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Factorization of uniformly holomorphic functions

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Abstract. Let E be a complex Hausdorff locally convex space such that the strong dual E' of E is sequentially complete, let F be a closed linear subspace of E and let U be a uniformly open subset of E. We denote by $\Pi : E \to E/F$ the canonical quotient mapping. In §1 we study the factorization of uniformly holomorphic functions through π. In §2 we study F-quotients of uniform type and introduce the concept of envelope of uF-holomorphy of a connected uniformly open subset U of E. The main result states that the pull-back $\varepsilon_{\mathrm{u}}^*(U)$ of the envelope of uniform holomorphy of $\Pi(U)$ constructed by Paques and Zaine [9] is the envelope of uF-holomorphy of U.

Introduction. We deal with the concept of uniform holomorphy (cf. [6]–[8]) of a holomorphic function $f : U \to \mathbb{C}$ in the case when U is a nonvoid uniformly open subset of a complex Hausdorff locally convex space E. Let F be a closed linear subspace of E, let $\Pi : E \to E/F$ be the canonical quotient mapping and let I_U be the set of all continuous seminorms α on E such that U is open in (E, α). Let $H_{\mathrm{u}}(U)$ be the set of all uniformly holomorphic functions from U into $\mathbb{C}$ and let $H_{\mathrm{u}F}(U)$ be the set of all $g \circ \Pi$ as g ranges over $H_{\mathrm{u}}(\Pi(U))$. It is easy to show that $H_{\mathrm{u}F} \subset H_{\mathrm{u}}(U)$. In §1 we prove that if U is a balanced uniformly open subset of E and F is a closed linear subspace of (E, α) for each $\alpha \in I_U$, then $g \circ \Pi$ is uniformly holomorphic if and only if g is uniformly holomorphic.

The concepts of Riemann domain of uniform type and F-quotient of a Riemann domain were introduced in [9] and [4] respectively. Given a uniformly open subset U of E it is easy to verify that $\Pi(U)$ is a uniformly open subset of E/F (cf. Ex. 3, §2). We have been unable to decide if an F-quotient of a Riemann domain of uniform type is always of uniform type. However, we give in §2 some non-trivial examples of F-quotients of a Riemann domain of uniform type which are of uniform type. In particular, we consider

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$(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi)$, the envelope of u-holomorphy of $\Pi(U)$ constructed in [9] and its pull-back $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$. We prove that there exists an open mapping ψ from $\varepsilon_{\mathrm{u}}^*(U)$ onto $\varepsilon_{\mathrm{u}}(\Pi(U))$ such that $(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi, \psi)$ is an F-quotient of uniform type of $\varepsilon_{\mathrm{u}}^*(U)$ satisfying the following: given $g \in H_{\mathrm{u}}(\Pi(U))$ there exists a uniform extension $\widetilde{f} \in H_{\mathrm{u}}(\varepsilon_{\mathrm{u}}^*(U))$ of $f = g \circ \Pi$ which is defined by $\widetilde{g} \circ \psi$ where $\widetilde{g} \in H_{\mathrm{u}}(\varepsilon_{\mathrm{u}}(\Pi(U)))$ is a uniform extension of g. We also find that $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$ is maximal in the sense of Definition 11.

We remark that the concept of envelope of F-holomorphy given in [4] of a connected open subset U of a Banach space E works also when E is an arbitrary locally convex space. In particular, this paper extends the results of [4] to locally convex spaces with $H_{\mathrm{u}}(U) = H(U)$ and $H_{\mathrm{u}}(\Pi(U)) = H(\pi(U))$. This is the case if E is a dual of a separable Fréchet space endowed with the compact-open topology (cf. [5] and [8]).

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Notation and terminology. Throughout this paper E is a complex Hausdorff locally convex space whose strong dual E' is sequentially complete, F is a closed linear subspace of E and $\Pi : E \to E/F$ is the canonical quotient mapping. We refer to [2] for the terminology in infinite-dimensional complex analysis.

Let $\mathrm{cs}(E)$ be the set of all continuous seminorms on E. For each $\alpha \in \mathrm{cs}(E)$, we denote by (E, α) the space E endowed with the topology generated by α, by E_α the normed space associated with (E, α), by $i_\alpha : E \to E_\alpha$ the canonical quotient mapping and by $B_\alpha(x, r)$ the open ball with center x and radius r in (E, α). Given an open subset U of E we write $i_\alpha(U) = U_\alpha$ and, as usual, $H(U)$ is the vector space of all holomorphic functions from U into $\mathbb{C}$.

An open subset U of E is said to be *uniformly open* if there exists $\alpha \in \mathrm{cs}(E)$ such that U is open in (E, α). Let I_U denote the set of all such $\alpha \in \mathrm{cs}(E)$. We remark that I_U is a directed subset of $\mathrm{cs}(E)$ that generates the topology of E.

If U is a uniformly open subset of E, a holomorphic function $f : U \to \mathbb{C}$ is said to be *uniformly holomorphic* on U if there exist $\alpha \in I_U$ and $f_\alpha \in H(U_\alpha)$ such that $f = f_\alpha \circ i_\alpha$. We denote by $H_{\mathrm{u}}(U)$ the vector space of all uniformly holomorphic functions from U into $\mathbb{C}$.

The following well known result will be useful:

PROPOSITION A. *If V is an open subset of a locally convex space M, M_S is the associated Hausdorff space of M, $Q : M \to M_S$ is the canonical* *mapping and* $V_S = Q(V)$*, then* $f \in H(V)$ *if and only if there exists* $f_S \in H(V_S)$ *such that* $f = f_S \circ Q$*.*

The pair (X, φ) is a *Riemann domain* over E if X is a nonvoid Hausdorff topological space and $\varphi : X \to E$ is a local homeomorphism. Instead of (X, φ) we often write X. Given $A \subseteq X$, we write $A \sim \varphi(A)$ to indicate that A is homeomorphic to $\varphi(A)$ under φ/A. A *chart* in X is a connected open subset V of X such that $\varphi/V : V \to \varphi(V)$ is a homeomorphism. An *atlas* on X is a collection $(V_i)_{i\in I}$ of charts which cover X. We recall that if U is an open subset of E and $i_U : U \to E$ is the inclusion mapping, then (U, i_U) is a Riemann domain over E.

A Riemann domain (X, φ) over E is said to be a *Riemann domain of uniform type* (or, simply, a domain of uniform type) if there exists $\alpha \in \mathrm{cs}(E)$ such that for each $x \in X$, there is a neighborhood $V(x)$ of x such that $V(x) \sim \varphi(V(x))$ and $\varphi(V(x))$ is open in (E, α). Let I_X denote the set of all such $\alpha \in \mathrm{cs}(E)$. For every $\alpha \in I_X$ let (X, α) be the set X endowed with the topology generated by the neighborhoods V that satisfy the above definition. We denote by X_α the Hausdorff space associated with (X, α), i.e., $X_\alpha = (X, \alpha)/R$ where R is the equivalence relation on X defined by: xRy if and only if $\alpha(\varphi(x) - \varphi(y)) = 0$ for all $x, y \in X$. For each $\alpha \in I_X$, let $I_\alpha : X \to X_\alpha$ be the canonical quotient mapping; it is clear that if we define $\varphi_\alpha : X_\alpha \to E_\alpha$ by $\varphi_\alpha \circ I_\alpha := i_\alpha \circ \varphi$, then φ_α is a local homeomorphism and $(X_\alpha, \varphi_\alpha)$ is a Riemann domain over E_α.

If (X, φ) is a Riemann domain over E and (Y, ϱ) is a Riemann domain over a Hausdorff locally convex space G, a continuous mapping $f : X \to Y$ is said to be *holomorphic* if there is an atlas $(V_i)_{i\in I}$ on X such that $\varrho \circ f \circ (\varphi/V_i)^{-1} : \varphi(V_i) \to G$ is holomorphic for each $i \in I$. We shall denote by $H(X, Y)$ the class of all mappings $f : X \to Y$ which are holomorphic. When $Y = \mathbb{C}$ we write $H(X)$ instead of $H(X; \mathbb{C})$.

If (X, φ) is a domain of uniform type, and G is a Hausdorff locally convex space, a holomorphic mapping $f : X \to G$ is said to be *uniformly holomorphic* if for each $\beta \in \mathrm{cs}(G)$ there exist $\alpha \in I_X$ and a holomorphic mapping $f_\alpha \in H(X_\alpha, G_\beta)$ such that $i_\beta \circ f = f_\alpha \circ I_\alpha$.

For other notations and basic results on uniform holomorphy we refer to [6] and [9].

1. Factorization of uniformly holomorphic mappings. Let U be a uniformly open subset of E. For each $\alpha \in \mathrm{cs}(E)$ define $\overline{\alpha}(\overline{x}) := \inf\{\alpha(x+y) : y \in F\}$ for $\overline{x} = \Pi(x) \in E/F$. It is well known that $\overline{\alpha} \in \mathrm{cs}(E/F)$ and the set $\{\overline{\alpha} : \alpha \in I_U\}$ generates the topology of E/F.

PROPOSITION 1. *Let U be a uniformly open subset of E. Then*:

(a) *$\Pi(U)$ is uniformly open and $\overline{\alpha} \in I_{\Pi(U)}$ for every $\alpha \in I_U$.*

(b) *If g is a uniformly holomorphic function on $\Pi(U)$, then $f = g \circ \Pi$ is uniformly holomorphic on U.*

P r o o f o f (b). Let $g \in H_{\mathrm{u}}(\Pi(U))$. Since $\{\overline{\alpha} : \alpha \in I_U\}$ generates the topology of E/F, there exist $\alpha \in I_U$ and $g_{\bar{\alpha}} \in H(\Pi(U)_{\bar{\alpha}})$ such that $g = g_{\bar{\alpha}} \circ i_{\bar{\alpha}}$ where $i_{\bar{\alpha}} : E/F \to (E/F)_{\bar{\alpha}}$ is the canonical quotient mapping and $\Pi(U)_{\bar{\alpha}} = i_{\bar{\alpha}}(\Pi(U))$. If $i : E \to (E, \alpha)$ is the identity mapping, $\Pi_\alpha : (E, \alpha) \to (E/F, \overline{\alpha})$ is the quotient mapping and $k_{\bar{\alpha}} : (E/F, \overline{\alpha}) \to (E/F)_{\bar{\alpha}}$ is the canonical quotient mapping, it is clear that $k_{\bar{\alpha}} \circ \Pi_\alpha \circ i = i_{\bar{\alpha}} \circ \Pi$. Consequently, $f = g \circ \Pi = g_{\bar{\alpha}} \circ i_{\bar{\alpha}} \circ \Pi = g_{\bar{\alpha}} \circ k_{\bar{\alpha}} \circ \Pi_\alpha \circ i$ and so there exists $f'_\alpha = g_{\bar{\alpha}} \circ k_{\bar{\alpha}} \circ \Pi_\alpha \in H(i(U))$ such that $f = f'_\alpha \circ i$. By Proposition A, there exists $f_\alpha \in H(U_\alpha)$ so that $f'_\alpha = f_\alpha \circ k_\alpha$ where $k_\alpha : (E, \alpha) \to E_\alpha$ is the canonical quotient mapping. So, $f = f'_\alpha \circ i = f_\alpha \circ k_\alpha \circ i = f_\alpha \circ i_\alpha$ and we have $f \in H_{\mathrm{u}}(U)$.

The next result gives us a reciprocal for Proposition 1(b) when F is a closed linear subspace of (E, α) for each $\alpha \in I_U$.

PROPOSITION 2. *Let U be a balanced uniformly open subset of E and let F be a closed linear subspace of (E, α) for each $\alpha \in I_U$. If f is uniformly holomorphic on U and $f = g \circ \Pi$ for some $g \in H(\Pi(U))$, then g is uniformly holomorphic on $\Pi(U)$.*

P r o o f. We define k_α, i, Π_α, $k_{\bar{\alpha}}$ and $i_{\bar{\alpha}}$ as in the proof of Proposition 1. By hypothesis there exist $\alpha \in I_U$ and $f_\alpha \in H(U_\alpha)$ such that $f = f_\alpha \circ i_\alpha$. If $f'_\alpha \in H(i(U))$ is defined by $f'_\alpha = f_\alpha \circ k_\alpha$ it follows that $f = f_\alpha \circ i_\alpha = f_\alpha \circ k_\alpha \circ i = f'_\alpha \circ i$. By Theorem 2.3 of [1], $f = g \circ \Pi$ if and only if $df(x)/F = 0$ for all $x \in U$. Consequently, $0 = df(x)(y) = df'_\alpha(i(x))(i(y))$ for all $y \in F$, i.e., $df'_\alpha(i(x))/F = 0$ or f'_α factors through $\Pi_\alpha(i(U))$. So there exists $g' \in H(\Pi_\alpha(i(U)))$ such that $f'_\alpha = g' \circ \Pi_\alpha$. If $k : E/F \to (E/F, \overline{\alpha})$ is the identity mapping, then $g = g' \circ k$ on $\Pi(U)$. Indeed, for every $x \in U$,

$$g(\Pi(x)) = f(x) = (f'_\alpha \circ i)(x) = (g' \circ \Pi_\alpha \circ i)(x) = (g' \circ k)(\Pi(x)).$$

Since $k_{\bar{\alpha}} \circ \Pi_\alpha \circ i = i_{\bar{\alpha}} \circ \Pi$, $\Pi_\alpha(i(U)) \subseteq (E/F, \overline{\alpha})$ and $g' \in H(\Pi_\alpha(i(U))$, by Proposition A, there exists $g'_{\bar{\alpha}} \in H(i_{\bar{\alpha}}(\Pi(U)))$ such that $g' = g'_{\bar{\alpha}} \circ k_{\bar{\alpha}}$ on $\Pi_\alpha(i(U)) = k(\Pi(U))$ and it follows that for every $\Pi(x) \in \Pi(U)$,

$$g(\Pi(x)) = (g' \circ k)(\Pi(x)) = (g'_{\bar{\alpha}} \circ k_{\bar{\alpha}} \circ k)(\Pi(x)) = g'_{\bar{\alpha}} \circ i_{\bar{\alpha}}(\Pi(x)).$$

So, there exist $\overline{\alpha} \in I_{\Pi(U)}$ and $g'_{\bar{\alpha}} \in H(\Pi(U)_{\bar{\alpha}})$ such that $g = g'_{\bar{\alpha}} \circ i_{\bar{\alpha}}$ on $\Pi(U)$, i.e., $g \in H_{\mathrm{u}}(\Pi(U))$.

2. Uniformly holomorphic continuation. Let (X, φ) be a Riemann domain over E. We say that (X_F, φ_F, ψ) is an F-*quotient* of X if (X_F, φ_F) is a Riemann domain over E/F and ψ is a continuous open mapping from X onto X_F such that $\varphi_F \circ \psi = \Pi \circ \varphi$. The concept of F-quotient of a

Riemann domain was introduced and studied in [4], where several examples are presented. Here we give some examples of Riemann domains of uniform type X over E which admit an F-quotient (X_F, φ_F, ψ) such that (X_F, φ_F) is also of uniform type. In this case we will say that (X_F, φ_F, ψ) is an *F-quotient of uniform type* of X.

EXAMPLE 3. Let U be a uniformly open subset of E, and $i_U : U \to E$ and $i_\Pi : \Pi(U) \to E/F$ the inclusion mappings. Then $\Pi(U)$ is a uniformly open subset of E/F (cf. Proposition 1(a)) and it is clear that $(\Pi(U), i_\Pi, \Pi)$ is an F-quotient of (U, i_U) which is of uniform type.

EXAMPLE 4. Let (X, φ) be a Riemann domain of uniform type over E, let R be the equivalence relation defined on X by xRy if and only if $\varphi(x) - \varphi(y) \in F$ for $x, y \in X$ and denote by X/R the quotient set by this equivalence, endowed with the quotient topology associated with the mapping ψ from X onto X/R defined by $\psi(x) := \overline{x}$ (where $\overline{x}$ denotes the equivalence class of x). We can define $\varphi_F : X/R \to E/F$ by $\varphi_F(\overline{x}) := \Pi(\varphi(x))$ for $\overline{x} \in X/R$ and it is easy to see that $(X/R, \varphi_F)$ is a Riemann domain over E/F. By hypothesis, there is $\alpha \in \mathrm{cs}(E)$ such that, for each $x \in X$, there exist a neighborhood $V(x)$ of x and an $r > 0$ satisfying $V(x) \sim \varphi(V(x)) = B_\alpha(\varphi(x), r)$. Since $\varphi_F \circ \psi = \Pi \circ \varphi$, we have $\varphi_F \circ \psi(V(x)) = \Pi \circ \varphi(V(x)) = \Pi(B_\alpha(\varphi(x), r)) = B_{\bar{\alpha}}(\varphi_F \circ \psi(x), r)$. Since $\psi(V(x))$ is a neighborhood of $\psi(x)$ and φ_F is injective on $\psi(V(x))$, it is clear that $(X/R, \varphi_F)$ is of uniform type and so $(X/R, \varphi_F, \psi)$ is an F-quotient of uniform type of X.

Let (X, φ) and (Y, ϱ) be two Riemann domains over E. A continuous mapping $j : X \to Y$ is said to be a *morphism* if $\varrho \circ j = \varphi$. The concept of envelope of uniform holomorphy of a Riemann domain of uniform type was introduced and studied in [9]. We recall that if U is a connected uniformly open subset of E and $(\varepsilon_{\mathrm{u}}(U), q)$ is constructed as in [9], the morphism $j' : U \to \varepsilon_{\mathrm{u}}(U)$ defined by $j'(u) := \widehat{u}$, where $\widehat{u}(f) := f(u)$ for $f \in H_{\mathrm{u}}(U)$, is the *envelope of uniform holomorphy* of U. Analogously $(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi)$ is constructed and the morphism $j_\Pi : \Pi(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ defined by $j_\Pi(\Pi(u)) := \widehat{\Pi(u)}$, where $\widehat{\Pi(u)}(g) := g(\Pi(u))$ for $g \in H_{\mathrm{u}}(\Pi(U))$, is the envelope of uniform holomorphy of $\Pi(U)$. Following the idea used in the proof of Propositions 6 and 7 and Corollary 8 of [4], we get a new construction of $(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi)$ and an open mapping $\psi : \varepsilon_{\mathrm{u}}(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ such that $\psi(\varepsilon_{\mathrm{u}}(U))$ is a connected topological subspace of $\varepsilon_{\mathrm{u}}(\Pi(U))$. We denote also by q_Π the restriction of q_Π to $\psi(\varepsilon_{\mathrm{u}}(U))$.

EXAMPLE 5. By using the definition of the topology of $\varepsilon_{\mathrm{u}}(\Pi(U))$, it is easy to verify that $(\psi(\varepsilon_{\mathrm{u}}(U)), q_\Pi)$ is a Riemann domain of uniform type over E/F. So, $(\psi(\varepsilon_{\mathrm{u}}(U)), q_\Pi, \psi)$ is an F-quotient of $(\varepsilon_{\mathrm{u}}(U), q)$ of uniform type.

The *pull-back* of $(\varepsilon_{\rm u}(\Pi(U)), q_\Pi)$ is, by definition, the Riemann domain $(\varepsilon_{\rm u}^*(U), \varphi^*)$ over E where $\varepsilon_{\rm u}^*(U) := \{(H,a) \in \varepsilon_{\rm u}(\Pi(U)) \times E : q_\Pi(H) = \Pi(a)\}$ endowed with the topology induced on $\varepsilon_{\rm u}^*(U)$ by the product topology on $\varepsilon_{\rm u}(\Pi(U)) \times E$ and $\varphi^*(H,a) := a$ for $(H,a) \in \varepsilon_{\rm u}^*(U)$ (cf. [3] and [10]).

Example 6. Let $\psi : \varepsilon_{\rm u}^*(U) \to \varepsilon_{\rm u}(\Pi(U))$ be defined by $\psi(H,a) := H$ for $(H,a) \in \varepsilon_{\rm u}^*(U)$. We claim that the Riemann domain $(\varepsilon_{\rm u}^*(U), \varphi^*)$ is of uniform type and $(\varepsilon_{\rm u}(\Pi(u)), q_\Pi, \psi)$ is an F-quotient of uniform type of $(\varepsilon_{\rm u}^*(U), \varphi^*)$.

Let $(H,a) \in \varepsilon_{\rm u}^*(U)$. By hypothesis there exist $\overline{\alpha} \in {\rm cs}(E/F)$, $r > 0$ and a basic neighborhood $N_{\bar\alpha}(H,r) = \{H_{\bar b} : \overline{b} \in B_{\bar\alpha}(\overline{0}, r)\}$ of H such that $N_{\bar\alpha}(H,r) \sim B_{\bar\alpha}(q_\Pi(H), r)$. We recall that $H_{\bar b}(g) := \sum (1/n!) H(\widehat{d^n_{\bar b}} g)$ for all $g \in H_{\rm u}(\Pi(U))$ and $q_\Pi(H_{\bar b}) = q_\Pi(H) + \overline{b}$ (cf. [9]). Let

$$V := (N_{\bar\alpha}(H,r) \times B_\alpha(a,r)) \cap \varepsilon_{\rm u}^*(U).$$

It is clear that V is a neighborhood of (H,a).

We claim φ^*/V is a homeomorphism between V and $B_\alpha(\varphi^*(H,a), r)$. The continuity of φ^*/V is clear. Let $(H_{\bar b_1}, c) \neq (H_{\bar b_2}, d)$ in V. If $c = d$, $H_{\bar b_1}$ must be different from $H_{\bar b_2}$ and consequently $\overline{b}_1 \neq \overline{b}_2$ and $\Pi(c) = q_\Pi(H_{\bar b_1}) = q_\Pi(H) + \overline{b}_1 \neq q_\Pi(H) + \overline{b}_2 = q_\Pi(H_{\bar b_2}) = \Pi(d)$, and we have a contradiction. So, we must have $c \neq d$ and it is clear that $\varphi^*(H_{\bar b_1}, c) \neq \varphi^*(H_{\bar b_2}, d)$. To prove that φ^*/V is onto $B_\alpha(a,r)$ it is enough to show that for each $c \in B_\alpha(a,r)$ there exists $\overline{b} \in B_{\bar\alpha}(\overline{0}, r)$ satisfying $q_\Pi(H_{\bar b}) = \Pi(c)$. Take $\overline{b} = \Pi(c-a)$ and it is done. This completes the proof that $(\varepsilon_{\rm u}^*(U), \varphi^*)$ is a Riemann domain of uniform type.

Now, we show that $(\varepsilon_{\rm u}(\Pi(U)), q_\Pi, \psi)$ is an F-quotient of $(\varepsilon_{\rm u}^*(U), \varphi^*)$ of uniform type. It is clear from the definitions that ψ is a continuous mapping from $\varepsilon_{\rm u}^*(U)$ onto $\varepsilon_{\rm u}(\Pi(U))$ such that $\Pi \circ \varphi^* = q_\Pi \circ \psi$. So, all we have to prove is that ψ is open. It is enough to show that given any $(H,a) \in \varepsilon_{\rm u}^*(U)$, for every basic neighborhood $N_{\bar\alpha}(H,r)$ of H, we have

$$\psi([N_{\bar\alpha}(H,r) \times B_\alpha(a,r)] \cap \varepsilon_{\rm u}^*(U)) = N_{\bar\alpha}(H,r).$$

Let $K \in N_{\bar\alpha}(H,r)$, i.e., $K = H_{\bar b}$ for some $\overline{b} \in B_{\bar\alpha}(\overline{0}, r)$. Since $\Pi(B_\alpha(0,r)) = B_{\bar\alpha}(\overline{0}, r)$, there exists $b_1 \in B_\alpha(0,r)$ such that $\Pi(b_1) = \overline{b}$. It is clear that $(H_{\bar b}, a + b_1) \in N_{\bar\alpha}(H,r) \times B_\alpha(a,r)$ and since $q_\Pi(H_{\bar b}) = \Pi(a) + \overline{b} = \Pi(a + b_1)$ implies $(H_{\bar b}, a + b_1) \in \varepsilon_{\rm u}^*(U)$ we get

$$N_{\bar\alpha}(H,r) \subseteq \psi([N_{\bar\alpha}(H,r) \times B_\alpha(a,r)] \cap \varepsilon_{\rm u}^*(U)).$$

The other inclusion is trivial.

Let (X_F, φ_F, ψ) be an F-quotient of uniform type of X and take any $\overline{\alpha} \in I_{X_F}$. We denote by $X_{F\bar\alpha}$ the space $(X_F)_{\bar\alpha}$ and by $\varphi_{F\bar\alpha}$ the local homeomorphism $\varphi_{F\bar\alpha} : X_{F\bar\alpha} \to (E/F)_{\bar\alpha}$. If $I_{\bar\alpha} : X_F \to X_{F\bar\alpha}$ is the canonical quotient mapping, let $\overline{x}_{\bar\alpha} := I_{\bar\alpha}(\overline{x})$ for all $\overline{x} \in X_F$, i.e., $\overline{x}_{\bar\alpha} = \{\overline{y} \in X_F : \overline{\alpha}(\varphi_F(\overline{x}) - \varphi_F(\overline{y})) = 0\}$. We recall that $\varphi_{F\bar\alpha}(\overline{x}_{\bar\alpha}) := \varphi_F(\overline{x}) + \overline{\alpha}^{-1}(\overline{0})$ for all $\overline{x} \in X_F$ and $(X_{F\bar\alpha}, \varphi_{F\bar\alpha})$ is a Riemann domain over $(E/F)_{\bar\alpha}$.

LEMMA 7. *Let U be a uniformly open subset of E. Suppose that (X,φ) is a Riemann domain of uniform type over E and (X_F, φ_F, ψ) is an F-quotient of uniform type of X. Then*:

(a) *With every $\alpha \in I_U$ and $\overline{\beta} \in I_{X_F}$ we can associate $\gamma \in I_U \cap I_X$ such that $\overline{\gamma} \geq \overline{\alpha}$, $\overline{\gamma} \geq \overline{\beta}$, and so $\overline{\gamma} \in I_{X_F}$.*

(b) *Given $\overline{\gamma}, \overline{\beta} \in I_{X_F}$ so that $\overline{\beta} \leq \overline{\gamma}$, if $\widetilde{g} = \widetilde{g}_{\bar\beta} \circ I_{\bar\beta}$ for some $\widetilde{g}_{\bar\beta} \in H(X_{F\bar\beta})$ then there exists $\widetilde{g}_{\bar\gamma} \in H(X_{F\bar\gamma})$ satisfying $\widetilde{g} = \widetilde{g}_{\bar\gamma} \circ I_{\bar\gamma}$.*

Proof. (a) Let $\alpha \in I_U$ and $\overline{\beta} \in I_{X_F}$. Since X is of uniform type we can choose $\delta \in I_X \neq \emptyset$. As $\{\overline{\lambda} : \lambda \in I_U\}$ generates the topology of E/F there exists $\alpha_0 \in I_U$ such that $\overline{\beta} \leq \overline{\alpha}_0$. But since I_U generates the topology of E there exists $\gamma \in I_U$ such that $\delta, \alpha, \alpha_0 \leq \gamma$. It is clear that $\overline{\gamma} \geq \overline{\alpha}, \overline{\beta}$, $\gamma \in I_U \cap I_X$ and $\overline{\gamma} \in I_{X_F}$.

(b) If we define $i_{\bar\gamma\bar\beta} : (E/F)_{\bar\gamma} \to (E/F)_{\bar\beta}$ by $i_{\bar\gamma\bar\beta} \circ i_{\bar\gamma} := i_{\bar\beta}$, it is easy to verify that $i_{\bar\gamma\bar\beta}$ is a well defined continuous linear mapping from $(E/F)_{\bar\gamma}$ onto $(E/F)_{\bar\beta}$. Consequently, it is a holomorphic mapping. Analogously we define $I_{\bar\gamma\bar\beta} : X_{F\bar\gamma} \to X_{F\bar\beta}$ by $I_{\bar\gamma\bar\beta}(\overline{x}_{\bar\gamma}) := \overline{x}_{\bar\beta}$ for $\overline{x}_{\bar\gamma} \in X_{F\bar\gamma}$. As $\overline{\gamma}(\varphi_F(x) - \varphi_F(y)) = 0$ implies $\overline{\beta}(\varphi_F(\overline{x}) - \varphi_F(\overline{y})) = 0$, it is easy to see that $I_{\bar\gamma\bar\beta}$ is well defined. It is also clear that $I_{\bar\gamma\bar\beta}$ is continuous and for every chart V of $X_{F\bar\gamma}$ we have $i_{\bar\gamma\bar\beta} = \varphi_{F\bar\beta} \circ I_{\bar\gamma\bar\beta} \circ (\varphi_{F\bar\gamma}/V)^{-1}$. Consequently, $I_{\bar\gamma\bar\beta} \in H(X_{F\bar\gamma}, X_{F\bar\beta})$. Now if $\widetilde{g} = \widetilde{g}_{\bar\beta} \circ I_{\bar\beta}$ with $\widetilde{g}_{\bar\beta} \in H(X_{F\bar\beta})$ it is enough to define $\widetilde{g}_{\bar\gamma} : X_{F\bar\gamma} \to \mathbb{C}$ by $\widetilde{g}_{\bar\gamma} := \widetilde{g}_{\bar\beta} \circ I_{\bar\gamma\bar\beta}$.

If (Y, ϱ) is a Riemann domain of uniform type over E/F, then a morphism $j : \Pi(U) \to Y$ is said to be a *uniform extension* of $\Pi(U)$ if for each $g \in H_{\mathrm{u}}(\Pi(U))$ there is a unique $\widetilde{g} \in H_{\mathrm{u}}(Y)$ such that $\widetilde{g} \circ j = g$. In this case $\widetilde{g}$ is said to be a *uniform extension* of g to Y.

DEFINITION 8. Let (X, φ) be a Riemann domain of uniform type over E. A morphism $\gamma : U \to X$ is said to be a u*F*-*extension* of U (uniform F-extension of U) if there exist an F-quotient of uniform type (X_F, φ_F, ψ) of X and a morphism $\gamma_\Pi : \Pi(U) \to X_F$ such that:

(a) $\psi \circ \gamma = \gamma_\Pi \circ \Pi$.

(b) γ_Π is a uniform extension of $\Pi(U)$.

Remark 9. In the above case, given $g \in H_{\mathrm{u}}(\Pi(U))$ there exists a uniform extension $\widetilde{f} \in H_{\mathrm{u}}(X)$ of $f = g \circ \Pi$ which is defined by $\widetilde{f} = \widetilde{g} \circ \psi$ where $\widetilde{g} \in H_{\mathrm{u}}(X_F)$ is a uniform extension of g. Indeed, since $g \in H_{\mathrm{u}}(\Pi(U))$, there exist $\alpha \in I_U$ and $g_{\bar\alpha} \in H(\Pi(U)_{\bar\alpha})$ such that $g = g_{\bar\alpha} \circ i_{\bar\alpha}$ (where $i_{\bar\alpha}, \Pi(U)_{\bar\alpha}$ and $g_{\bar\alpha}$ are defined as in the proof of Proposition 1(b)). If $\widetilde{g} \in H_{\mathrm{u}}(X_F)$ is the uniform extension of g to X_F (whose existence is proved in [9]), then there exist $\overline{\beta} \in I_{X_F}$ and $\widetilde{g}_{\bar{\beta}} \in H(X_{F\bar{\beta}})$ such that $\widetilde{g} = \widetilde{g}_{\bar{\beta}} \circ I_{\bar{\beta}}$. By Lemma 7 there exists $\gamma \in I_U \cap I_X$ so that $\overline{\gamma} \geq \overline{\alpha}$ and $\overline{\gamma} \geq \overline{\beta}$ and there exists $\widetilde{g}_{\bar{\gamma}} \in H(X_{F\bar{\gamma}})$ satisfying $\widetilde{g} = \widetilde{g}_{\bar{\gamma}} \circ I_{\bar{\gamma}}$. Let $\widetilde{f} := \widetilde{g} \circ \psi$. It is clear that $\widetilde{f}$ is continuous.

We claim that there exists a holomorphic mapping $\psi_{\bar{\gamma}} : X_\gamma \to X_{F\bar{\gamma}}$ such that $\psi_{\bar{\gamma}} \circ I_\gamma = I_{\bar{\gamma}} \circ \psi$ (recall that $I_\gamma : X \to X_\gamma$ is the canonical quotient mapping). If this is true, there exists $\widetilde{f}_\gamma := \widetilde{g}_{\bar{\gamma}} \circ \psi_{\bar{\gamma}}$ such that $\widetilde{f}_\gamma \in H(X_\gamma)$ and $\widetilde{f}_\gamma \circ I_\gamma = \widetilde{g}_{\bar{\gamma}} \circ \psi_{\bar{\gamma}} \circ I_\gamma = \widetilde{g}_{\bar{\gamma}} \circ I_{\bar{\gamma}} \circ \psi = \widetilde{g} \circ \psi = \widetilde{f}$ and consequently $\widetilde{f} \in H_{\mathrm{u}}(X)$. So, it is clear that $\widetilde{f} = \widetilde{g} \circ \psi$ is a uniform extension of $f = g \circ \Pi$.

Now we are going to prove the claim. Let $\psi_{\bar{\gamma}}(I_\gamma(x)) := I_{\bar{\gamma}}(\psi(x))$ for $x \in X$. It is clear from the definition that $\psi_{\bar{\gamma}}(X_\gamma) \subseteq X_{F\bar{\gamma}}$. Given $x, y \in X$ such that $I_\gamma(x) = I_\gamma(y)$, we have $\gamma(\varphi(x) - \varphi(y)) = 0$. Let $\xi = \varphi(x) - \varphi(y) \in \gamma^{-1}(0)$. From $\Pi \circ \varphi = \varphi_F \circ \psi$ and $\overline{\gamma}(\Pi(\xi)) = 0$ we get

$$\overline{\gamma}[\varphi_F(\psi(x)) - \varphi_F(\psi(y))] = \overline{\gamma}[\Pi(\varphi(x) - \varphi(y))] = \overline{\gamma}(\Pi(\xi)) = 0$$

and so $I_{\bar{\gamma}}(\psi(x)) = I_{\bar{\gamma}}(\psi(y))$, i.e., $\psi_{\bar{\gamma}}(I_\gamma(x)) = \psi_{\bar{\gamma}}(I_\gamma(y))$. To prove the continuity of $\psi_{\bar{\gamma}}$ we take $x_\gamma = I_\gamma(x) \in X_\gamma$ and an arbitrary open neighborhood $V_{\bar{\gamma}}$ of $\psi_{\bar{\gamma}}(x_\gamma)$. We recall that $\overline{\gamma} \in I_{X_F}$. So there exist $r_1 > 0$ and an open neighborhood $V_{\bar{\gamma}}^{r_1}$ of $\psi_{\bar{\gamma}}(x_\gamma)$ such that $V_{\bar{\gamma}}^{r_1} \sim B_{\bar{\gamma}}(\varphi_{F\bar{\gamma}} \circ \psi_{\bar{\gamma}}(x_\gamma), r_1)$ and $V_{\bar{\gamma}}^{r_1} \subseteq V_{\bar{\gamma}}$; it is clear that for all $s \leq r_1$ there exists an open neighborhood $V_{\bar{\gamma}}^s$ of $\psi_{\bar{\gamma}}(x_\gamma)$ such that $V_{\bar{\gamma}}^s \subseteq V_{\bar{\gamma}}^{r_1}$ and $V_{\bar{\gamma}}^s \sim B_{\bar{\gamma}}(\varphi_{F\bar{\gamma}} \circ \psi_{\bar{\gamma}}(x_\gamma), s)$. Since $\psi(x) \in X_F$ there exist $r_2 > 0$ and an open neighborhood $U_{\bar{\gamma}}^{r_2}$ of $\psi(x)$ such that $U_{\bar{\gamma}}^{r_2} \sim B_{\bar{\gamma}}(\varphi_F(\psi(x)), r_2)$; for all $s \leq r_2$ there is an open neighborhood $U_{\bar{\gamma}}^s$ of $\psi(x)$ so that $U_{\bar{\gamma}}^s \subseteq U_{\bar{\gamma}}^{r_2}$ and $U_{\bar{\gamma}}^s \sim B_{\bar{\gamma}}(\varphi_F(\psi(x)), s)$. Let $r = \min\{r_1, r_2\}$. Then $U_{\bar{\gamma}}^r$ is the open neighborhood of $\psi(x)$ such that $U_{\bar{\gamma}}^r \sim B_{\bar{\gamma}}(\varphi_F(\psi(x)), r))$. Now,

$$\begin{aligned} i_{\bar{\gamma}}[B_{\bar{\gamma}}(\varphi_F(\psi(x)), r)] &= B_{\bar{\gamma}}(i_{\bar{\gamma}} \circ \varphi_F(\psi(x)), r) = B_{\bar{\gamma}}(\varphi_{F\bar{\gamma}}(I_{\bar{\gamma}} \circ \psi(x)), r) \\ &= B_{\bar{\gamma}}(\varphi_{F\bar{\gamma}}(\psi_{\bar{\gamma}}(x_\gamma)), r) \end{aligned}$$

implies $I_{\bar{\gamma}}(U_{\bar{\gamma}}^r) = V_{\bar{\gamma}}^r$. On the other hand, given $B_\gamma(\varphi(x), r)$ it is clear that $\Pi(B_\gamma(\varphi(x), r)) = B_{\bar{\gamma}}(\Pi(\varphi(x)), r) = B_{\bar{\gamma}}(\varphi_F(\psi(x)), r)$. Since $\gamma \in I_X$ there are $r_0 \leq r$ and an open neighborhood W of x in X such that $\varphi(W) \sim B_\gamma(\varphi(x), r_0)$. As $\Pi \circ \varphi = \varphi_F \circ \psi$ it follows that

$$\begin{aligned} \psi(W) &= (\varphi_F/U_{\bar{\gamma}}^r)^{-1} \circ \Pi \circ \varphi(W) = (\varphi_F/U_{\bar{\gamma}}^r)^{-1}(B_{\bar{\gamma}}(\varphi_F(\psi(x)), r_0)) \\ &\subseteq (\varphi_F/U_{\bar{\gamma}}^r)^{-1}(B_{\bar{\gamma}}(\varphi_F(\psi(x)), r)) = U_{\bar{\gamma}}^r. \end{aligned}$$

So $I_\gamma(W)$ is an open subset of X_γ containing $I_\gamma(x)$ such that $\psi_{\bar{\gamma}}(I_\gamma(W)) = I_{\bar{\gamma}}(\psi(W)) \subseteq V_{\bar{\gamma}}^r \subseteq V_{\bar{\gamma}}$ and we have the continuity of $\psi_{\bar{\gamma}}$.

Finally, $\psi_{\bar{\gamma}}$ is holomorphic if there exists a holomorphic mapping $\Pi_{\bar{\gamma}} : E_\gamma \to (E/F)_{\bar{\gamma}}$ satisfying $\Pi_{\bar{\gamma}} = \varphi_{F\bar{\gamma}} \circ \psi_{\bar{\gamma}} \circ (\varphi_\gamma/V)^{-1}$ for every chart V of X_γ. Define $\Pi_{\bar{\gamma}}(i_\gamma(x)) := i_{\bar{\gamma}}(\Pi(x)) \in (E/F)_{\bar{\gamma}}$ for $x \in X$. It is clear that

$\Pi_{\bar{\gamma}}$ is a well defined mapping from E_γ onto $(E/F)_{\bar{\gamma}}$. The linearity of $\Pi_{\bar{\gamma}}$ follows from the linearity of i_γ, $i_{\bar{\gamma}}$ and Π. Now, for all $i_\gamma(x) \in E_\gamma$ we have $\overline{\gamma}[\Pi_{\bar{\gamma}}(i_\gamma(x))] = \overline{\gamma}[i_{\bar{\gamma}}(\Pi(x))] = \overline{\gamma}(\Pi(x)) \leq \gamma(x) = \gamma(i_\gamma(x))$ and consequently $\Pi_{\bar{\gamma}}$ is continuous. (We remark that we denote by γ the norm in E_γ associated with γ since $\inf\{\gamma(x+y) : y \in \gamma^{-1}(0)\} = \gamma(x)$; analogously for $\overline{\gamma}$.) Since $\Pi_{\bar{\gamma}}$ is a continuous linear mapping, it is holomorphic. It is easy to verify that $\Pi_{\bar{\gamma}} \circ \varphi_\gamma = \varphi_{F\bar{\gamma}} \circ \psi_{\bar{\gamma}}$ and this completes the proof.

Example 10. The morphism $j' : U \to \varepsilon_{\mathrm{u}}(U)$ defined by $j'(u) := \widehat{u}$ is a uF-extension of U. Indeed, in Example 5 we define $(\psi(\varepsilon_{\mathrm{u}}(U)), q_\Pi, \psi)$ and prove that it is an F-quotient of uniform type of $(\varepsilon_{\mathrm{u}}(U), q)$ such that $\psi(\varepsilon_{\mathrm{u}}(U)) \subseteq \varepsilon_{\mathrm{u}}(\Pi(U))$. Since $j_\Pi : \Pi(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ is a uniform extension of $\Pi(U)$ such that $j_\Pi(\Pi(U)) \subseteq \psi(\varepsilon_{\mathrm{u}}(U))$, it is easy to show that j_Π is a uF-extension of $\Pi(U)$. From the definitions it is also clear that $\psi \circ j' = j_\Pi \circ \Pi$.

Definition 11. Let (X, φ) be a Riemann domain of uniform type over E. A morphism $\gamma : U \to X$ is said to be an *envelope of* uF*-holomorphy* of U if:

(a) γ is a uF-extension of U.

(b) If $\mu : U \to Z$ is a uF-extension of U, then there is a morphism $\nu : Z \to X$ such that $\nu \circ \mu = \gamma$.

It is clear that if $\gamma : U \to X$ and $\gamma' : U \to X'$ are two envelopes of uF-holomorphy of U then the Riemann domains X and X' are isomorphic. In other words, the envelope of uF-holomorphy of U, if it exists, is unique up to isomorphism.

Theorem 12. *Let U be a connected uniformly open subset of E and let $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$ be the pull-back of $(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi)$. Then the mapping $\gamma : U \to \varepsilon_{\mathrm{u}}^*(U)$ defined by $\gamma(u) := (\widehat{\Pi(u)}, u)$ for $u \in U$ is an envelope of* uF*-holomorphy of U.*

Proof. It is clear that $\varphi^* \circ \gamma = i_U$ where $i_U : U \to E$ is the inclusion. So, γ is a morphism if it is continuous. Given $u \in U$, take a neighborhood of $(\widehat{\Pi(u)}, u)$ in $\varepsilon_{\mathrm{u}}^*(U)$ of the form $(V \times W) \cap \varepsilon_{\mathrm{u}}^*(U)$ where V is a neighborhood of $\widehat{\Pi(u)}$ in $\varepsilon_{\mathrm{u}}(\Pi(U))$ and W is a neighborhood of u in E. Without loss of generality, we can suppose $W \subset U$. Since $j_\Pi : \Pi(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ is an extension of $\Pi(U)$ there is an open set $V_1 \subset \Pi(U)$ such that $\Pi(u) \in V_1$ and $j_\Pi(V_1) \subseteq V$. Let $V_2 := W \cap \Pi^{-1}(V_1)$. It is clear that for every $a \in V_2$ we have $\gamma(a) \in (V \times W) \cap \varepsilon_{\mathrm{u}}^*(U)$ and this gives the continuity of γ in u. From Example 6, $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$ is a Riemann domain of uniform type over E and $(\varepsilon_{\mathrm{u}}(\Pi(U)), q_\Pi, \psi)$ is an F-quotient of uniform type of $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$. Since $j_\Pi : \Pi(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ is a uniform extension of $\Pi(U)$ (cf. [9]) and clearly $\psi \circ \gamma = j_\Pi \circ \Pi$, it follows that $(\varepsilon_{\mathrm{u}}^*(U), \varphi^*)$ is a uF-extension of U.

Now, if (Z, ϱ) is a Riemann domain of uniform type over E and $\mu : U \to Z$ is a uF-extension of U there are an F-quotient of uniform type (Z_F, ϱ_F, ψ_F) of Z and $\mu_\Pi : \Pi(U) \to Z_F$ such that $\psi_F \circ \mu = \mu_\Pi \circ \Pi$ and μ_Π is a uniform extension of $\Pi(U)$. From the maximality of $\varepsilon_{\mathrm{u}}(\Pi(U))$ (cf. [9]) there is a morphism $\mu_F : Z_F \to \varepsilon_{\mathrm{u}}(\Pi(U))$ such that $\mu_F \circ \mu_\Pi = j_\Pi$. We define $\nu : Z \to \varepsilon^*_{\mathrm{u}}(U)$ by $\nu(z) := ((\mu_F \circ \psi_F)(z), \varrho(z))$. Since $(q_\Pi \circ \mu_F \circ \psi_F)(z) = \Pi \circ \varrho(z)$, we have $\nu(z) \in \varepsilon^*_{\mathrm{u}}(U)$ for every $z \in Z$. It is easy to verify that ν is a morphism and $\nu \circ \mu = \gamma$.

Remark 13. We have the following generalization: Let G be a complete Hausdorff locally convex space and $f \in H_{\mathrm{u}}(U, G)$ such that $f = g \circ \Pi$ where $g \in H_{\mathrm{u}}(\Pi(U), G)$. From Theorem 2.5 of [9], there exists a uniform extension $\widetilde{g} : \varepsilon_{\mathrm{u}}(\Pi(U)) \to G$ of g. If $\widetilde{f} := \widetilde{g} \circ \psi$, where $\psi : \varepsilon^*_{\mathrm{u}}(U) \to \varepsilon_{\mathrm{u}}(\Pi(U))$ is defined as in Example 6, then a small change in the argument used in Remark 9 shows that $\widetilde{f}$ is a uniform extension of f.

Finally, we establish the relation between $\varepsilon^*_{\mathrm{u}}(U)$ and $\varepsilon_{\mathrm{u}}(U)$.

Remark 14. There exists a morphism $\delta : \varepsilon_{\mathrm{u}}(U) \to \varepsilon^*_{\mathrm{u}}(U)$ satisfying $\delta \circ j' = \gamma$ (where γ is defined in Theorem 12 and j' in Example 10).

Proof. From Example 10 we know that j' is a uF-extension of U. Since, by Theorem 12, $\gamma : U \to \varepsilon^*_{\mathrm{u}}(U)$ is an envelope of uF-holomorphy of U, the existence of such δ follows from the maximality of $\varepsilon^*_{\mathrm{u}}(U)$.

References

[1] R. Aron, L. Moraes and R. Ryan, *Factorization of holomorphic mappings in infinite dimensions*, Math. Ann. 277 (1987), 617–628.

[2] S. Dineen, *Complex Analysis in Locally Convex Spaces*, North-Holland Math. Stud. 57, North-Holland, Amsterdam, 1981.

[3] P. Hilton, *Tópicos de Álgebra Homológica*, 8º Colóquio Brasileiro de Matemática, IME–Universidade de São Paulo, Brasil, 1971.

[4] L. Moraes, O. W. Paques and M. C. F. Zaine, *F-quotients and envelope of F-holomorphy*, J. Math. Anal. Appl. 163 (2) (1992), 393–405.

[5] J. Mujica, *Domain of holomorphy in (DFC)-spaces*, in: Functional Analysis, Holomorphy and Approximation Theory, Lecture Notes in Math. 843, Springer, Berlin, 1980, 500–533.

[6] L. Nachbin, *Uniformité d'holomorphie et type exponentiel*, in: Séminaire P. Lelong 1970, Lectures Notes in Math. 205, Springer, Berlin, 1971, 216–224.

[7] —, *Recent developments in infinite dimensional holomorphy*, Bull. Amer. Math. Soc. 79 (1973), 625–640.

[8] —, *On pure uniform holomorphy in spaces of holomorphic germs*, Results in Math. 8 (1985), 117–122.

[9] O. W. Paques and M. C. Zaine, *Uniformly holomorphic continuation*, J. Math. Anal. Appl. 123 (2) (1987), 448–454.

[10] M. Schottenloher, *The Levi problem for domains spread over locally convex spaces with a finite dimensional Schauder decomposition*, Ann. Inst. Fourier (Grenoble) 26 (4) (1976), 207–237.

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