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INTEGRATED DESIGN OF OBSERVER BASED FAULT DETECTION FOR A CLASS OF UNCERTAIN NONLINEAR SYSTEMS

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Integrated design of observer based Fault Detection (FD) for a class of uncertain nonlinear systems with Lipschitz nonlinearities is studied. In the context of norm based residual evaluation, the residual generator and evaluator are designed together in an integrated form, and, based on it, a trade-off FD system is finally achieved in the sense that, for a given Fault Detection Rate (FDR), the False Alarm Rate (FAR) is minimized. A numerical example is given to illustrate the effectiveness of the proposed design method.

Keywords: fault detection, observers, nonlinear systems, optimization, robustness.

1. Introduction

The observer based fault detection and isolation technology is currently receiving much attention (Ding, 2008; Blanke *et al.*, 2003; Chen and Patton, 1999; Patton *et al.*, 2002). Generally speaking, an observer-based fault detection system consists of an observer-based residual generator and a residual evaluator. For linear systems, the observer-based FD technology has been well developed. Ding *et al.* (1993) as well as Qiu and Gertler (1993) proposed residual generator design schemes based on the H_∞-optimization technique. Further on, the so-called H_-/H_∞ design of residual generator, initiated by Ding *et al.* (1993) as well as Hou and Patton (1996), attracted lots of interest (Wang *et al.*, 2007; Henry and Zolghadri, 2005). Recently, Ding *et al.* (2000a) and Zhang *et al.* (2005) proposed a unified solution which solves the H_i/H_∞ (including H_-/H_∞ and H_∞/H_∞) optimization problem, where H_i represents all nonzero singular values of the transfer matrix from faults to the residual signal. For residual evaluation, there are two major strategies. One is statistic testing, which deals with the systems with stochastic behavior (Basseville and Nikiforov, 1993; Lai and Shan, 1999), and the other is norm based residual evaluation, which focuses on deterministic disturbance and model uncertainty (Frank and Ding, 1997).

In applications, an optimal trade-off between the False Alarm Rate (FAR) and Fault Detection Rate (FDR) is of practical interest in designing an FD system. Ding *et al.* (2000b) extended the FAR and FDR concepts from the statistic context to characterize the performance of an FD system with norm based residual evaluation. As the FAR and FDR depend not only on the performance of the residual generator but also on the residual evaluator, an optimal trade-off between the FAR and FDR requires an integrated design of the residual generator and residual evaluator, which optimizes the performance of the whole FD system. This trade-off problem has been formulated in two ways: (1) given the FDR, minimize the FAR (Zhang and Ding, 2008), (2) given the FAR, maximize the FDR (Ding *et al.*, 2000b).

On the other hand, since nonlinear systems are more common in practice, observer based FD techniques for nonlinear systems have also been studied extensively (Frank, 1994; Hammouri *et al.*, 1999; Ferrari *et al.*, 2007; Narasimhan *et al.*, 2007; Shumsky, 2007; Edelmayer *et al.*, 2004). There are many works dealing with Lipschitz nonlinear systems (Pertew *et al.*, 2007; Chen and Saif, 2007; Rajamani and Ganguli, 2004; Yaz and Azemi, 1998; de Souza *et al.*, 1993; Xie *et al.*, 1996; Abbaszadeh and Marquez, 2008), since, under some conditions, more general nonlinear systems can be transformed into Lipschitz nonlinear systems (Rajamani, 1998). In this paper, we extend the integrated FD system design approach which has been well developed for linear systems to uncertain Lipschitz nonlinear systems, based on the following formulation: Given the FDR, minimize the FAR.

The paper is organized as follows. After preliminaries addressed in Section 2, the problem of minimizing the FAR for a given FDR is formulated in Section 3. In Section 4, a solution to the integrated FD system design problem is proposed. Finally, in Section 5, the achieved results are illustrated with an example.

2. Preliminaries

Consider the following uncertain nonlinear systems:

$$\Sigma_{\mathscr{S}}:\begin{cases}\dot{x} &= \bar{A}x+\phi(x,u)+\bar{B}u+\bar{E}_d d+E_f f,\\ y &= \bar{C}x+\bar{D}u+\bar{F}_d d+F_f f,\end{cases} \tag{1}$$

where $x\in\mathbb{R}^n$ is the state vector, $u\in\mathbb{R}^m$ is the control input, $y\in\mathbb{R}^p$ is the output vector, $f\in\mathbb{R}^l$ is the fault vector to be detected, and $d\in\mathbb{R}^q$ is the unknown input vector. Moreover, the matrices $\bar{A}$, $\bar{B}$, $\bar{E}_d$, $\bar{C}$, $\bar{D}$, $\bar{F}_d$ in (1) are uncertain of the form $\bar{X}=X+\Delta X$, where $X\in\{A,B,E_d,C,D,F_d\}$ are known matrices with appropriate dimensions. Similarly, the matrices E_f and F_f are also known. $\Delta X\in\{\Delta A,\Delta B,\Delta E_d,\Delta C,\Delta D,\Delta F_d\}$ are norm bounded uncertainties and can be expressed as

$$\begin{bmatrix}\Delta A & \Delta B & \Delta E_d\\ \Delta C & \Delta D & \Delta F_d\end{bmatrix}=\begin{bmatrix}E\\ F\end{bmatrix}\Delta(t)\begin{bmatrix}G & H & K\end{bmatrix},$$

where E, F, G, H, K are known matrices with appropriate dimensions and $\Delta(t)$ is bounded by

$$\Delta(t)^T\Delta(t)\le I.$$

The nonlinear function $\phi(x,u)$ is assumed to be Lipschitz in x with a Lipschitz constant γ, i.e., $\forall x,\hat{x},u$:

$$\|\phi(x,u)-\phi(\hat{x},u)\|\le\gamma\|x-\hat{x}\|.$$

In addition, the following assumptions should be made throughout:

1. $A+\Delta A$ is asymptotically stable for all ΔA.
2. (C,A) is detectable.

The first assumption can be checked by the standard Lyapunov approach with LMI tools. The nonlinear observer based fault detection filter is designed as

$$\Sigma_{\mathscr{F}}:\begin{cases}\dot{\hat{x}}=A\hat{x}+\phi(\hat{x},u)+Bu+L(y-C\hat{x}-Du),\\ r=y-C\hat{x}-Du,\end{cases}$$

where $r\in\mathbb{R}^p$ is the residual signal and $L\in\mathbb{R}^{n\times p}$ is the observer gain. Denoting by $e=x-\hat{x}$ the estimation error, we have the following observer error dynamics:

$$\Sigma_{\mathscr{E}}:\begin{cases}\dot{e}=(A-LC)e+\Psi+(\Delta A-L\Delta C)x\\ \quad+(\Delta B-L\Delta D)u+(\bar{E}_d-L\bar{F}_d)d\\ \quad+(E_f-LF_f)f,\\ r=Ce+\Delta Cx+\Delta Du+\bar{F}_d d+F_f f,\end{cases} \tag{2}$$

where

$$\Psi=\phi(x,u)-\phi(\hat{x},u).$$

Combining the system (1) and the error dynamics (2), the residual generator dynamics are as follows:

$$\Sigma_{\mathscr{R}}:\begin{cases}\dot{x}_0=\bar{A}_0x_0+\Psi_0+\bar{E}_0d_0+E_{0,f}f,\\ r=\bar{C}_0x_0+\bar{F}_0d_0+F_{0,f}f,\end{cases} \tag{3}$$

where

$$x_0=\begin{bmatrix}x\\ e\end{bmatrix},\quad d_0=\begin{bmatrix}u\\ d\end{bmatrix},$$

$$\begin{bmatrix}\bar{A}_0 & \bar{E}_0\\ \bar{C}_0 & \bar{F}_0\end{bmatrix}=\begin{bmatrix}A_0 & E_0\\ C_0 & F_0\end{bmatrix}+\begin{bmatrix}\Delta A_0 & \Delta E_0\\ \Delta C_0 & \Delta F_0\end{bmatrix},$$

$$A_0=\begin{bmatrix}A & 0\\ 0 & A-LC\end{bmatrix},\quad E_0=\begin{bmatrix}B & E_d\\ 0 & E_d-LF_d\end{bmatrix},$$

$$E_{0,f}=\begin{bmatrix}E_f\\ E_f-LF_f\end{bmatrix},\quad \Psi_0=\begin{bmatrix}\phi(x,u)\\ \phi(x,u)-\phi(\hat{x},u)\end{bmatrix},$$

$$C_0=\begin{bmatrix}0 & C\end{bmatrix},\quad F_0=\begin{bmatrix}0 & F_d\end{bmatrix},\quad F_{0,f}=F_f,$$

$$\Delta A_0=\begin{bmatrix}\Delta A & 0\\ \Delta A-L\Delta C & 0\end{bmatrix},$$

$$\Delta C_0=\begin{bmatrix}0 & \Delta C\end{bmatrix},$$

$$\Delta E_0=\begin{bmatrix}\Delta B & \Delta E_d\\ \Delta B-L\Delta D & \Delta E_d-L\Delta F_d\end{bmatrix},$$

$$\Delta F_0=\begin{bmatrix}\Delta D & \Delta F_d\end{bmatrix},$$

$$\begin{bmatrix}\Delta A_0 & \Delta E_0\\ \Delta C_0 & \Delta F_0\end{bmatrix}=\begin{bmatrix}\bar{E}\\ \bar{F}\end{bmatrix}\Delta(t)\begin{bmatrix}\bar{G} & \bar{H}\end{bmatrix},$$

where

$$\bar{E}=\begin{bmatrix}E^T & (E-LF)^T\end{bmatrix}^T,\quad \bar{F}=F,$$
$$\bar{G}=\begin{bmatrix}G & 0\end{bmatrix},\quad \bar{H}=\begin{bmatrix}H & K\end{bmatrix},$$

and d_0 is assumed to be bounded by

$$\|d_0\|_2\le\delta_{d,\max}. \tag{4}$$

In the following, we will define the H_- index to measure the influence of the faults f on the residual signal r in (3).

Definition 1. (Khan *et al.*, 2009) Given the system $\Sigma_{\mathscr{R}}$ (3), assume that $d_0 = 0$. Then the H_- index can be defined as

$$\|\Sigma_{\mathscr{R}}\|_- = \inf_{f\neq 0} \frac{\|r\|_2}{\|f\|_2}.$$

For the H_- index to be larger than some positive number, β can be defined as

$$\|\Sigma_{\mathscr{R}}\|_- = \inf_{f\neq 0} \frac{\|r\|_2}{\|f\|_2} \geq \beta$$

or

$$\|r\|_2 \geq \beta \|f\|_2.$$

For the purpose of residual evaluation, the $\mathcal{L}_2$ norm of the residual signal is often adopted as the evaluation function and is also used in this paper,

$$J = \|r\|_2.$$

The decision logic of fault detection is as follows:

$$\begin{aligned} J &> J_{th} \Longrightarrow \textit{faulty} \\ J &\leq J_{th} \Longrightarrow \textit{fault}-\textit{free} \end{aligned}$$

so that a false alarm is created if

$$J > J_{th} \ \text{ for } \ f = 0$$

and a fault is detected if

$$J > J_{th} \ \text{ for } \ f \neq 0.$$

Moreover, the following lemma is very useful.

Lemma 1. (Wang *et al.*, 1992) *Let $G, L, E, F(t)$ be real matrices of appropriate dimensions with $F(t)$ being a matrix function and $F(t)^T F(t) \leq I$. Then, for any $\epsilon > 0$,*

$$LF(t)E + E^T F(t) L^T \leq \frac{1}{\epsilon} LL^T + \epsilon E^T E.$$

3. Problem formulation

The objective of FD system design in this paper is to minimize the FAR under a given FDR. Ding (2008) defines the FDR in the norm based framework as

$$\text{FDR} = \frac{\beta \delta_{f,\min}}{J_{th}}$$

where β is the H_- gain from faults to the residual signal with the assumption that there are no disturbances and uncertainties. In this case, the residual generator dynamics (3) become

$$\begin{cases} \dot{x}_f = (A - LC)x_f + \Psi + (E_f - LF_f)f, \\ r_f = Cx_f + F_f f, \end{cases} \tag{5}$$

so β fulfills

$$\|r_f\|_2 \geq \beta \|f\|_2,$$

and $\delta_{f,\min}$ is the minimum size of the f vector which is defined as a fault to be detected. We have

$$\|f\|_2 \geq \delta_{f,\min}.$$

The physical meaning of this definition is that the larger the faults ($\delta_{f,\min}$ is larger), the larger the FDR and the larger the threshold (J_{th} is larger), the smaller the FDR.

Based on the definition of the FDR, when it is given (FDR $\neq 0$), the threshold should be set as

$$J_{th} = \beta \theta_{\text{FDR}}, \tag{6}$$

where

$$\theta_{\text{FDR}} = \frac{\delta_{f,\min}}{\text{FDR}}.$$

In the work of Ding (2008), given a residual generator r and J_{th}, the set Ω_{FA} defined by

$$\Omega_{\text{FA}} = \{d \mid J > J_{th} \quad \text{for} \quad f = 0\}$$

is called the set of disturbances that cause false alarms (SDFA). Since false alarms are created when $f = 0$, in this case, the residual generator dynamics (3) become

$$\begin{cases} \dot{x}_{0,d} = \bar{A}_0 x_{0,d} + \Psi_0 + \bar{E}_0 d_0, \\ r_d = \bar{C}_0 x_{0,d} + \bar{F}_0 d_0. \end{cases} \tag{7}$$

So when the FDR is given and the threshold is set as (6), Ω_{FA} can be expressed as

$$\Omega_{\text{FA}} = \{d_0 \mid \ \|r_d\|_2 > \beta \theta_{\text{FDR}}\}.$$

The size of Ω_{FA} is a reasonable measurement of the rate of the false alarms. In this context, the size of Ω_{FA} is interpreted as the FAR ($0 \leq \text{FAR} \leq 1$). Since the $\mathcal{L}_2$ norm of the disturbances considered is bounded by (4), when the FAR is small, only relative large disturbances will cause false alarms. So in the norm-based framework, the FAR can be expressed as

$$\|d_0\|_2 > (1 - \text{FAR})\delta_{d,\max} \tag{8}$$

$$\Longleftrightarrow \|r_d\|_2 - \beta \theta_{\text{FDR}} > 0. \tag{9}$$

Note that, when FAR $= 0$, since $\|d_0\|_2$ is bounded by (4), the condition $\|d_0\|_2 > \delta_{d,\max}$ will never be fulfilled, which leads to the smallest size of Ω_{FA}. When FAR $=$ 1, the condition $\|d_0\|_2 > 0$ is almost always true, which leads to the largest size of Ω_{FA}.

Based on the relationship between the FDR and FAR which is represented in (8) and (9), our problem can be formulated as follows: Given the FDR, find an observer gain L so that the FAR is minimized.

4. Solution to the integrated design problem

Since the FDR is given, (9) can be transformed into

$$\frac{\|r_d\|_2}{\beta\theta_{\mathrm{FDR}}} > 1. \tag{10}$$

Then, based on (10), one sufficient condition for (8) is

$$\|d_0\|_2 \geq (1-\mathrm{FAR})\delta_{d,\max}\frac{\|r_d\|_2}{\beta\theta_{\mathrm{FDR}}}. \tag{11}$$

In (11), the FAR and FDR are connected, which is the key step for optimization. Since (11) is a sufficient condition, for the given FDR, the FAR calculated by (11) will be larger than or equal to its actual value. So a suboptimal solution can be achieved by minimizing the FAR based on (11). The FAR has been defined as

$$0 \leq \mathrm{FAR} \leq 1.$$

In the following study, it is assumed that

$$0 \leq \mathrm{FAR} < 1.$$

Since the FAR should be minimized, this assumption will not lead to a conservative result. Based on it, (11) can be transformed into

$$\|r_d\|_2 - \beta\theta_{\mathrm{FDR}}\frac{\|d_0\|_2}{(1-\mathrm{FAR})\delta_{d,\max}} \leq 0, \tag{12}$$

where β fulfills

$$\|r_f\|_2 \geq \beta\|f\|_2. \tag{13}$$

Based on (12) and (13), for a given FDR, the minimization of FAR can be solved in an iterative way as follows:

Step 1. Set the initial value of FAR.

Step 2. If there exist β and an observer gain L which fulfill (12) and (13), decrease (otherwise increase) the value of FAR till we get a minimum FAR.

The above algorithm gives an elegant tool for the minimization of the FAR provided the FDR is given. Now the question is how to check whether β and L exist in Step 2. To this end, the following theorem gives sufficient conditions for the existence of β and L based on the LMI technique.

Theorem 1. *Given the residual generator dynamics* (5) *and* (7), *assume that* $x_f(0)=0$, $x_{0,d}(0)=0$. *Then*

$$\|r_d\|_2 - \beta\theta_{df}\|d_0\|_2 \leq 0, \tag{14}$$

$$\|r_f\|_2 \geq \beta\|f\|_2, \tag{15}$$

where

$$\theta_{df} = \frac{\theta_{\mathrm{FDR}}}{(1-\mathrm{FAR})\delta_{d,\max}}$$

if there exist some $\epsilon > 0$, $\beta > 0$, L *and symmetric matrices* $P_1 > 0$, $P_2 > 0$, $Q \leq 0$ *so that*

$$\begin{bmatrix} \Omega_1 & \Omega_2 \\ * & \Omega_3 \end{bmatrix} \leq 0, \tag{16}$$

$$\begin{bmatrix} N_5 & Q(E_f - LF_f) + C^T F_f & \gamma Q \\ * & F_f^T F_f - \beta^2 I & 0 \\ * & 0 & I \end{bmatrix} \geq 0, \tag{17}$$

where

$$\Omega_1 = \begin{bmatrix} N_1 & 0 & N_3 & P_1E_d + \varepsilon G^T K \\ 0 & N_2 & 0 & P_2(E_d - LF_d) \\ * & * & N_4 & \varepsilon H^T K \\ * & * & * & -\beta^2\theta_{df}^2 I + \varepsilon K^T K \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0 & P_1E & 0 & \gamma P_1 \\ C^T & P_2(E-LF) & \gamma P_2 & 0 \\ 0 & 0 & 0 & 0 \\ F_d^T & 0 & 0 & 0 \end{bmatrix},$$

$$\Omega_3 = \begin{bmatrix} -I & -F & 0 & 0 \\ * & -\varepsilon I & 0 & 0 \\ * & * & -I & 0 \\ * & * & * & -I \end{bmatrix},$$

$$N_1 = P_1A + A^TP_1 + \varepsilon G^TG + I,$$
$$N_2 = P_2(A-LC) + (A-LC)^TP_2 + I,$$
$$N_3 = P_1B + \varepsilon G^TH,$$
$$N_4 = -\beta^2\theta_{df}^2 I + \varepsilon H^TH,$$
$$N_5 = Q(A-LC) + (A-LC)^TQ - I + C^TC.$$

Proof. The proof includes two parts.
Part 1: Let

$$V_d = x_{0,d}^T P x_{0,d}, \qquad P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} > 0.$$

We have that

$$\begin{aligned} & r_d^T r_d - \beta^2\theta_{df}^2 d_0^T d_0 + \dot{V}_d \leq 0 \\ \Longrightarrow & \int_0^\infty r_d^T r_d - \beta^2\theta_{df}^2 \int_0^\infty d_0^T d_0 + V_d(\infty) \leq 0 \\ \Longrightarrow & \|r_d\|_2 - \beta\theta_{df}\|d_0\|_2 \leq 0. \end{aligned} \tag{18}$$

Thus (18) is a sufficient condition for (14).

We have

$$\dot{V}_d = 2x_{0,d}^T P[\bar{A}_0 x_{0,d} + \bar{E}_0 d_0] + 2x_{0,d}^T P\Psi_0. \tag{19}$$

Using the Cauchy–Schwarz inequality,

$$2x_{0,d}^T P\Psi_0 \leq 2\|Px_{0,d}\|\|\Psi_0\|, \tag{20}$$

and with the Lipschitz property of Ψ_0, we have

$$\begin{aligned} 2\|Px_{0,d}\|\|\Psi_0\| &\leq 2\gamma\|Px_{0,d}\|\|x_{0,d}\| \\ &\leq \gamma^2 x_{0,d}^T PPx_{0,d} + x_{0,d}^T x_{0,d}. \end{aligned} \tag{21}$$

Substituting (20) and (21) into (19) we get

$$\begin{aligned}\dot{V}_d \le\ & 2x_{0,d}^T P[\bar{A}_0 x_{0,d} + \bar{E}_0 d_0] \\ & + \gamma^2 x_{0,d}^T PP x_{0,d} + x_{0,d}^T x_{0,d}. \end{aligned} \tag{22}$$

Then a sufficient condition for (18) is

$$\begin{aligned} & (\bar{C}_0 x_{0,d} + \bar{F}_0 d_0)^T (\bar{C}_0 x_{0,d} + \bar{F}_0 d_0) - \beta^2 \theta_{df}^2 d_0^T d_0 \\ & \quad + 2x_{0,d}^T P[\bar{A}_0 x_{0,d} + \bar{E}_0 d_0] + \gamma^2 x_{0,d}^T PP x_{0,d} \\ & \quad + x_{0,d}^T x_{0,d} \le 0 \qquad (23) \\ & \Longleftrightarrow \begin{bmatrix} x_{0,d} \\ d_0 \end{bmatrix}^T \chi_1 \begin{bmatrix} x_{0,d} \\ d_0 \end{bmatrix} \le 0, \end{aligned}$$

where

$$\chi_1 = \begin{bmatrix} \bar{C}_0^T \\ \bar{F}_0^T \end{bmatrix} \begin{bmatrix} \bar{C}_0 & \bar{F}_0 \end{bmatrix} + \begin{bmatrix} N_6 & P\bar{E}_0 \\ * & -\beta^2 \theta_{df}^2 I \end{bmatrix}$$

and

$$N_6 = P\bar{A}_0 + \bar{A}_0^T P + \gamma^2 PP + I,$$

so that

$$\chi_1 \le 0 \Longrightarrow \|r_d\|_2 - \beta\theta_{df} \|d_0\|_2 \le 0. \tag{24}$$

Applying the Schur complement, we can rewrite (24) as

$$\begin{bmatrix} N_6 & P\bar{E}_0 & \bar{C}_0^T \\ \bar{E}_0^T P & -\beta^2\theta_{df}^2 I & \bar{F}_0^T \\ \bar{C}_0 & \bar{F}_0 & -I \end{bmatrix} \le 0 \tag{25}$$

$$\begin{aligned} \Longleftrightarrow & \begin{bmatrix} PA_0 + A_0^T P + \gamma^2 PP + I & PE_0 & C_0^T \\ E_0^T P & -\beta^2\theta_{df}^2 I & F_0^T \\ C_0 & F_0 & -I \end{bmatrix} \\ & + \begin{bmatrix} P\Delta A_0 + \Delta A_0^T P & P\Delta E_0 & \Delta C_0^T \\ \Delta E_0^T P & 0 & \Delta F_0^T \\ \Delta C_0 & \Delta F_0 & 0 \end{bmatrix} \le 0. \end{aligned}$$

Split the second matrix in the above inequality into

$$\begin{bmatrix} P\Delta A_0 + \Delta A_0^T P & P\Delta E_0 & \Delta C_0^T \\ \Delta E_0^T P & 0 & \Delta F_0^T \\ \Delta C_0 & \Delta F_0 & 0 \end{bmatrix} = \chi_2 + \chi_2^T,$$

where

$$\chi_2 = \begin{bmatrix} P\bar{E} \\ 0 \\ F \end{bmatrix} \Delta(t) \begin{bmatrix} \bar{G} & \bar{H} & 0 \end{bmatrix}.$$

Then, according to Lemma 1, (25) holds if there exists $\varepsilon > 0$ so that

$$\begin{aligned} & \begin{bmatrix} N_7 & PE_0 & C_0^T \\ E_0^T P & -\beta^2\theta_{df}^2 I & F_0^T \\ C_0 & F_0 & -I \end{bmatrix} + \frac{1}{\varepsilon} \begin{bmatrix} P\bar{E} \\ 0 \\ F \end{bmatrix} \begin{bmatrix} P\bar{E} \\ 0 \\ F \end{bmatrix}^T \\ & + \varepsilon \begin{bmatrix} \bar{G} & \bar{H} & 0 \end{bmatrix}^T \begin{bmatrix} \bar{G} & \bar{H} & 0 \end{bmatrix} \le 0. \end{aligned}$$

where

$$N_7 = PA_0 + A_0^T P + \gamma^2 PP + I.$$

Applying the Schur complement yields

$$\begin{bmatrix} N_8 & PE_0 + \varepsilon \bar{G}^T \bar{H} & C_0^T & P\bar{E} \\ * & -\beta^2\theta_{df}^2 I + \varepsilon \bar{H}^T \bar{H} & F_0^T & 0 \\ * & * & -I & F \\ * & * & * & -\varepsilon I \end{bmatrix} \le 0,$$

where

$$N_8 = PA_0 + A_0^T P + \gamma^2 PP + I + \varepsilon \bar{G}^T \bar{G}.$$

Substituting

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}$$

into the above inequality, we get

$$\begin{bmatrix} N_9 & 0 & N_{10} & N_{11} & 0 & P_1 E \\ * & N_{12} & 0 & N_{13} & C^T & P_2(E - LF) \\ * & * & N_{14} & \varepsilon H^T K & 0 & 0 \\ * & * & * & N_{15} & F_d^T & 0 \\ * & * & * & * & -I & -F \\ * & * & * & * & * & -\varepsilon I \end{bmatrix} \le 0,$$

where

$$\begin{aligned} & N_9 = P_1 A + A^T P_1 + \varepsilon G^T G + \gamma^2 P_1 P_1 + I, \\ & N_{10} = P_1 B + \varepsilon G^T H, N_{11} = P_1 E_d + \varepsilon G^T K, \\ & N_{12} = P_2(A - LC) + (A - LC)^T P_2 + \gamma^2 P_2 P_2 + I, \\ & N_{13} = P_2(E_d - LF_d), N_{14} = -\beta^2\theta_{df}^2 I + \varepsilon H^T H, \\ & N_{15} = -\beta^2\theta_{df}^2 I + \varepsilon K^T K. \end{aligned}$$

Finally, applying the Schur complement again, we get (16) of Theorem 1, which is a sufficient condition for (14).

Part 2: Let

$$V_f(x) = x_f^T Q x_f, \quad Q \le 0.$$

It holds that

$$\begin{aligned} & r_f^T r_f - \beta^2 f^T f + \dot{V}_f \ge 0 \qquad (26) \\ & \Longrightarrow \int_0^\infty r_f^T r_f - \beta^2 \int_0^\infty f^T f + V_f(\infty) \ge 0 \\ & \Longrightarrow \|r_f\|_2 \ge \beta \|f\|_2, \end{aligned}$$

so (26) is a sufficient condition for (15). We have

$$\begin{aligned} \dot{V}_f =\ & 2x_f^T Q[(A - LC)x_f + (E_f - LF_f)f] \\ & + 2x_f^T Q\Psi. \end{aligned} \tag{27}$$

Using the Cauchy–Schwarz inequality,

$$2x_f^T Q\Psi \ge -2\|Qx_f\| \|\Psi\|, \tag{28}$$

and with the Lipschitz property of Ψ, we have

$$2\|Qx_f\|\|\Psi\| \leq 2\gamma\|Qx_f\|\|x_f\| \leq \gamma^2 x_f^T QQx_f + x_f^T x_f. \quad (29)$$

Substituting (28) and (29) into (27), we get

$$\dot{V}_f \geq 2x_f^T Q[(A - LC)x_f + (E_f - LF_f)f] - \gamma^2 x_f^T QQx_f - x_f^T x_f.$$

Then a sufficient condition for (26) is

$$(Cx_f + F_f f)^T(Cx_f + F_f f) - \beta^2 f^T f + 2x_f^T Q(A - LC)x_f + 2x_f^T Q(E_f - LF_f)f - \gamma^2 x_f^T QQx_f - x_f^T x_f \geq 0$$

$$\Longleftrightarrow \begin{bmatrix} x_f \\ f \end{bmatrix}^T \begin{bmatrix} N_{16} & N_{17} \\ * & F_f^T F_f - \beta^2 I \end{bmatrix} \begin{bmatrix} x_f \\ f \end{bmatrix} \geq 0,$$

where

$$N_{16} = Q(A - LC) + (A - LC)^T Q - \gamma^2 QQ - I + C^T C,$$
$$N_{17} = Q(E_f - LF_f) + C^T F_f.$$

Applying the Schur complement yields (17) of Theorem 1, which is a sufficient condition for (15). This completes the proof. ∎

In Theorem 1, (16) and (17) are NMIs which can be approached by an advanced nonlinear optimization technique. A conservative solution could be achieved by setting

$$Q = -P_2, \quad Y = P_2 L.$$

Then (16) and (17) are transformed into standard LMIs.

5. Example

In this section, an example is given to illustrate the achieved results.

Consider the FD problem of a system in the form of (1) with coefficient matrices:

$$A = \begin{bmatrix} -6.5 & 3.9 & 5.2 \\ 0 & -9.1 & 3.9 \\ 1.3 & 3.9 & -7.8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 1.5 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 3 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 \\ 0.3 \end{bmatrix},$$

$$Ed = \begin{bmatrix} -0.3 & 1 & 0.6 \\ 0 & 0.3 & 0.5 \\ 0.4 & 0 & -0.2 \end{bmatrix},$$

$$Ef = \begin{bmatrix} 1.3 & 0.65 \\ -0.39 & 1.04 \\ 0.78 & -1.17 \end{bmatrix}, \quad F = \begin{bmatrix} 0.35 \\ 0.1 \end{bmatrix},$$

$$Fd = \begin{bmatrix} 0.7 & 1 & -0.3 \\ 0 & 0.6 & 0.2 \end{bmatrix},$$

$$Ff = \begin{bmatrix} 1.6 & 0 \\ 0 & -1.6 \end{bmatrix},$$

$$E = \begin{bmatrix} 0.2 \\ 0.3 \\ 0.15 \end{bmatrix},$$

$$\phi(x, u) = 0.5 \begin{bmatrix} \sin(x_1) \\ \cos(x_2) \\ 0 \end{bmatrix}$$

$$G = \begin{bmatrix} 0.25 & 0.1 & 0.33 \end{bmatrix}, \qquad H = 0.12,$$
$$K = \begin{bmatrix} 0.16 & 0.23 & 0.31 \end{bmatrix}.$$

The faults which are defined to be detected are bounded by

$$\|f\|_2 \geq \delta_{f,\min} = 5,$$

and the disturbances are bounded by

$$\|d_0\|_2 \leq \delta_{d,\max} = 6.$$

Set FDR $= 1$. Then, following the design procedure in Section 4 to minimize FAR, we get the optimal observer gain matrix as

$$L_{\text{opt}} = \begin{bmatrix} 0.8175 & -0.3244 \\ -0.1393 & -0.4087 \\ 0.3407 & 0.5156 \end{bmatrix}.$$

The corresponding H_- gain β in (12) is $\beta = 1.5038$, so, according to (6), the threshold is set as

$$J_{th} = \beta \frac{\delta_{f,\min}}{\text{FDR}} = 7.5191.$$

In the simulation study, the simulation time is set to be 3000 seconds and the control input is a step signal $u = 0.4$ (step time at 0). The unknown disturbances are, respectively, a sine wave $0.14\sin(10t)$, a step signal (step time at 0) of amplitude 0.25, and a continuous signal taking value randomly from a uniform distribution between $[-0.2, 0.2]$. The residual signal is evaluated in a time window of 10 seconds $J(t) = (\int_{t-10}^{t} r^T r \mathrm{d}t)^{\frac{1}{2}}$. Fault 1 appears at the 1750-th second as a step function $f_1 = 0.5$. Fault 2 appears at the 1950-th second as a step function $f_2 = -0.4$. The simulation results are shown in Fig. 1. We can see that, when there are no faults, J is always less than the threshold, and hence no false alarm is created. When faults appear, J is turned to be greater than the threshold, which means that the faults are successfully detected. This demonstrates the achieved results.

6. Conclusions

In this paper, an integrated design of observer based fault detection for a class of uncertain nonlinear systems has been developed, which is a trade-off design between the

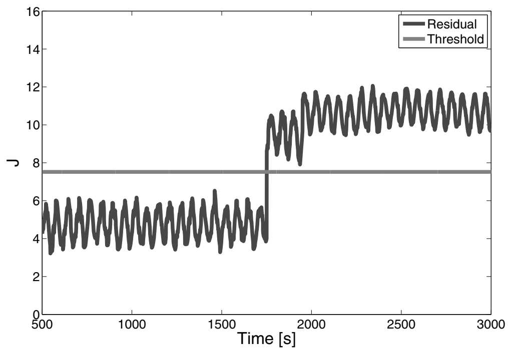

Fig. 1. Evaluated residual signal J.

norm based FAR and FDR. This problem is formulated as minimizing the FAR under a given FDR. The extension of the proposed approach to more general nonlinear systems will be the focus of our future work in this area.

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