Generic properties of generalized hyperbolic partial differential equations

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Abstract. The existence and uniqueness of solutions and convergence of successive approximations are considered as generic properties for generalized hyperbolic partial differential equations with unbounded right-hand sides.

1. Introduction. In this note we consider the Darboux problem

\[
\begin{aligned}
& u_{xy} = f(x, y, u, u_x, u_y, u_{xy}), \\
& u(0, y) = \psi(y), \quad u(x, 0) = \phi(x),
\end{aligned}
\]

where \( f : [0, a] \times [0, b] \times \mathbb{R}^4 \to \mathbb{R} \) is a continuous function, \( \phi : [0, a] \to \mathbb{R} \) and \( \psi : [0, b] \to \mathbb{R} \) are continuously differentiable on \([0, a]\) and \([0, b]\) respectively and satisfy \( \phi(0) = \psi(0) \). The family of all pairs of functions \((\phi, \psi)\) satisfying the above conditions is denoted by \( Z \).

By a solution of the problem (1) we mean a continuous function \( u : [0, a] \times [0, b] \to \mathbb{R} \) with continuous derivatives \( u_x, u_y, u_{xy} = u_{yx} \) which satisfies (1) for \( x \in [0, a] \) and \( y \in [0, b] \).

The problem (1) was investigated by Goebel [6]. Using Darbo’s fixed point theorem for \( \alpha \)-contractions [4] and Bielecki’s norms [2] he showed that under the additional assumptions that \( f \) is bounded and satisfies the Lipschitz condition

\[
|f(x, y, u, p, q, s) - f(x, y, u, \overline{p}, \overline{q}, \overline{s})| \leq M|p - \overline{p}| + N|q - \overline{q}| + k|s - \overline{s}|,
\]

where \( k < 1 \), the problem (1) has at least one solution.

Much attention was paid to the quasilinear hyperbolic equation

\[
\begin{aligned}
& u_{xy} = f(x, y, u, u_x, u_y), \\
& u(0, y) = \psi(y), \quad u(x, 0) = \phi(x), \quad x \in [0, a], \quad y \in [0, b],
\end{aligned}
\]

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where \((\phi, \psi) \in \mathcal{Z}\) (see [2] and [7]). It is well known that the continuity of \(f\) is not sufficient to guarantee the existence of solution of the problem (2). However, Costello [3] proved that the existence, uniqueness, and continuous dependence of the solution for the problem (2) is a generic property in the space \(\mathcal{H}\) of all continuous bounded \(f : [0, a] \times [0, b] \times \mathbb{R}^3 \rightarrow \mathbb{R}\) endowed with the norm of uniform convergence. This result is a generalization of an earlier paper of Alexiewicz and Orlicz [1]. Costello’s method of proof is similar to that of Lasota and Yorke [8]. His result was strengthened by De Blasi and Myjak [5] who proved that the convergence of successive approximations is a generic property in \(\mathcal{H}\). In our approach, applying Bielecki’s norms, we are able to study generic properties of the problem (1) in two cases: with unbounded right-hand sides under the metric of uniform convergence on bounded sets or with bounded right-hand sides under the norm of uniform convergence.

2. Results. The space of all continuous \(v : [0, a] \times [0, b] \rightarrow \mathbb{R}\) is denoted by \(\mathcal{C}\). The family of Bielecki’s norms in \(\mathcal{C}\) for \(\lambda \geq 0\) is defined by

\[
\|v\|_{\lambda} = \sup \{ \exp(-\lambda(x + y)) | v(x, y) | : x \in [0, a], y \in [0, b] \}.
\]

Note that all these norms are equivalent.

We associate with every continuous \(f : [0, a] \times [0, b] \times \mathbb{R}^4 \rightarrow \mathbb{R}\) and \((\phi, \psi) \in \mathcal{Z}\) the continuous mapping \(\hat{f}_{\phi, \psi} : \mathcal{C} \rightarrow \mathcal{C}\) and its successive approximations given as follows:

\[
\hat{f}_{\phi, \psi}^1(v)(x, y) = f(x, y, \int_0^x \int_0^y v(\xi, \eta) d\xi d\eta + \phi(x) + \psi(y) - \phi(0),
\]

\[
\int_0^y v(x, \eta) d\eta + \phi'(x), \int_0^x v(\xi, y) d\xi + \psi'(y) + v(x, y))
\),

\[
\hat{f}_{\phi, \psi}^{i+1}(v) = \hat{f}_{\phi, \psi}^1(\hat{f}_{\phi, \psi}^i(v)), \quad v \in \mathcal{C}.
\]

Putting \(v(x, y) = u_{xy}(x, y)\) we can now write the problem (1) in an equivalent form

\[
\hat{f}_{\phi, \psi}^1(v) = v, \quad v \in \mathcal{C}.
\]

By \((\mathcal{F}, d)\) we denote the complete metric space of all continuous \(f : [0, a] \times [0, b] \times \mathbb{R}^4 \rightarrow \mathbb{R}\) satisfying the condition

\[
|f(x, y, u, p, q, s) - f(x, y, u, p, q, \bar{s})| \leq |s - \bar{s}|
\],

for \(x \in [0, a], y \in [0, b], u, p, q, s, \bar{s} \in \mathbb{R}\), endowed with the metric of uniform convergence on bounded sets given by
\[d(f, g) = \sum_{n=1}^{\infty} 2^{-n} \sup \left\{ \frac{|f(x, y, u, p, q, s) - g(x, y, u, p, q, s)|}{1 + |f(x, y, u, p, q, s) - g(x, y, u, p, q, s)|} : x \in [0, a], \ y \in [0, b], \ |u|, |p|, |q|, |s| \leq n \right\}.

Recall that a subset of a metric space is said to be residual if its complement is of the first category.

**Theorem 1.** Let \(F_0\) be the subset of all \(f \in F\) such that (i) for every \((\phi, \psi) \in Z\) the problem (1) (or (3)) has exactly one solution; (ii) for every \(v \in C\) and \((\phi, \psi) \in Z\) the sequence \(\hat{f}_{\phi, \psi}^i(v)\) is convergent in \(C\) as \(i \to \infty\). Then \(F_0\) is a residual set in the space \((F, d)\).

**Proof.** Let \(G\) be the subset of all \(g \in F\) satisfying the following Lipschitz condition:

\[|g(x, y, u, p, q, s) - g(x, y, \overline{p}, \overline{q}, \overline{s})| \leq L|u - \overline{u}| + M|p - \overline{p}| + N|q - \overline{q}| + k|s - \overline{s}|,
\]
for \(x \in [0, a], \ y \in [0, b], \ u, p, q, s, \overline{p}, \overline{q}, \overline{s} \in \mathbb{R}\), where \(L, M, N, k\) are some constants and \(k < 1\).

We will see that for \(g \in G\) there exist constants \(K(g) < 1\) and \(\lambda(g) \geq 0\) such that

\[\|\hat{g}_{\phi, \psi}^1(v) - \hat{g}_{\phi, \psi}^1(w)\|_{\lambda(g)} \leq K(g)\|v - w\|_{\lambda(g)}, \quad v, w \in C, \ (\phi, \psi) \in Z.
\]

In fact, from (4) we get

\[\exp(-\lambda(x + y))\|\hat{g}_{\phi, \psi}^1(v)(x, y) - \hat{g}_{\phi, \psi}^1(w)(x, y)\|
\]
\[\leq \exp(-\lambda(x + y))\left( L \int_0^x \int_0^y |v(\xi, \eta) - w(\xi, \eta)| \, d\xi \, d\eta \right.
\]
\[+ M \int_0^y |v(x, \eta) - w(x, \eta)| \, d\eta + N \int_0^x |v(\xi, y) - w(\xi, y)| \, d\xi + k|v(x, y) - w(x, y)|
\]
\[\leq \exp(-\lambda(x + y))\left( L \int_0^x \int_0^y \exp(\lambda(\xi + \eta)) \, d\xi \, d\eta + M \int_0^y \exp(\lambda(x + \eta)) \, d\eta + N \int_0^x \exp(\lambda(\xi + y)) \, d\xi + k \exp(\lambda(x + y))\right)\|v - w\|_{\lambda}
\]
\[\leq (2L\lambda^{-2} + (M + N)\lambda^{-1} + k)\|v - w\|_{\lambda},
\]
Note that \( \gamma_n : \mathbb{R}^4 \to [0, \infty) \) be such that \( \int_{\mathbb{R}^4} \gamma_n \, d\mu = 1 \) and \( \sup \{ |z| : \gamma_n(z) > 0 \} \to 0 \) as \( n \to \infty \). For \( n = 1, 2, \ldots \) let continuously differentiable functions \( \gamma_n \in \mathcal{F} \) be such that \( \int_{\mathbb{R}^4} \gamma_n \, d\mu = 1 \) and \( \sup \{ |z| : \gamma_n(z) > 0 \} \to 0 \) as \( n \to \infty \). For \( n = 1, 2, \ldots \) consider the nonexpansive retractions \( r_n : \mathbb{R} \to [-n, n] \) given by

\[
r_n(u) = \begin{cases} u & \text{if } |u| \leq n, \\
-1000 & \text{if } |u| > n.
\end{cases}
\]

Define \( g_n : [0, a] \times [0, b] \times \mathbb{R}^4 \to \mathbb{R} \) by

\[
g_n(x, y, u, p, q, s) = \frac{n}{n+1} \int_{\mathbb{R}^4} f(x, y, r_n(\overline{u}), r_n(\overline{p}), r_n(\overline{q}), r_n(\overline{s})) \gamma_n(u - \overline{u}, p - \overline{p}, q - \overline{q}, s - \overline{s}) \, d\mu.
\]

One checks that \( d(f, g_n) \to 0 \) as \( n \to \infty \) and \( g_n \) satisfies (4) with some constants \( L, M, N, \) and \( k = n/(n+1) \).

We denote by \( Z_n \) the subset of all pairs \( (\phi, \psi) \in \mathcal{Z} \) such that \( |\phi(x)|, |\phi'(x)| \leq n \) for \( x \in [0, a] \) and \( |\psi(y)|, |\psi'(y)| \leq n \) for \( y \in [0, b] \).

Note that \( \bigcup_{n=1}^{\infty} Z_n = \mathcal{Z} \). Put

\[
\mathcal{B}_\lambda(0, R) = \{ v \in \mathcal{C} : \|v\|_\lambda \leq R \}.
\]

For every \( g \in \mathcal{G} \) and \( n = 1, 2, \ldots \) define

\[
R(g, n) = (1 - K(g))^{-1} \left( nK(g) + G_n \right),
\]

where

\[
G_n = \sup \{ |g(x, y, u, p, q, s)| : x \in [0, a], \ y \in [0, b], \ |u| \leq 3n, \ |p|, |q| \leq n, \ s = 0 \}.
\]

We claim that for \( (\Phi, \Psi) \in Z_n \),

\[
\mathcal{B}_{\lambda(n)}^\Delta(0, R(g, n) + n)) \subset \mathcal{B}_{\lambda(n)}(0, R(g, n)).
\]

In fact, if \( \|v\|_{\lambda(n)} \leq R(g, n) + n \) and \( (\Phi, \Psi) \in Z_n \), then from (5) and (6) we obtain

\[
\|g_{\Phi, \Psi}(v)\|_{\lambda(n)} \leq \|g_{\Phi, \Psi}(v) - g_{\Phi, \Psi}(0)\|_{\lambda(n)} + \|g_{\Phi, \Psi}(0)\|_{\lambda(n)} \\
\leq K(g)\|v\|_{\lambda(n)} + \|g_{\Phi, \Psi}(0)\|_{\lambda(n)} \leq K(g)(R(g, n) + n) + G_n \\
= R(g, n).
\]

Since \( d \) is the metric of uniform convergence on bounded sets, it is possible to find for every \( g \in \mathcal{G} \) and \( n = 1, 2, \ldots \) a positive number \( \varepsilon(g, n) \) such that for \( f \in \mathcal{F} \), \( d(f, g) < \varepsilon(g, n) \) and for \( x \in [0, a], \ y \in [0, b] \),

\[
|u| \leq ab\exp(\lambda(g)(a + b))(R(g, n) + n) + 3n,
\]
we have
\[ |p| \leq b \exp(\lambda(g)(a + b))(R(g, n) + n) + n, \]
\[ |q| \leq a \exp(\lambda(g)(a + b))(R(g, n) + n) + n, \]
\[ |s| \leq \exp(\lambda(g)(a + b))(R(g, n) + n), \]
This means that for \( g \in G \), \( d(f, g) < \varepsilon(g, n) \) we have
\[ v \in B_{\lambda(g)}(0, R(g, n) + n), \quad (\phi, \psi) \in \mathbb{Z}_n \]
\[ \Rightarrow \| \hat{f}^1_{\phi, \psi}(v) - \hat{g}^1_{\phi, \psi}(v) \|_0 \leq (3n)^{-1}(1 - K(g)) \exp(-\lambda(g)(a + b)). \]
Now we show that
\[ \bigcap_{n=1}^{\infty} \bigcup_{g \in G} B(g, \varepsilon(g, n)) \subset \mathcal{F}_0, \]
where \( B(g, \varepsilon(g, n)) \) denotes the open ball in \( \mathcal{F} \) with center \( g \) and radius \( \varepsilon(g, n) \).
Suppose that \( f \) belongs to the left side of the inclusion (9). This implies that for every \( n = 1, 2, \ldots \) there exists \( g_n \in G \) satisfying \( d(g_n, f) < \varepsilon(g_n, n) \).
First, we prove that the equation (3) for \((\phi, \psi) \in \mathbb{Z}\) has at most one solution in \( \mathcal{C} \). To this end it suffices to check that \( I - \hat{f}^1_{\phi, \psi} \) is injective (\( I \)
\[ \| (I - \hat{f}^1_{\phi, \psi})(v) - (I - \hat{f}^1_{\phi, \psi})(w) \|_{g_n, n} \]
\[ \geq \| v - w \|_{g_n, n} - \| \hat{g}^1_{\phi, \psi}(v) - \hat{f}^1_{\phi, \psi}(w) \|_{g_n, n} \]
\[ \geq (1 - K(g_n)) \| v - w \|_{g_n, n} - 2(3n)^{-1}(1 - K(g_n)) \exp(-\lambda(g_n)(a + b)) \]
\[ \geq (3n)^{-1}(1 - K(g_n)) \exp(-\lambda(g_n)(a + b)) > 0. \]
Since \( n \) can be chosen arbitrarily large, for \( \| v \|_0 > 0 \) and \((\phi, \psi) \in \mathbb{Z}\) we get
\[ (I - \hat{f}^1_{\phi, \psi})(v) \neq (I - \hat{f}^1_{\phi, \psi})(w), \]
so \( I - \hat{f}^1_{\phi, \psi} \) is injective.
Now we prove that the sequence of successive approximations \( \hat{f}^1_{\phi, \psi}(v) \)
is convergent in \( \mathcal{C} \) as \( i \to \infty \) for all \( v \in \mathcal{C} \) and \((\phi, \psi) \in \mathbb{Z}\). To this end fix \( \phi, \psi \) and \( v \) and consider any positive integer \( n \) satisfying \((\phi, \psi) \in \mathbb{Z}_n\) and \( \| v \| \leq n \). Observe that
\[ \hat{f}^1_{\phi, \psi}(B_{\lambda(g_n)}(0, R(g, n) + n)) \subset B_{\lambda(g_n)}(0, R(g, n) + n). \]
In fact, for \( \|w\|_{\lambda(g_n)} \leq R(g_n, n) + n \) from (7) and (8) we obtain
\[
\begin{align*}
\|\hat{f}_{\phi, \psi}^i(w)\|_{\lambda(g_n)} & \leq \|\hat{f}_{\phi, \psi}^i(w) - \hat{g}_{\phi, \psi}^i(w)\|_{\lambda(g_n)} + \|\hat{g}_{\phi, \psi}^i(w)\|_{\lambda(g_n)} \\
& \leq (3n)^{-1}(1 - K(g_n)) \exp(-\lambda(g_n)(a + b)) + R(g_n, n) < R(g_n, n) + n.
\end{align*}
\]
Since \( v \in B_0(0, n) \subset B_{\lambda(g_n)}(0, R(g_n, n) + n) \), by (10) we see that for every \( i = 1, 2, \ldots \),
\begin{equation}
\hat{f}_{\phi, \psi}^i(v) \in B_{\lambda(g_n)}(0, R(g_n, n) + n).
\end{equation}

Now we show by induction that for every positive integer \( i \) we have
\begin{equation}
\|\hat{f}_{\phi, \psi}^{i+1}(v) - \hat{g}_{\phi, \psi}^{i+1}(v)\|_{\lambda(g_n)} \leq (3n)^{-1}(1 - K(g_n)^i) \exp(-\lambda(g_n)(a + b)).
\end{equation}
For \( i = 1 \), (12) follows from (8). If (12) is true for a fixed \( i \), then by (5), (8) and (11) we have
\[
\begin{align*}
\|\hat{f}_{\phi, \psi}^{i+1}(v) - \hat{g}_{\phi, \psi}^{i+1}(v)\|_{\lambda(g_n)} & \leq \|\hat{f}_{\phi, \psi}^{i+1}(v) - \hat{f}_{\phi, \psi}^{i}(v)\|_{\lambda(g_n)} \\
& + \|\hat{f}_{\phi, \psi}^{i}(v) - \hat{g}_{\phi, \psi}^{i}(v)\|_{\lambda(g_n)} \\
& \leq (3n)^{-1}(1 - K(g_n) + (3n)^{-1}K(g_n)(1 - K(g_n)^i)) \exp(-\lambda(g_n)(a + b)) \\
& = (3n)^{-1}(1 - K(g_n)^{i+1}) \exp(-\lambda(g_n)(a + b)),
\end{align*}
\]
so (12) is true for \( i + 1 \).

From (5) and (12) it follows that
\[
\begin{align*}
\|\hat{f}_{\phi, \psi}^i(v) - \hat{f}_{\phi, \psi}^{i+j}(v)\|_0 & \leq \exp(\lambda(g_n)(a + b)) \|\hat{f}_{\phi, \psi}^i(v) - \hat{f}_{\phi, \psi}^{i+j}(v)\|_{\lambda(g_n)} \\
& \leq \exp(\lambda(g_n)(a + b)) \|\hat{f}_{\phi, \psi}^i(v) - \hat{f}_{\phi, \psi}^{i+j}(v)\|_{\lambda(g_n)} \\
& + \|\hat{f}_{\phi, \psi}^{i+j}(v) - \hat{g}_{\phi, \psi}^{i+j}(v)\|_{\lambda(g_n)} + \|\hat{g}_{\phi, \psi}^{i+j}(v) - \hat{g}_{\phi, \psi}^{i+j}(v)\|_{\lambda(g_n)} \\
& \leq 2(3n)^{-1} + K(g_n)^i(1 - K(g_n))^{-1} \|v - \hat{g}_{\phi, \psi}^{i+j}(v)\|_{\lambda(g_n)}.
\end{align*}
\]
Therefore, for sufficiently large \( i \) and every \( j \in \mathbb{N} \) we have
\[
\|\hat{f}_{\phi, \psi}^i(v) - \hat{f}_{\phi, \psi}^{i+j}(v)\|_0 < 1/n.
\]
Since one can choose \( n \) arbitrarily large, the sequence \( \{\hat{f}_{\phi, \psi}^i(v)\} \) is Cauchy in the Banach space \( \mathcal{C} \), so it is convergent. As \( \hat{f}_{\phi, \psi}^1 : \mathcal{C} \to \mathcal{C} \) is continuous it has at least one fixed point \( \lim_{i \to \infty} \hat{f}_{\phi, \psi}^i(v) \).

From (9) it follows that \( \mathcal{F}_0 \) contains a dense \( G_\delta \) subset of \( \mathcal{F} \), so \( \mathcal{F}_0 \) is a residual subset of \( \mathcal{F} \). This completes the proof.
Remark. Theorem 1 is not true when the metric \( d \) is replaced by the metric \( \tilde{d} \) of uniform convergence given as follows:

\[
\tilde{d}(f, g) = \sup \left\{ \frac{|f(x, y, u, p, q, s) - g(x, y, u, p, q, s)|}{1 + |f(x, y, u, p, q, s) - g(x, y, u, p, q, s)|} : \right. \\
x \in [0, a], \ y \in [0, b], \ u, p, q, s \in \mathbb{R} \left. \right\}.
\]

In fact, consider the open ball \( B(f_0, 1/2) \) in \((\mathcal{F}, \tilde{d})\), where \( f_0(x, y, u, p, q, s) = s + 1 \). One can check that for every \( f \in B(f_0, 1/2) \) there is no solution of the equation (3).

Now consider the Banach space \((\tilde{\mathcal{F}}, \| \cdot \|_\infty)\) of all bounded \( f \in \mathcal{F} \) with the norm of uniform convergence \( \| f \|_\infty = \sup \{ |f(x, y, u, p, q, s)| : x \in [0, a], \ y \in [0, b], \ u, p, q, s \in \mathbb{R} \} \).

Theorem 2. Let \( \tilde{\mathcal{F}}_0 \) be the subset of all \( f \in \tilde{\mathcal{F}} \) such that (i) for every \( (\phi, \psi) \in \mathcal{Z} \) the problem (1) (or (3)) has exactly one solution; (ii) for every \( v \in \mathcal{C} \) and \( (\phi, \psi) \in \mathcal{Z} \) the sequence \( \hat{f}_i^{\phi, \psi}(v) \) is convergent in \( \mathcal{C} \) as \( i \to \infty \). Then \( \tilde{\mathcal{F}}_0 \) is a residual set in the Banach space \((\tilde{\mathcal{F}}, \| \cdot \|_\infty)\).

Proof. Let \( \tilde{G} \) be the subset of all \( g \in \tilde{\mathcal{F}} \) satisfying, on every bounded subset of \([0, a] \times [0, b] \times \mathbb{R}^4\), the Lipschitz condition (4) with some constants \( k < 1, L, M, N \). We show that \( \tilde{G} \) is dense in \( \tilde{\mathcal{F}} \). Suppose that \( f \in \tilde{\mathcal{F}} \) and \( \varepsilon > 0 \). As we have seen in the proof of Theorem 1, for every positive integer \( n \) there exists \( f_n \in G \) such that

\[
\sup \{ |f(x, y, u, p, q, s) - f_n(x, y, u, p, q, s)| : x \in [0, a], \ y \in [0, b], \ u, p, q, s \leq n \} < 2^{-n} \varepsilon.
\]

Let us find continuously differentiable \( \beta_m : \mathbb{R}^3 \to [0, \infty) \) such that \( \text{supp}(\beta_1) \subset B(0, 1), \text{supp}(\beta_m) \subset B(0, m) - B(0, m-2) \) for \( m \geq 2 \) and \( \sum_{m=1}^{\infty} \beta_m(z) = 1 \) for \( z \in \mathbb{R}^3 \). Define \( h_n : [0, a] \times [0, b] \times \mathbb{R}^4 \to \mathbb{R} \) for \( n = 1, 2, \ldots \) by

\[
h_n(x, y, u, p, q, s) = \sum_{m=1}^{\infty} \beta_m(u, p, q)f_{n+m}(x, y, u, p, q, s).
\]

Note that for \( n = 1, 2, \ldots \),

\[
\sup \{ |h_n(x, y, u, p, q, s) - f(x, y, u, p, q, s)| : x \in [0, a], \ y \in [0, b], \ u, p, q \in \mathbb{R}, \ |s| \leq n \} \leq 2^{-n-1} \varepsilon.
\]

A function \( g \in \tilde{G} \) such that \( \| f - g \|_\infty < \varepsilon \) may be given by
\[ g(x, y, u, p, q, s) = h_{|s|+1}(x, y, u, p, q, s) + \sum_{i=1}^{[s]} (h_i(x, y, u, p, q, i \text{ sgn}(s))) \\
- h_{i+1}(x, y, u, p, q, i \text{ sgn}(s)) \]

(we assume that the sum \( \sum_{i=1}^{[s]} \) vanishes for \( s \in (-1, 1) \)).

For every \( g \in \tilde{G} \) define

\[ R(g) = \sup \{ |g(x, y, u, p, q, s)| : x \in [0, a], \ y \in [0, b], \ u, p, q, s \in \mathbb{R} \}. \]

Notice that for \((\phi, \psi) \in \mathcal{Z}\) we have

\[ \| \tilde{g}_{\phi, \psi}(v) - \tilde{g}_{\phi, \psi}(w) \|_{\tilde{\lambda}(g)} \leq \tilde{K}(g) \| v - w \|_{\tilde{\lambda}(g)}. \]

We set

\[ \varepsilon(g, n) = (3n)^{-1}(1 - \tilde{K}(g)) \exp(-\tilde{\lambda}(g)(a + b)). \]

Now as in the proof of Theorem 1 one can show that

\[ \bigcap_{n=1}^\infty \bigcup_{g \in \tilde{G}} B(g, \varepsilon(g, n)) \subset \tilde{F}_0, \]

where \( B(g, \varepsilon(g, n)) = \{ f \in \tilde{F} : \| f \|_{\infty} \leq \varepsilon(g, n) \}. \) This completes the proof.

References


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