j \in U(\{(f, c_1)\}, \widetilde{K})$. Inclusions (4) and (5) give

$$(f_1^{-1}(|W|) \cup f^{-1}(|W|)) \cap (\overline{E^p} \times \partial E^{n-p}) = \emptyset.$$

If $f_1^{-1}(W_j) \cap E^n = \emptyset$ then by (4), $f_1^{-1}(W_j) \cap E(1, 1+\varepsilon) = \emptyset$. So, according to (6), $f^{-1}(W_j) \cap E(r/2, 1+\varepsilon/2) = \emptyset$. Thus, by Remmert's theorem we have $f^{-1}(W_j) \cap E^n = \emptyset$. Similarly $f^{-1}(W_j) \cap E^n = \emptyset \Rightarrow f_1^{-1}(W_j) \cap E^n = \emptyset$, which gives $\{j : f^{-1}(W_j) \cap E^n \neq \emptyset\} = \{j : f_1^{-1}(W_j) \cap E^n \neq \emptyset\}$.

By (1),

$$\mu(f_1^*(W_j)) = \deg(f_1^{-1}(W_j) \wedge (\{0\}^p \times E^{n-p})).$$

By [Wi, Theorem 9.1] and (8), (9),

$$\deg_{E^n}(f_1^{-1}(W_j) \wedge (\{0\}^p \times E^{n-p})) = \deg_{E^n}(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times \mathbb{C}^{n-p})).$$

From (7),

$$\begin{aligned} &\deg_{E^n}(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times \mathbb{C}^{n-p})) \\ &\qquad = \deg_{E^n}(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times E^{n-p})). \end{aligned}$$

By (4) and (6),

$$\begin{aligned} &\deg_{E^n}(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times E^{n-p})) \\ &\qquad = \deg(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times E^{n-p})), \\ &\deg(f_1^{-1}(W_j) \wedge (f_1^{-1} \circ f)(\{0\}^p \times E^{n-p})) = \deg(W_j \wedge f(\{0\}^p \times E^{n-p})) \\ &\qquad = \deg(f^{-1}(W_j) \wedge (\{0\}^p \times E^{n-p})) = \mu(f^*(W_j)). \end{aligned}$$

We have obtained $Z \in U(\{(f,c_1)\},\widetilde{K}) \subset U(\{(f_1,c_1)\},K)$. Density of $\mathcal{A}_{\mathbb{Q}}(\Omega)$ in $\mathcal{A}(\Omega)$ ends the proof.

**3. Metric on $\mathcal{G}^p(\Omega)$.** Let $Z \in \mathcal{G}^p(\Omega)$. For each compact subset $K$ of $\Omega$ we fix $0 < d_K < \min\{1, \operatorname{dist}(K,\partial\Omega)\}$ and define $H(K) = \bigcup_{x\in K} B(x,d_K)$.

DEFINITION 3.1.

$$d(Z,K) = \begin{cases} \operatorname{dist}(|Z| \cap H(K), K) & \text{if } |Z| \cap H(K) \neq \emptyset,\\ d_K & \text{if } |Z| \cap H(K) = \emptyset.\end{cases}$$

LEMMA 3.2. *$d(\cdot,K)$ is continuous.*

Proof. Let $Z^\nu \to Z$ and $d(Z,K) > 0$. Fix $\widetilde{d} < d(Z,K)$. Then we have $|Z^\nu| \cap \overline{\bigcup_{x\in K} B(x,\widetilde{d})} = \emptyset$ for almost all $\nu$. We obtain $\liminf_{\nu\to\infty} d(Z^\nu,K) \geq d(Z,K)$. If $|Z| \cap H(K) = \emptyset$ then $d(Z,K) = d_K$ and the lemma follows.

If $|Z| \cap H(K) \neq \emptyset$ then $\operatorname{dist}(|Z| \cap H(K), K) = |z-y|$ where $y \in K$, $z \in |Z| \cap \overline{H(K)}$. By Rückert's lemma there is a sequence $\{z_\nu\}$, $z_\nu \in |Z^\nu|$, $z_\nu \to z$, which gives

$$\limsup_{\nu\to\infty} d(Z^\nu,K) \leq d(Z,K).$$

By the same argument the previous inequality holds when $d(Z,K) = 0$.

Let $l \in \mathbb{Z}$ and let $\overline{E^n} \subset \Omega$.

DEFINITION 3.3. If $|Z| \cap (\overline{E^p} \times \partial E^{n-p}) = \emptyset$, $|Z| \cap E^n \neq \emptyset$, $\mu(Z) = l$ we define $m_l(Z) = d(Z, \overline{E^p} \times \partial E^{n-p})$. We put $m_l(Z) = 0$ otherwise.

LEMMA 3.4. *$m_l$ is continuous.*

Proof. Let $Z^\nu \to Z$. If $m_l(Z) \neq 0$ then $m_l(Z^\nu) = d(Z^\nu, \overline{E^p} \times \partial E^{n-p})$ for sufficiently large $\nu$ and we can use Lemma 3.2 . If $m_l(Z) = 0$ and $|Z| \cap (\overline{E^p} \times \partial E^{n-p}) \neq \emptyset$ then $|m_l(Z^\nu)| \leq |d(Z^\nu, \overline{E^p} \times \partial E^{n-p})| \to 0$. Suppose that $|Z| \cap (\overline{E^p} \times \partial E^{n-p}) = \emptyset$ and $|Z| \cap E^n = \emptyset$. By Remmert's theorem $|Z^\nu| \cap E^n = \emptyset$ for almost all $\nu$. If $m_l(Z) = 0$, $|Z| \cap (\overline{E^p} \times \partial E^{n-p}) = \emptyset$ and $|Z| \cap E^n \neq \emptyset$, then

$$\mu(Z^\nu) = \mu(Z) \neq l$$

for sufficiently large $\nu$.

Set $\mathcal{P}(\Omega) = \{m_l \circ f^* : f \in \mathcal{A}_{\mathbb{Q}}(\Omega),\ l \in \mathbb{Z}\}$, and observe that we have

$$\Big\{ \prod_{h\in J} h \cdot d(\cdot,K) : J \subset \mathcal{P}(\Omega),\ J \text{ is finite},\ K \in \mathcal{K} \Big\},$$

a countable family of continuous functions. Let $\{G_j\}$ denote a sequence of all its elements.

Definition 3.5. Let $X, Z \in \mathcal{G}^p(\Omega)$. We define

$$\varrho(X,Z) = \sum_{j=0}^{\infty} \frac{1}{2^j} |G_j(X) - G_j(Z)|.$$

Theorem 3.6. *$\varrho$ is a metric on $\mathcal{G}^p(\Omega)$. The topology induced by $\varrho$ coincides with the topology of $p$-chains.*

Proof. It is sufficient to prove that the sequence $\{G_j\}$ gives an embedding of $\mathcal{G}^p(\Omega)$ in the Hilbert cube. According to [En, 2.3, Theorem 10] we need to prove that:

1. $\{G_j\}_{j\in\mathbb{N}}$ separates elements of $\mathcal{G}^p(\Omega)$,
2. $\{G_j\}_{j\in\mathbb{N}}$ separates elements of $\mathcal{G}^p(\Omega)$ from closed subsets of $\mathcal{G}^p(\Omega)$.

1) We can assume that $|Z| \neq \emptyset$. If $|X| \neq |Z|$ then there is $K \in \widetilde{K}$ satisfying $|X| \cap K = \emptyset$, $|Z| \cap K \neq \emptyset$. We obtain

$$0 = d(Z,K) \neq d(X,K).$$

Suppose $|X| = |Z|$. There is $z \in \operatorname{Reg}|X|$ satisfying $m(z,X) \neq m(z,Z)$. Fix $g \in \mathcal{A}_{\mathbb{Q}}(\Omega)$ such that $\mu(\pi|g^{-1}(|Z|)) = 1$. Consequently,

$$(m_{m(z,Z)} \circ g^*)(Z) \neq (m_{m(z,Z)} \circ g^*)(X) = 0.$$

2) Let $X \in U(\{(f_1,c_1),\ldots,(f_m,c_m)\},K) \subset \mathcal{G}^p(\Omega)\setminus C$, where $C$ is a closed subset of $\mathcal{G}^p(\Omega)$. Without loss of generality $U(\{(f_1,c_1),\ldots,(f_m,c_m)\},K) \in \mathcal{U}_{\mathbb{Q}}(\Omega)$.

If $|X| \neq \emptyset$ set $G_n = \prod_{j=1}^m (m_{c_j} \circ f_j^*) \cdot d(\ ,K)$. If $|X| = \emptyset$ choose $G_n = d(\ ,\widetilde{K})$ where $\widetilde{K} \in \mathcal{K}$ and

$$U(\widetilde{K}) \subset U(\{(f_1,c_1),\ldots,(f_m,c_m)\},K).$$

In both cases we obtain $G_n|_C = 0, G_n(X) \neq 0$.

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