

FUZZY FEEDBACK LINEARIZING CONTROLLER AND ITS EQUIVALENCE WITH THE FUZZY NONLINEAR INTERNAL MODEL CONTROL STRUCTURE

REDA BOUKEZZOULA, SYLVIE GALICHET, LAURENT FOULLOY

Laboratoire d'Informatique, Systèmes, Traitement de l'Information et de la Connaissance
Université de Savoie, B.P. 80439, 74944 Annecy le Vieux Cedex, France

e-mail: {Reda.Boukezzoula, Sylvie.Galichet, Laurent.Foullouy}@univ-savoie.fr

This paper examines the inverse control problem of nonlinear systems with stable dynamics using a fuzzy modeling approach. Indeed, based on the ability of fuzzy systems to approximate any nonlinear mapping, the nonlinear system is represented by a Takagi-Sugeno (TS) fuzzy system, which is then inverted for designing a fuzzy controller. As an application of the proposed inverse control methodology, two popular control structures, namely, feedback linearization and Nonlinear Internal Model Control (NIMC) are investigated. Moreover, the paper points out that, under some conditions, both of the control structures are equivalent and naturally implement a Smith predictor in the presence of time delays.

Keywords: fuzzy control, inverse control, feedback linearization, internal model control

1. Introduction

Inverting systems is an important issue in engineering applications, especially in linear and nonlinear control problems (Baoming *et al.*, 2002; Boukezzoula *et al.*, 2001; Boukezzoula *et al.*, 2003; Boukezzoula *et al.*, 2006; Cabrera and Narendra, 1999; Devanathan *et al.*, 2000; Li and Deng, 2006; Rivals and Personnaz, 2000). The underlying principle of inverse control is based on the following remark: Since a plant model can be viewed as a mapping from control inputs to future outputs according to the process history, one can use the inverse mapping from the desired outputs to the inputs as a design control procedure. Actually, even when not clearly specified, an inversion mechanism is often included in nonlinear control structures. Feedback linearizing control (Devanathan *et al.*, 2000; Kang *et al.*, 1998; Kwanghee, 1999; Ying, 1999) as well as internal model control (Babuska, 1998; Baoming *et al.*, 2002; Boukezzoula *et al.*, 2001; Boukezzoula *et al.*, 2003; Fang and Rad, 2000; Li and Deng, 2006; Lightbody and Irwin, 1997) are typical application examples of such as an inversion based control methodology.

If both control strategies are promising approaches for controlling input-output stable nonlinear systems, their applicability is related to an accurate representation of the nonlinear plants under consideration. As control problems arising in a large variety of engineering fields are characterized by uncertain environments and nonlinearities,

the identification of accurate process models according to first principles remains difficult or at least time-consuming. Moreover, for an efficient implementation of inverse based techniques, an exact inversion procedure must be available. This latter requirement represents a major obstacle in handling general nonlinear systems. Indeed, as analytic inverse solutions cannot generally be determined, one must resort to numerical inversion techniques (Economou *et al.*, 1986; Lightbody and Irwin, 1997; Nahas *et al.*, 1992), which necessarily introduce difficulties inherent in iterative algorithms (initialization values, convergence conditions, approximation errors). These drawbacks weaken the potential use of inverse based methods in nonlinear control strategies. It follows that many proposed methodologies are usually restricted to affine-in-control nonlinear systems that can be analytically inverted (Jagannathan, 1998; Jagannathan, 1999; Wang, 1993).

Overpassing the limitation of having an accurate representation of nonlinear plants is the first motivation for introducing fuzzy systems and/or artificial neural networks in recent control architectures. Indeed, both system categories are now known as being universal approximators and can thus be used to represent nonlinear systems and/or to implement nonlinear regulators (Narendra and Mukhopadhyay, 1997; Sugeno, 1999; Zeng and Singh, 1996b). In this framework, inverse control based tech-

niques have been extensively applied to nonlinear control. In many cases, the control signals are computed by numerically inverting the neural network and/or the fuzzy model, which induces the restrictions mentioned previously in implementing the corresponding control strategies. One way of avoiding the problems associated with the use of iterative inversion algorithms consists in suppressing the on-line inversion by applying an inverse control learning method (Cabrera and Narendra, 1999; Park and Han, 2000; Rivals and Personnaz, 2000). In that case, the inverse system is trained by minimizing the deviation between the controlled system output and the desired one by means of adaptive techniques. Unfortunately, the flaw of this alternative approach resides in the need of a cautious initialization of the inverse controller. In other words, special attention must be paid to this preliminary task, which has an important repercussion on the control signal quality and can even lead to unstable situations when the inverse system structure is not correctly chosen or when the corresponding parameters are randomly initialized.

Finally, even if neural networks or fuzzy systems are efficient tools for modeling complex systems, they are not especially well suited for implementing the on-line model inversion. This weakness clearly appears in several published papers that mostly consider simplified models when inverse control is concerned. This restriction implies the use of specific neural networks or fuzzy models whose approximation capacity is usually limited to affine-in-control systems (Jagannathan, 1998; Jagannathan, 1999; Leland, 1998; Nahas et al., 1992). This remark concerns two main classes of inversion based control structures, i.e., feedback linearizing control and internal model control. Indeed, many referenced works are restricted to affine-in-control nonlinear systems even when discrete-time systems are considered (Jagannathan, 1998; Jagannathan, 1999) or when particular fuzzy systems are implemented, as in (Kang et al., 1998), where a PDC (Parallel Distributed Compensation) structure is chosen, or in (Leland, 1998), where fuzzy inputs are considered.

In this paper, the inverse control methodology proposed in (Boukezzoula et al., 2003; Galichet et al., 2004) for handling nonaffine-in-control nonlinear systems is used. In this framework, the proposed controller design is based on the on-line analytic inversion of the fuzzy plant model. The chosen fuzzy formalism guarantees fuzzy model decomposability and consequently makes analytic processing possible. Indeed, as the fuzzy system can be viewed as a collection of elementary subsystems (Rovatti, 1998; Ying and Chen, 1997; Zeng and Singh, 1996a), its inversion can be addressed in a local way, i.e., on all elementary subsystems that may provide an inverse solution. The proposed inversion methodology is exact, noniterative and makes the determination of multiple solutions possible, thus allowing the handling

of more general systems than the ones usually considered in the fuzzy control literature. A detailed discussion concerning our inversion method with regard to other fuzzy inversion techniques (Babuska, 1998; Baranyi et al., 1998) is given in (Boukezzoula et al., 2003; Galichet et al., 2004). Consequently, the developed inversion mechanism can be easily used for implementing inverse control strategies, as shown in the paper for dealing with feedback linearizing control and internal model control. Besides, the equivalence between both the studied control structures is established.

The paper is structured in the following way: After a brief statement of the inverse control problem in Section 2, the formalism associated with decomposable TS fuzzy systems is introduced in Section 3. As an exact inversion of such TS fuzzy models is possible, an inverse model control strategy, together with the practical implementation of the analytic inversion, is proposed in Section 4. The sequel of the paper is devoted to the application of the inverse principle to nonlinear control. In this context, two classical control structures, namely, feedback linearization and nonlinear internal model control are investigated in Section 5, where the equivalence between both strategies is exhibited. Section 6 gives simulation examples to demonstrate the feasibility of the proposed method. Although all developments are presented for systems without effective delay in Sections 5 and 6, it is possible to generalize the approach to systems with general relative degree. That is the purpose of Section 7, which shows that the proposed fuzzy inverse methodology implicitly introduces a Smith predictor. Finally, discussions and remarks are summarized in Section 8.

2. Problem Statement

Let us consider a nonlinear plant in the form of a general Nonlinear Auto Regressive Moving Average model (NARMA) given by the following equation:

$$y(k+r) = \Psi[y(k-n+1), \dots, y(k), u(k), \dots, u(k-m+1)] \quad (1)$$

with $m \leq n$, where y and u are the output and input variables, respectively. Ψ represents a nonlinear mapping and r is the relative degree of the system.

For the sake of simplicity, let us first consider systems without effective delay, i.e., $r = 1$ (an extension to general relative degree systems will be developed in a further section). In this case, the system (1) is reduced to

$$y(k+1) = \Psi[Z(k)] = \Psi[y(k-n+1), \dots, y(k), u(k), \dots, u(k-m+1)]. \quad (2)$$

In control applications, the objective consists in forcing the plant output to follow a specified desired setpoint

$sp(k)$. In other words, one has to determine a control action u which guarantees that y converges to sp . If the system (2) is invertible, there exists a function Ω such that the input $u(k)$ can be expressed in the form

$$u(k) = \Omega[y(k+1), y(k-n+1), \dots, y(k), u(k-1) \dots, u(k-m+1)] = \Omega[y(k+1), Z_1(k)], \quad (3)$$

where $Z_1(k) = [y(k-n+1), \dots, y(k), u(k-1), \dots, u(k-m+1)]$.

In this case, since the control objective is to reach the setpoint sp , the control $u(k)$ can be obtained by replacing the next system output by the desired one in (3), i.e.,

$$u(k) = \Omega[sp(k+1), y(k-n+1), \dots, y(k), u(k-1), \dots, u(k-m+1)]. \quad (4)$$

When in the system under consideration Ψ can be written in an affine-in-control form, i.e.,

$$\Psi[Z(k)] = \alpha[Z_1(k)] + \beta[Z_1(k)]u(k), \quad (5)$$

where $\beta[Z_1(k)] \neq 0$ for $Z_1 \in \mathbb{R}^{n+m-1}$, the mapping Ω is easily determined and the controller can be implemented using the analytical inversion of (5), i.e.,

$$u(k) = \frac{sp(k+1) - \alpha[Z_1(k)]}{\beta[Z_1(k)]}. \quad (6)$$

However, this simple method cannot be directly applied to nonaffine-in-control systems in the general form (2), even when the dynamics of Ψ are exactly known. Indeed, when the analytic inverse of (2) cannot be determined, one must resort to numerical inversion techniques, which necessarily introduce difficulties inherent in iterative algorithms (initialization values, convergence conditions, approximation errors).

Moreover, when the dynamics of (2) are unknown, it is even more difficult to design the controller. As an alternative, when the plant dynamics are represented by a fuzzy system, an exact and noniterative inverse model controller can then be designed based on the local inversion of the fuzzy model. Indeed, *TS* fuzzy systems with singleton consequences (Sugeno, 1999) can be easily and exactly inverted provided that the inputs are described using strict triangular partitions (Boukezzoula *et al.*, 2003; Galichet *et al.*, 2004).

Before detailing the proposed control strategies, the next section briefly presents the formalism associated with the *TS* fuzzy system that will be used in the global control structure.

3. Decomposable Takagi-Sugeno Fuzzy Systems

The attractive benefit of using fuzzy systems is that the precise understanding and the development of a mathematical model are not necessary. In return, the successful determination of a fuzzy model requires a significant

amount of informative data, and the resulting model is unreliable outside the operating domains to which learning data belong. In addition, the fuzzy model input-output relationship does not contain any physical knowledge about the plant. Despite these limitations, the nonlinear plant (2) is represented by a *TS* system identified from input-output data.

Let *TS* be a zeroth-order Takagi-Sugeno fuzzy model (sometimes also called the Wang-Mendel model (Wang, 1994)) of (2) whose rulebase consists of fuzzy rules with singleton consequences in the form

$$\begin{aligned} &\text{If } y(k) \text{ is } A_1^{i_1} \text{ and } y(k-1) \text{ is } A_2^{i_2} \text{ and } \dots \text{ and} \\ &\quad y(k-n+1) \text{ is } A_n^{i_n} \text{ and } u(k) \text{ is } B_1^{j_1} \text{ and } \dots \text{ and} \\ &\quad u(k-m+1) \text{ is } B_m^{j_m} \\ &\text{then } y_f(k+1) = \phi^{i_1, \dots, i_n, j_1, \dots, j_m}, \end{aligned} \quad (7)$$

where $A_p^{i_p}$ and $B_q^{j_q}$ are linguistic terms associated respectively with the variables $y(k-p+1)$ and $u(k-q+1)$, $\phi^{i_1, \dots, i_n, j_1, \dots, j_m}$ being a real constant.

As fuzzy systems are universal approximators, it can be guaranteed that as long as the plant evolves on a compact set, there is always a fuzzy system in the form (7) capable of approximating its input-output mapping for any tolerance level. Accordingly, as far as the *TS* fuzzy model (7) is considered, the plant output can be rewritten as follows:

$$y(k+1) = y_f(k+1) + \Delta(k+1), \quad (8)$$

where $y_f(k+1)$ is the output generated by the fuzzy system and Δ is the fuzzy approximation error, which is assumed to be bounded by Δ_0 , i.e., $|\Delta| \leq \Delta_0$.

When assuming a strict partitioning of the different input universes of discourse with triangular membership functions, at each sampling time an input variable is described with at most two linguistic terms (Zeng and Singh, 1996a). Thus, in the case of a system with $n+m$ inputs, at most 2^{n+m} rules are activated for any vector of inputs. The output generated by the fuzzy system is then reduced to that produced by the 2^{n+m} fired rules. This property permits computational efficiency and offers an advantage over neural-fuzzy approaches.

The global fuzzy system can thus be viewed as a collection of elementary fuzzy subsystems. Each of them is defined on a fuzzy mesh built from the modal input values delimiting it. In this context, the study of a fuzzy system can be handled from a generic analysis of an elementary fuzzy subsystem. Indeed, according to the decomposition principle given in (Zeng and Singh, 1996a), when considering N_p and M_q possible linguistic terms for describing respectively the inputs $y(k-p+1)$ and $u(k-q+1)$, i.e., $i_p \in I_p = \{1, 2, \dots, N_p\}$, $p = 1, \dots, n$ and $j_q \in J_q = \{1, 2, \dots, M_q\}$, $q = 1, \dots, m$, the fuzzy

system (7) is composed of a collection of

$$\text{Nb_Subsyst} = \prod_{p=1}^n (N_p - 1) \prod_{q=1}^m (M_q - 1)$$

subsystems, each being defined by a collection of 2^{n+m} fired rules. In this case, at each sampling time, the output of the global fuzzy system (7) is equal to the output generated by one active subsystem.

According to the multi-linearity property of fuzzy subsystems as established in (Rovatti, 1998; Ying and Chen, 1997; Zeng and Singh, 1996a), it can be shown (Boukezzoula et al., 2003; Galichet et al., 2004) that the output of a fired fuzzy subsystem can be rewritten as follows:

$$y_f(k+1) = \alpha^{(\lambda)} [Z_1(k)] + \beta^{(\lambda)} [Z_1(k)] u(k), \quad (9)$$

where $\lambda \in \{1, 2, \dots, \text{Nb_Subsyst}\}$. In this case, at each sampling time, the output of the fuzzy system (7) is given by (9).

4. Fuzzy Model Inverse Control Strategy

When a TS fuzzy model (7) is used, an exact and noniterative inverse model can be designed based on the local inversion of the fuzzy model (Boukezzoula et al., 2003; Galichet et al., 2004). Indeed, when considering $u(k)$ as the inversion control variable, the problem consists in finding all $u(k)$ -solutions, the variables $y(k), \dots, y(k-n+1), u(k-1), \dots, u(k-m+1)$ having known values. According to the fuzzy system decomposition, the global task can be decomposed into elementary subtasks which aim at analyzing subsystems that may provide solutions. The first remark relative to subsystem exploration concerns the limited number of subsystems that must be studied. Indeed, as $y(k), \dots, y(k-n+1), u(k-1), \dots, u(k-m+1)$ have known values, it is possible to determine the fuzzy meshes to which they belong. In other words, as $u(k)$ is the single free variable, $M_1 - 1$ subsystems are concerned if M_1 represents the number of symbols used to describe $u(k)$. In this case, at each sampling time, if the objective is to reach a desired signal $v(k)$, the inverse control law $u(k)$ is computed from (9) as

$$u(k) = \frac{v(k) - \alpha^{(\lambda)} [Z_1(k)]}{\beta^{(\lambda)} [Z_1(k)]}. \quad (10)$$

When considering a subsystem λ , the control law (10) is only defined on the corresponding subsystem. Although this subsystem can be determined, it is easier to evaluate (10) and to post-validate the obtained result with regard to its belonging to this subsystem. Moreover, the existence of the inverse control law is related to $\beta^{(\lambda)} [Z_1(k)]$.

As analyzed in (Galichet et al., 2004), when $\beta^{(\lambda)} [Z_1(k)] = 0$, two cases have to be distinguished according to the respective values of $v(k)$ and $\alpha^{(\lambda)} [Z_1(k)]$.

Indeed, if $v(k) \neq \alpha^{(\lambda)} [Z_1(k)]$, the inverse control law (10) has no solution. On the contrary, when the desired output $v(k) = \alpha^{(\lambda)} [Z_1(k)]$, there is an infinity of solutions. Generally, the inverse problem requires $\beta^{(\lambda)} [Z_1(k)] \neq 0$ (no fuzzy model singularities).

Moreover, the fuzzy model inversibility is related to the zero dynamic stability. For a given fuzzy model in the form (7), the zero dynamic behavior can be characterized by a control sequence $u_0(k)$ which is chosen so that $v(k), y(k), \dots, y(k-n+1)$ are equal to zero. In this case, according to (10), the zero dynamics are given by

$$u_0(k) = -\frac{\alpha^{(\lambda)} [0, \dots, 0, u_0(k-1), \dots, u_0(k-m+1)]}{\beta^{(\lambda)} [0, \dots, 0, u_0(k-1), \dots, u_0(k-m+1)]}. \quad (11)$$

If the fuzzy model has no zero dynamics, the feedback control inputs $u(k-1), \dots, u(k-m+1)$ are not used by the fuzzy inverse. In this case, if no singularities are present, i.e., $\beta^{(\lambda)} [Z_1(k)] \neq 0$, Eqn. (11) is stable and the system is always invertible.

In the case where the fuzzy model has zero dynamics, the control input $u(k)$ is dependent not only on $v(k), y(k), \dots, y(k-n+1)$, but also on the past values of $u(k)$. Then the system is invertible if Eqn. (11) is stable. A detailed study concerning this problem is given in (Boukezzoula et al., 2003).

Finally, if the model is invertible, the inverse problem has always one or several solutions. In the presence of multiple solutions, a criterion has to be considered. As studied in (Boukezzoula et al., 2003), the latter can influence the control signal behavior and the stability of the zero dynamics. Criteria leading to abrupt switching between fuzzy subsystems (high frequency inputs) should be avoided because they can damage the actuators. In this context, the minimal energy criterion which is relevant to the physical properties of the actuators is frequently used.

5. Application in Nonlinear Control

5.1. Fuzzy Feedback Linearization Controller (FLC).

The objective of this section is to illustrate the proposed inverse control methodology for the implementation of feedback linearization. Generally, two approaches to feedback linearization can be distinguished: feedback linearization which results in an input-output relation and input-state linearization, where the input-state mapping is linear. In this paper, only the first approach is considered because the fuzzy systems used are input-output models.

The basic idea of feedback linearization is to transform a nonlinear system into new coordinates where nonlinearities can be canceled fully or partially by feedback. Indeed, in an ideal situation, when the system inverse is placed upstream with the original system, the input-output behavior becomes linear. The simplicity of this approach, in addition to the advantage of being

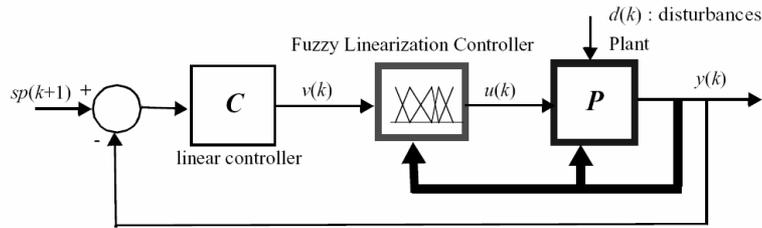


Fig. 1. Fuzzy control structure augmented with a linear controller.

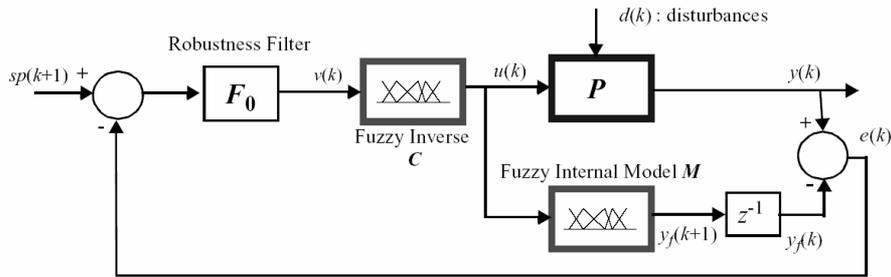


Fig. 2. Fuzzy NIMC implementation.

able to use linear control theory on the resulting system once feedback is linearized, has fostered its application in many areas (Knospe and Lindlau, 2000). Generally, feedback linearization is deeply connected to the inverse control methodology (Chen and Khalil, 1995; Slotine and Li, 1991) and can be thought of as its generalization.

Under the assumptions that the fuzzy model and its inverse are stable and by adopting the feedback linearization principle (Slotine and Li, 1991), if the control law is chosen as (10), then the input-output behavior of the plant output (8) becomes the following linear one:

$$y(k + 1) = v(k) + \Delta(k + 1). \tag{12}$$

If no model mismatch and no disturbances are present, it can be demonstrated that perfect control is obtained when the input $v(k)$ is chosen as the desired setpoint. However, in the presence of uncertainties and disturbances, the linearization controller is not able to cancel them in the steady state. In this case, a linear controller can be used. Indeed, as the controlled system obtained by implementing the control law (10) is linear, a linear control strategy can be used to guarantee desired control performances. Then the control structure is augmented by a linear controller as illustrated in Fig. 1.

According to the fuzzy plant representation and based on the exact fuzzy inversion property, it can be shown that if the control loop contains an integrator, then constant disturbances and the model mismatch can be canceled in the steady state. In order to ensure the open loop integrator presence, the linear controller C is taken as

$$C(z^{-1}) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z - 1}. \tag{13}$$

The parameter T in (13) determines the transient state form and must be chosen to provide a compromise between stability-robustness and performance.

5.2. Fuzzy Nonlinear Internal Model Control Structure (FNIMC). The principle of the NIMC methodology, introduced by Economou *et al.* (1986), is based on the inclusion of a nonlinear plant model within the control structure. Indeed, when a plant model is available, the nonlinear controller can be directly obtained by model inversion. By doing so, an offset-free response can be achieved even in the presence of constant disturbances acting on the plant output. A robustness filter is generally designed to alleviate sensitivity problems (Boukezzoula *et al.*, 2003; Lightbody and Irwin, 1997; Morari and Zafiriou, 1989; Nahas *et al.*, 1992; Rivals and Personnaz, 2000). The NIMC advantages, which are the motivating factors for using this technique in control applications, are summarized by Economou *et al.* (1986).

Based on this control principle, a fuzzy version of the NIMC structure can be obtained as illustrated in Fig. 2, where the analytical model is replaced by a fuzzy model. By applying the inversion mechanism, the control input $u(k)$ is then determined in the presence of the input v . Under the assumptions that the fuzzy model and its inverse are stable, the effect of constant disturbances and the model mismatch are compensated in the steady state. In this case, an integral action is achieved.

The filter F_0 , which guarantees the robustness of stability with respect to the plant-model mismatch, is generally chosen as follows (Boukezzoula *et al.*, 2001; Nahas

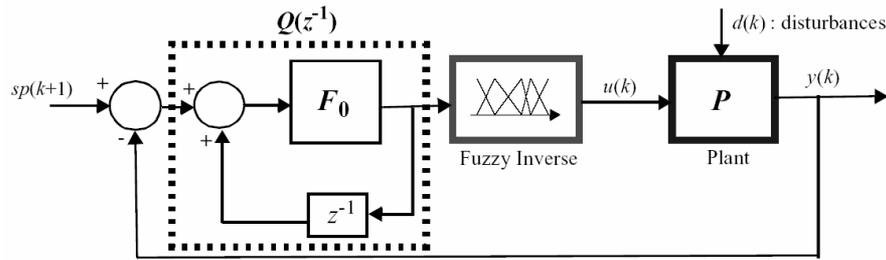


Fig. 3. Transformation of the FNIMC structure.

et al., 1992):

$$F_0(z^{-1}) = \frac{T}{1 - (1 - T)z^{-1}} = \frac{Tz}{z - (1 - T)}. \quad (14)$$

By comparing linear (Morari and Zafiriou, 1989) and non-linear IMC cases, it can be stated that the filter F_0 plays the same role as the robustness filter in the linear IMC.

5.3. Equivalence Between Feedback Linearization and the NIMC Structure. The objective of this section is to demonstrate that the developed fuzzy feedback linearization controller is equivalent to the FNIMC strategy. It can be shown that the FNIMC structure of Fig. 2 can be transformed into a closed-loop feedback control structure according to the scheme given in Fig. 3.

When the robustness filter F_0 is chosen as (14), the transfer function $Q(z^{-1})$ is given by

$$Q(z^{-1}) = \frac{T}{1 - z^{-1}} = \frac{Tz}{z - 1}. \quad (15)$$

Now, the FNIMC structure can be compared with the feedback control structure of Fig. 1. Thus, as $Q = C$, the equivalence between both control structures is established.

From the control point of view, both of the control structures (FLC and FNIMC) are strictly equivalent. However, in practical implementations, the FNIMC is slightly greedier for computations than FLC. Indeed, using a direct implementation of the FNIMC structure, at each sampling time it is required not only to determine the control value by inverting the fuzzy model, but also to compare the fuzzy model output with the process output in order to compute the fuzzy model error. This second step is avoided in the FLC implementation, where only the fuzzy model inversion is needed. In other words, from an implementation point of view, the FLC implementation is preferable.

6. Simulation Examples

In this section, simulation results using the proposed control structures are presented. Two nonaffine nonlinear

plants are considered. The first example, which represents a classical control problem (see, e.g., (Hunt and Sbarbaro, 1991; Kambhampati et al., 2000; Narendra and Parthasarathy, 1990), where inverse learning control is used), is dedicated to an illustration of the fuzzy inverse control methodology. The second one is a complex system (Rivals and Personnaz, 2000) used to show the potential of the proposed technique for the control of complex nonlinear plants. The inversion principle is implemented by means of Simulink S-functions. Indeed, Matlab/Simulink tools are widely used by the control community, but they do not provide specific functions for dealing with the fuzzy inversion control problem. Therefore, we have implemented a minimal set of blocks that are needed to experimentally verify the previous concepts:

- *SymbFuz* is an S-function that implements the fuzzification procedure. The S-function parameter is a text file providing the definition of the fuzzy partitions.
- *SymbInfSug* is an S-function that computes the output of a Takagi-Sugeno inference. The S-function parameter is a text file providing the definition of the rule base.
- *FuzzyInversion* is an S-function that computes the fuzzy inverse solutions.

It can be noted that *SymbFuz* and *SymbInfSug* are two components of the Matlab/Simulink toolbox *Floulib* freely available at

<http://www.listic.univ-savoie.fr>
see (Foulloy et al., 2006).

In all simulation examples, if the inversion algorithm produces multiple solutions, the minimal energy choice criterion, which is relevant to the physical properties of the actuators, is used (Boukezzoula et al., 2003).

6.1. First Example. The main objective of this example is to detail step-by-step how the above-mentioned generic fuzzy components can be combined for implementing FLC and FNIMC control structures. We thus focus on the possible integration of a *FuzzyInversion* Simulink block into control structures developed using the *Floulib* toolbox (Foulloy et al., 2006).

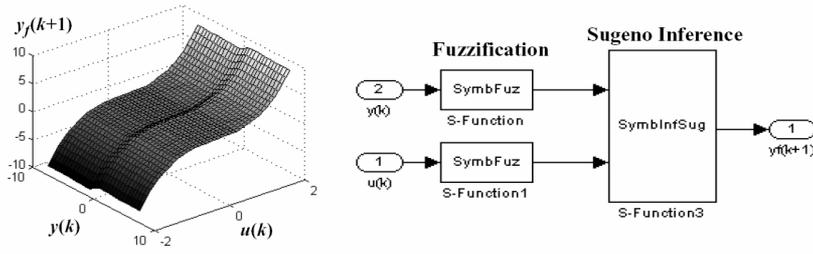


Fig. 4. Fuzzy model input-output mapping.

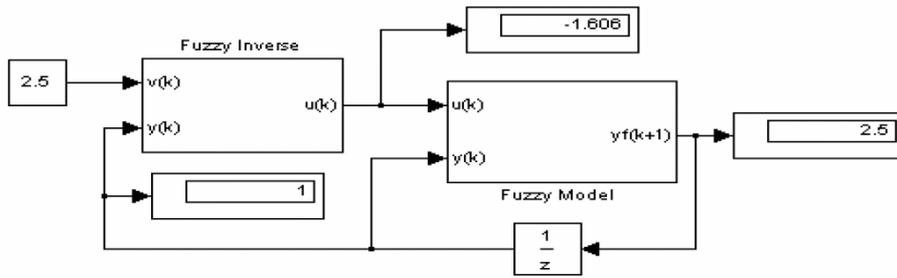


Fig. 5. Fuzzy inverse in series with its model.

The plant under consideration is described by the nonlinear equation

$$y(k + 1) = \frac{y(k)}{1 + y^2(k)} + u^3(k). \quad (16)$$

Let us assume that a fuzzy model has been identified (the identification method will be detailed in the second example). The identified fuzzy system is composed of 45 rules in the form

If $y(k)$ is $A_1^{i_1}$ and $u(k)$ is $B_1^{j_1}$ then

$$y_f(k + 1) = \phi^{i_1, j_1}, \quad i_1 = 1, \dots, 5 \text{ and } j_1 = 1, \dots, 9. \quad (17)$$

The input and output universes of discourse are respectively $[-2.0, 2.0]$ and $[-7.5, 7.5]$. In this case, 5 and 9 linguistic terms are defined for $y(k)$ and $u(k)$, respectively. When the input is a random one over the interval $[-2, 2]$, the maximum magnitude of the identification error is about 0.3, which represents 4% of the output maximum magnitude.

The identified fuzzy model input-output mapping when $y(k)$ and $u(k)$ belong to the corresponding universes of discourse is illustrated in Fig. 4. According to the decomposition principle, the fuzzy system can be written as a collection of 32 fuzzy subsystems. At each sampling time, the fuzzy system output is then delivered by the fired subsystem indexed by λ , i.e.,

$$y_f(k + 1) = \alpha^{(\lambda)} [y(k)] + \beta^{(\lambda)} [y(k)]u(k), \quad (18)$$

where $\lambda \in \{1, \dots, 32\}$. Our objective now is to apply the inversion principle in order to determine the control law

$u(k)$ so that the fuzzy system output follows the given desired setpoint.

Let us start with a simple illustration of the inversion implementation. Figure 5 shows the inversion algorithm results for $v(k) = 2.50$ and $y(k) = 1$ when the fuzzy inverse is placed in series with its model. It can be demonstrated that the inversion is exact, i.e., $y_f(k + 1) = v(k) = 2.5$.

Let us now assume that the inputs $v(k)$ and $y(k)$ both vary in their corresponding universes of discourse. The control mapping obtained by using the proposed inversion method is illustrated in Fig. 6. Now, the fuzzy model and

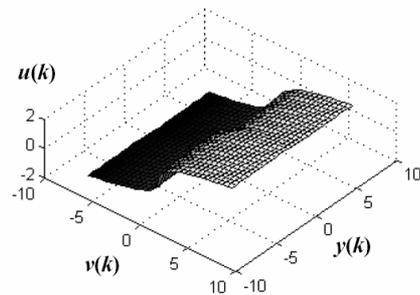


Fig. 6. Inverse model input-output mapping.

its inverse are used to implement the proposed fuzzy feedback linearizing controller and the FNIMC structure according to Figs. 7 and 8.

In both cases, the same simulation results illustrated in Fig. 9 are obtained, which is in accordance with the proved equivalence between both control structures. The first plot (Fig. 9(a)) shows the system response to changes

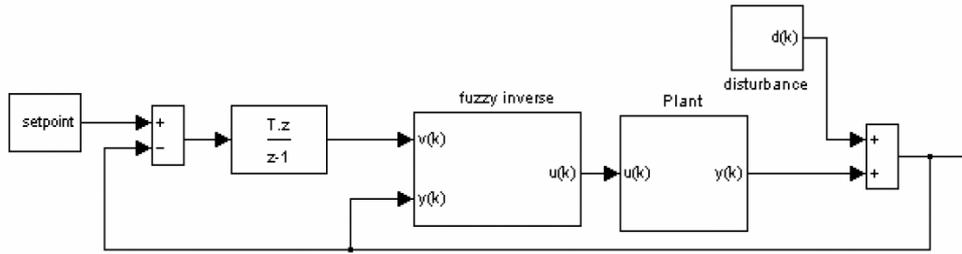


Fig. 7. Implementation of fuzzy feedback linearization.

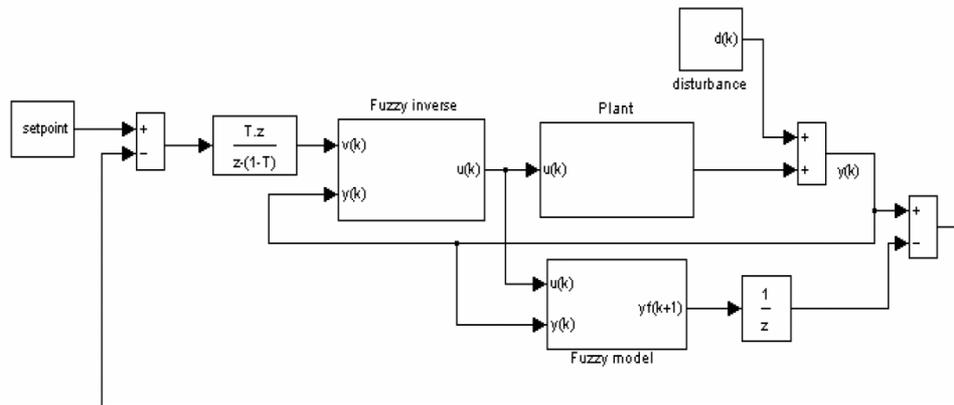


Fig. 8. Fuzzy NIMC implementation.

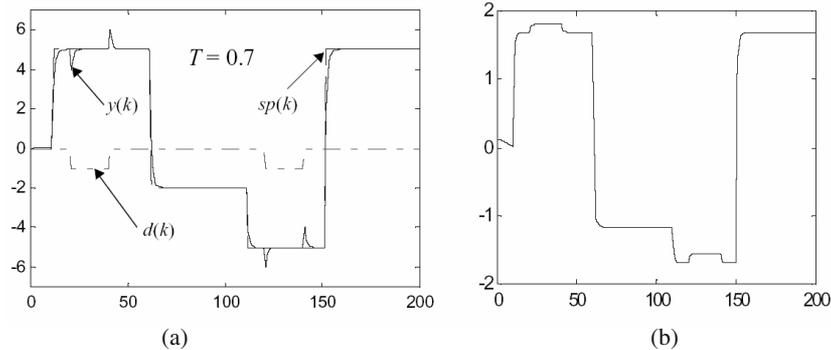


Fig. 9. Control performance: (a) the response to changes in the reference signal with an additive disturbance $d(k)$, (b) the control action.

in the reference signal in the presence of an additive disturbance. The evolution of the corresponding control law is illustrated in Fig. 9(b).

From these results, it is seen that good control performances (comparable to those reported in (Hunt and Sbarbaro, 1991; Kambhampati *et al.*, 2000; Narendra and Parthasarathy, 1990) for various reference signals) are obtained. Indeed, the system output converges rapidly to the setpoint. The errors observed after each transition are due to the modeling errors or to some disturbance transitions. The effect of constant disturbances is compensated in the steady state.

6.2. Second Example. The objective of this example is to illustrate the performance of the proposed technique and

to compare it with the neural network internal model control given in (Rivals and Personnaz, 2000), where an inverse model controller obtained by a learning technique is used. In order to be able to compare the performances of the two techniques, the same simulation conditions are considered.

The plant under consideration is described by

$$\begin{cases} x(k) = \frac{24+x(k-1)}{30}x(k-1) \\ \quad -0.8\frac{u^2(k-1)}{1+u^2(k-1)}x(k-2) + 0.5u(k-1), \\ y(k) = x(k) = d(k). \end{cases}$$

It can be shown that the system input-output behavior can be written in the following form:

$$y(k+1) = \Psi[y(k), y(k-1), u(k)]. \quad (19)$$

6.2.1. Model Identification and Validation. The identification input consists of a pulse sequence of random amplitude uniformly distributed over the interval $[-5, 5]$ with a duration of 10 sampling periods.

Let us briefly describe the employed fuzzy identification method given in (Nakoula *et al.*, 1997). The algorithm is composed of two steps. The first one is an initialization phase, during which the extreme training points are selected to create an initial partition of the input universes of discourse and to generate an initial fuzzy rule base. The second step is an iterative process, which refines the initial input partitions and the rule base. At each iteration, the model is tested over the training set. The training point that provides the maximal approximation error is learned, i.e., the fuzzy system is modified so that it produces a zero or at least very small approximation error for the point considered. This modification simply consists in generating a new linguistic term on the analysed input partitions whose modal value is set to the corresponding component of the training point that has to be learned. A new rule is then added to the rule base whose premise involves the newly created symbols and whose conclusion is the output component of the selected training point. Furthermore, a completion method is elaborated in order to determine the conclusion of all possible rules that have been artificially produced by the generation of new input symbols. This method makes it possible to obtain nonregular partitions with a different number of symbols on each universe of discourse. This property is well suited for a nonlinear system representation.

By using this identification technique, 7, 8 and 5 linguistic terms were generated for describing $y(k-1)$, $y(k)$ and $u(k)$, respectively. In this case, the identified fuzzy model was thus composed of 280 rules of the form

$$\begin{aligned} &\text{If } y(k) \text{ is } A_1^{i_1} \text{ and } y(k-1) \text{ is } A_2^{i_2} \text{ and } u(k) \text{ is } B_1^{j_1} \\ &\text{then } y_f(k+1) = \phi^{i_1, i_2, j_1}, \end{aligned} \quad (20)$$

with $i_1 = 1, \dots, 7$, $i_2 = 1, \dots, 8$ and $j_1 = 1, \dots, 5$.

In order to assess the quality of the identified fuzzy model, a pulse sequence of a random amplitude in the interval $[-5, 5]$ with a duration of 10 sampling periods was used as the control input. The identification results are shown in Fig. 10, which shows the performance of the identified fuzzy model of the plant.

6.2.2. Control Strategy Implementation. As the feedback linearization controller and the FNIMC structure are strictly equivalent from the point of view of the

achieved control performances, only the latter (also studied in (Rivals and Personnaz, 2000) with a neural network based approach) is presented in this example.

The reference model used (Rivals and Personnaz, 2000) is given by the following linear equation:

$$\begin{aligned} y_r(k+1) - 0.803y_r(k) + 0.160y_r(k-1) \\ = 0.232sp(k) + 0.126sp(k-1), \end{aligned} \quad (21)$$

where $sp(k)$ consists of pulses of an amplitude in the interval $[-2, 1.75]$. The total length of the sequence is 330. The plant is affected by an output disturbance $d(k)$, which consists of randomly occurring pulses of an amplitude of -1 .

The control structure obtained by the application of FNIMC according to Fig. 2 is illustrated in Fig. 11. The simulation results are illustrated in Fig. 12. It can be seen that good control performances are obtained. Indeed, the mean square error between the plant output and the reference signal is equal to 0.0286. In (Rivals and Personnaz, 2000), where neural internal model control was used, similar results were obtained (the mean square error was equal to 0.03).

Let us now discuss the advantages of our method compared with those given in (Rivals and Personnaz, 2000), where the inverse controller is directly trained in a closed-loop control structure. If the latter has the advantage to learn the controller in the desired operating regions and solves problems encountered when numerical inversion algorithms are used, it may cause nonconvergence or instability problems, which can be dangerous for the controlled plant. Indeed, if the controller initialisation parameters are chosen in an arbitrary way, unstable situations may be produced. In this case, it is necessary to spend significant time on the parameter preparation of the inverse initial controller. As the plant dynamics are supposed to be unknown, it is difficult to deal with this task.

In the proposed method the controller design is based on the local inversion of a decomposable fuzzy model identified off-line from plant input-output data. Moreover, the developed inversion procedure is exact and noniterative.

From the control implementation point of view, as the identified fuzzy system is decomposable, at each sampling time only one subsystem is used to elaborate the controller (composed of 8 fired rules in this example). This property permits computational efficiency, especially in real-time applications, and represents an advantage with regard to neural or fuzzy-neural control approaches.

7. Generalization for Systems with General Relative Degree

7.1. Control Architecture. In control applications, many plants contain time delays which can affect the control synthesis. In other words, if the relative degree is

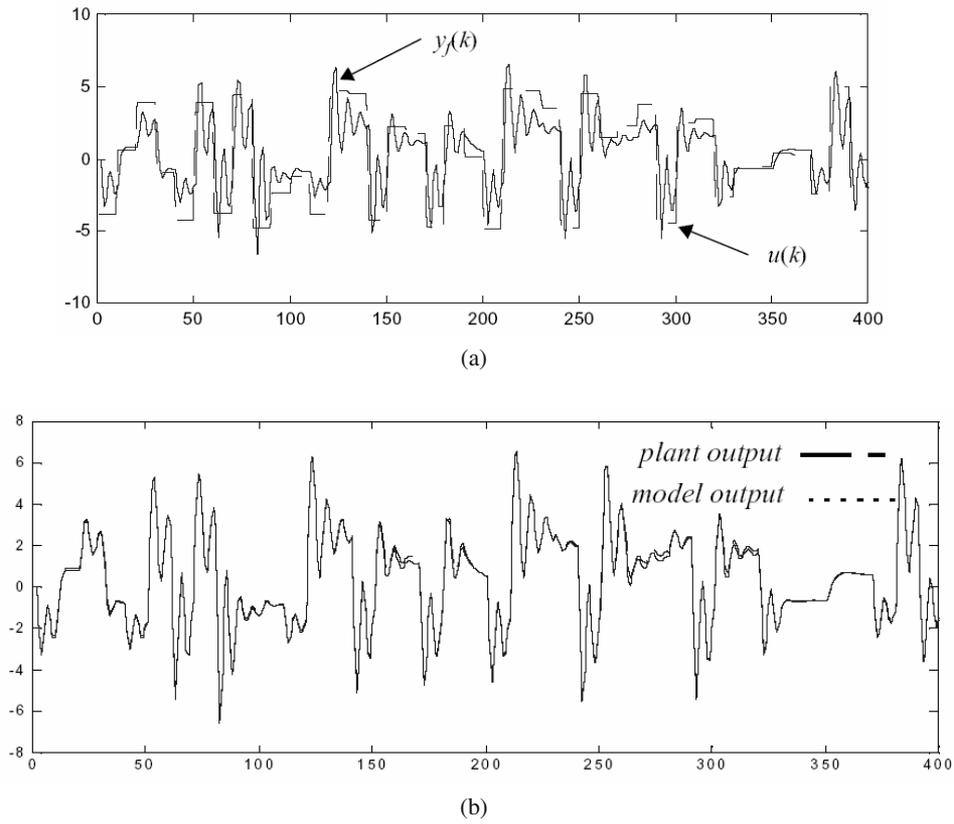


Fig. 10. Fuzzy identification performances: (a) the control input and the fuzzy model output, (b) the fuzzy model output and the plant output.

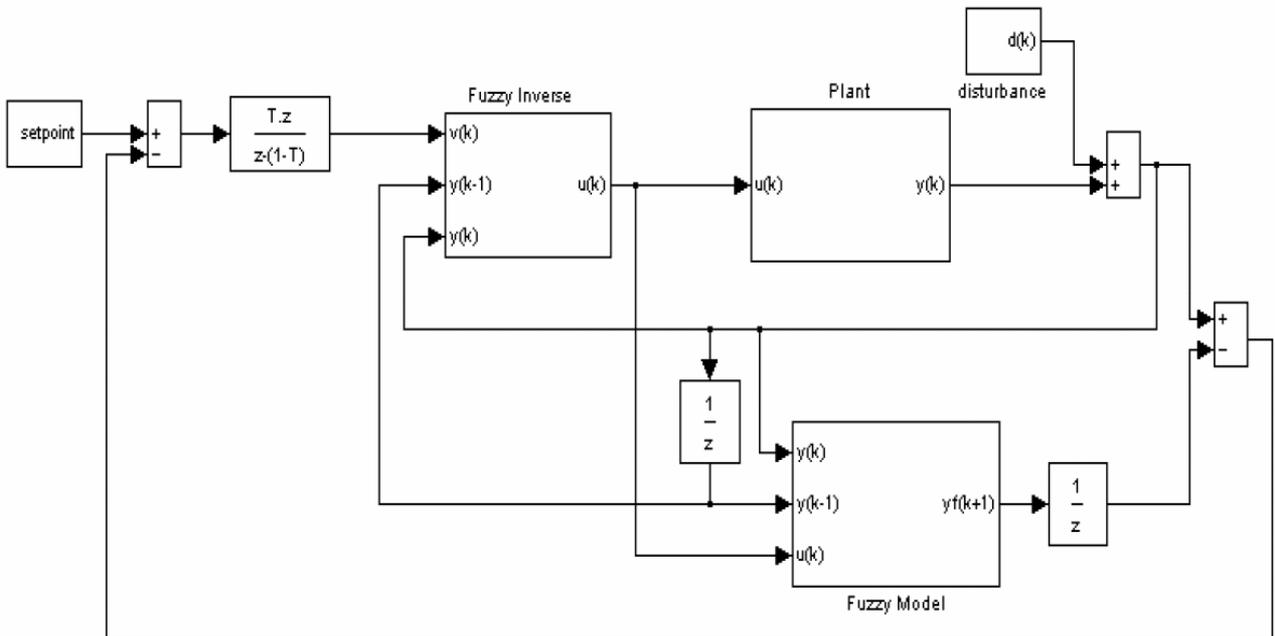


Fig. 11. Fuzzy NIMC implementation.

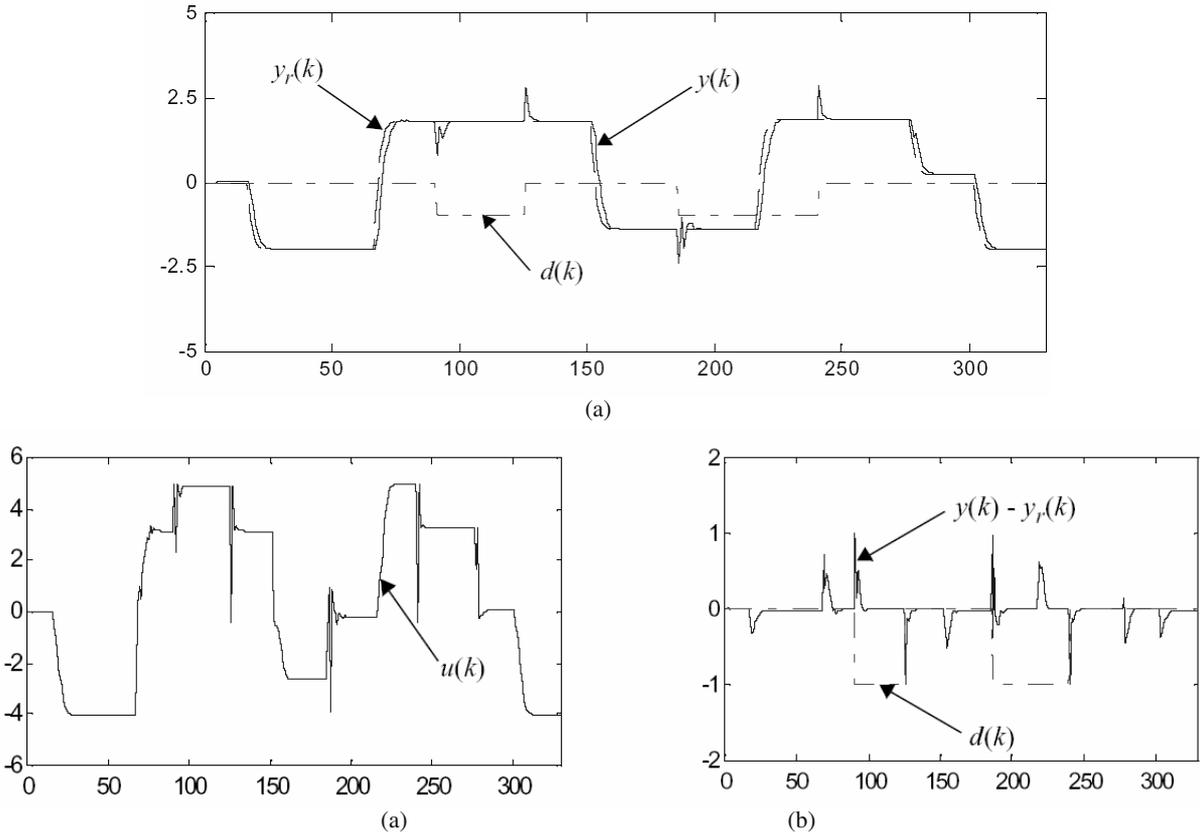


Fig. 12. Control NIMC performances: (a) the response to changes in the reference signal with an additive disturbance $d(k)$, (b) the control signal, (c) the evolution of the error signal.

greater than one, it may have a significant effect on the control system performances.

A useful method to overcome the effect of time delays consists in using the prediction approach. In this framework, it is important to determine the delay elements within the model. Several techniques have been developed to deal with this problem. For example, in (Liao and Chen, 1997), a dispersion function technique based on an orthogonal projection is used. In this paper, this task is not considered. In other words, the system time delay is supposed to be known or to have been identified off-line from input-output data.

According to the general representation given in (1) and by generalizing the fuzzy representation (8), at each sampling time the fuzzy system output can be reformulated as

$$\begin{aligned} y(k+r) &= y(k+\theta+1) \\ &= y_f(k+\theta+1) + \Delta(k+\theta+1) \\ &= \alpha^{(\lambda)} [Z_1(k)] + \beta^{(\lambda)} [Z_1(k)] u(k) \\ &\quad + \Delta(k+\theta+1), \end{aligned} \quad (22)$$

where $\theta = r - 1$ is the effective time-delay. The nonlinear functions $\alpha^{(\lambda)}$ and $\beta^{(\lambda)}$ are deduced from the decomposi-

tion of the *TS* fuzzy model given by a collection of rules of the form

$$\begin{aligned} \text{If } y(k) \text{ is } A_1^{i_1} \text{ and } y(k-1) \text{ is } A_2^{i_2} \text{ and } \dots \text{ and} \\ y(k-n+1) \text{ is } A_n^{i_n} \text{ and } u(k) \text{ is } B_1^{j_1} \text{ and } \dots \text{ and} \\ u(k-m+1) \text{ is } B_m^{j_m} \\ \text{then } y_f(k+\theta+1) = \phi^{i_1, \dots, i_n, j_1, \dots, j_m}. \end{aligned} \quad (23)$$

Let us now study both of the control structures given in the previous sections. In the same spirit of synthesis, if the feedback linearizing control law $u(k)$ is chosen as (10), then the following linear equation is obtained:

$$y(k+r) = y(k+\theta+1) = v(k) + \Delta(k+\theta+1). \quad (24)$$

In order to ensure the desired control performances in the presence of uncertainties and disturbances, the linearization controller is augmented by a linear controller. As before, one can design the linear controller C so that the open loop contains an integrator.

In this case, the controller C is chosen as

$$C(z^{-1}) = \frac{T}{1 - (1-T)z^{-1} - Tz^{-\theta-1}}. \quad (25)$$

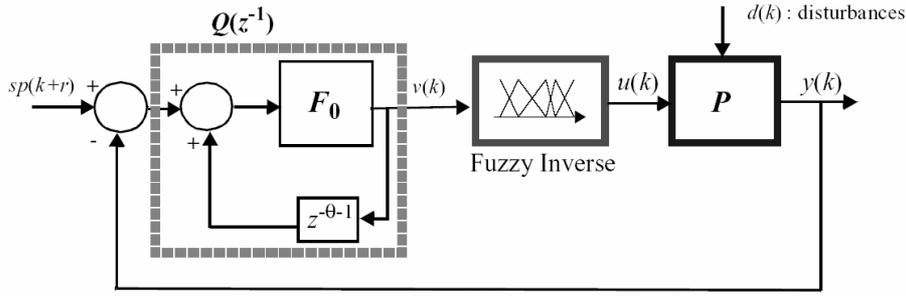


Fig. 13. Transformation of the fuzzy internal model control structure.

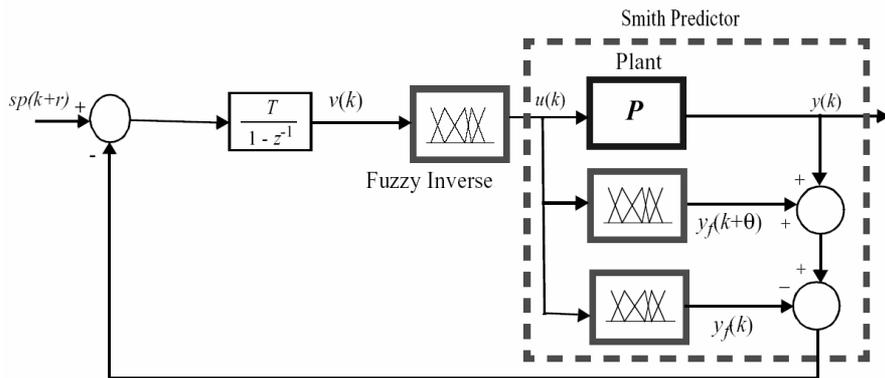


Fig. 14. Fuzzy IMC structure with a Smith predictor.

As for the FNIMC structure, in the presence of θ time delays, the control structure can be transformed into the feedback scheme given in Fig. 13.

If the robustness filter F_0 is chosen as (14), it follows that

$$Q(z^{-1}) = \frac{T}{1 - (1 - T)z^{-1} - Tz^{-\theta-1}}. \quad (26)$$

From (25) and (26), the equivalence between the FNIMC and the feedback linearization controller is established.

Let us now show that, in the presence of time delays, the FNIMC structure includes naturally a Smith predictor. The robustness filter output can be written as

$$v(k) = v(k - 1) + T \{ [sp(k + r) - e(k)] - v(k - 1) \}. \quad (27)$$

As the model inversion is exact, i.e., $y_f(k + r) = y(k + \theta + 1) = v(k)$, it follows that $v(k - 1) = y_f(k + \theta)$, which implies

$$v(k) = v(k - 1) + T \{ [sp(k + r) - e(k)] - y_f(k + \theta) \} = v(k - 1) + T \{ sp(k + r) - [e(k) + y_f(k + \theta)] \}. \quad (28)$$

According to the definition of the Smith predictor, the FNIMC control structure can be transformed into that

of Fig. 14. From this it appears clear that the control structure integrates a Smith predictor. Indeed, the input is fed to the true plant, and to the fuzzy model with and without time delays. In this case, it can be stated that the effect of the time-delay has been eliminated and the predictor output can be used as a feedback signal in the controller design.

7.2. Illustrative Example. The plant under consideration is the same as the one studied in Section 6.2, but now with 10 time delays (Rivals and Personnaz, 2000):

$$\begin{cases} X(k) = \frac{24 + x(k - 1)}{30}x(k - 1) \\ \quad - 0.8 \frac{u^2(k - 9)}{1 + u^2(k - 9)}x(k - 2) + 0.5u(k - 9), \\ y(k) = x(k) + d(k). \end{cases}$$

It follows that the system input-output behavior can be written in the following form:

$$y(k + \theta + 1) = y(k + 10) = \Psi [y(k), y(k - 1), u(k - 9)]. \quad (29)$$

By adopting the fuzzy identification strategy detailed in Section 6.2.1, a fuzzy model of the plant is determined.

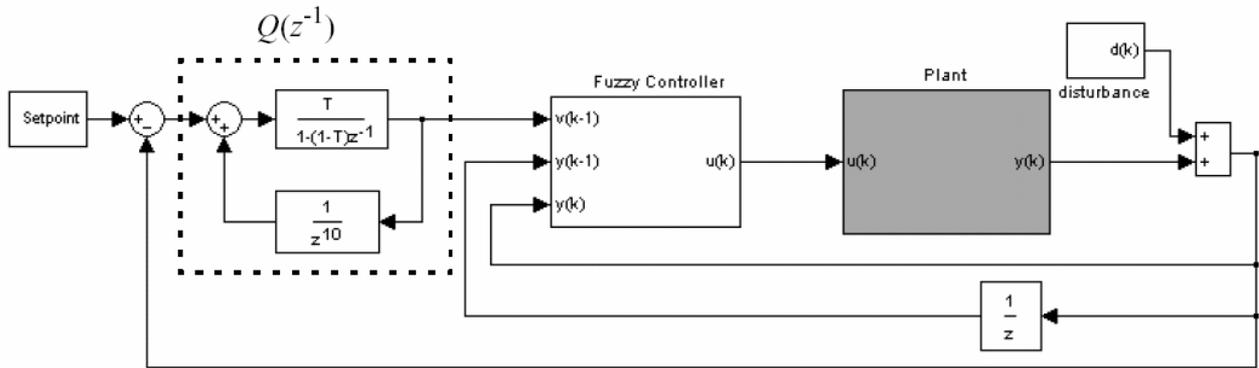


Fig. 15. FLC implementation.

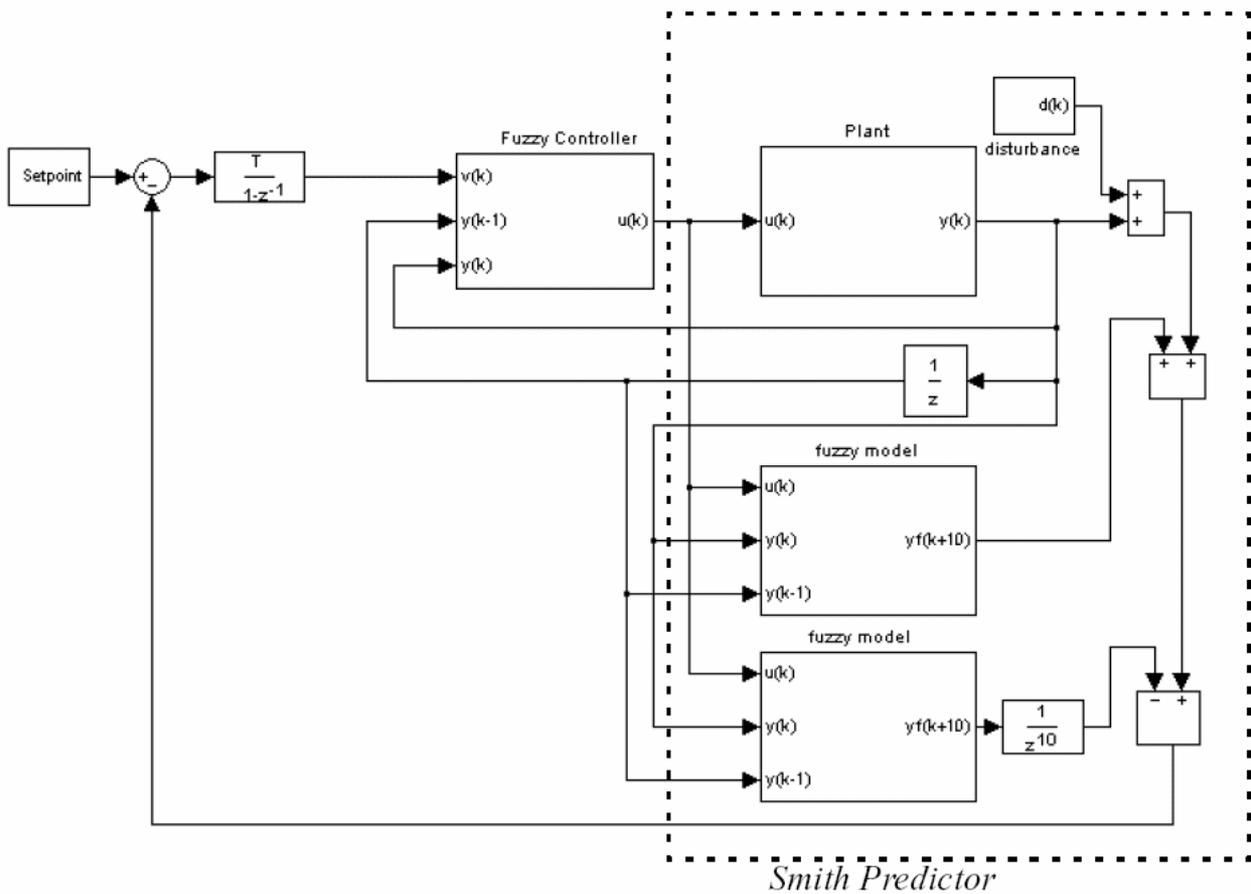


Fig. 16. FNIMC implementation.

The latter is used to synthesize a fuzzy inverse controller which is implemented either in the FLC or in the FNIMC structure. The implemented control structures are illustrated in Figs. 15 and 16. Figure 15 shows the FLC implementation with the introduction of the required time delay as expressed by (26). Figure 16 illustrates the FNIMC implementation and exhibits the occurrence of a Smith predictor.

As expected, the obtained simulation results (displayed in Figs. 17 and 18) are exactly the same for both architectures. If we accept the errors due to the pure time delay and the disturbance transitions, it can be deduced that the control performances are good (comparable to those obtained in (Rivals and Personnaz, 2000)), which illustrates that the proposed technique is able to control systems even in the presence of time delays. In other words,

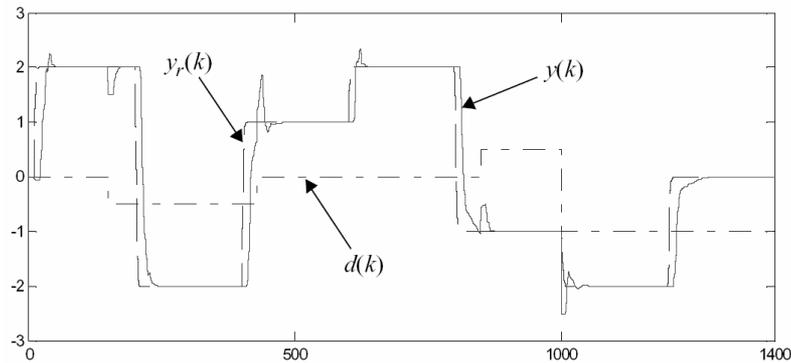


Fig. 17. Response to changes in the reference signal with an additive disturbance $d(k)$.

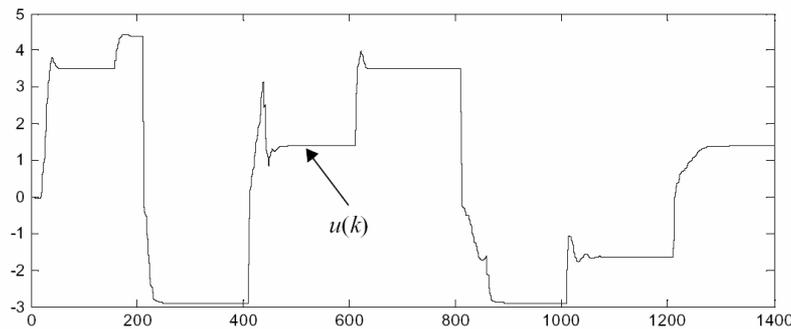


Fig. 18. Control signal.

the advantages of the fuzzy inverse approach expressed at the end of Section 6.2.2 are still valid.

8. Discussion and Conclusions

This paper has demonstrated how fuzzy systems and their inverses can be used in inverse model based control for stable nonlinear systems. In this context, two main approaches have been studied according to the proposed inversion principle.

Some remarks and design considerations can now be outlined concerning the implementation of the control methodologies previously presented:

- For the implementation of the proposed control techniques, some prior knowledge concerning the input-output behavior of the plant is needed for identification. The required information includes the system order, the relative degree of the system and the stability of the zero dynamics. A study of the fuzzy model zero dynamic stability was performed in (Boukezzoula et al., 2003).
- In this paper, the fuzzy identification procedure was implemented using the technique developed in (Nakoula et al., 1997). But other methods published

in the literature, which guarantee the system decomposability property, can also be used (Johansen and Foss, 1995).

- Whatever the chosen identification methodology, it is necessary to select a rich excitation signal plant input. Generally, a random signal with a uniformly distributed amplitude is used to generate the input-output identification data. If it had to be employed in industrial applications, a more realistic excitation signal would have to be chosen.
- In order to overcome the effect of time delays on control performances, prediction approaches are frequently used, especially the Smith predictor. In this context, it was shown that in the presence of time-delays, the FNIMC structure, which is equivalent to the fuzzy feedback linearization controller, includes a Smith predictor.
- From the controller design point of view, the proposed inversion technique is exact and noniterative, and it can be applied to nonaffine-in-control nonlinear systems. These properties are major advantages compared with other nonlinear fuzzy and/or neural techniques given in the literature.
- The use of *TS* fuzzy systems with singleton consequences may produce a significant number of rules

compared with *TS* fuzzy systems with nonlinear input membership functions and/or polynomial conclusions. In the proposed methodology, this problem does not have a significant impact because when assuming decomposable fuzzy systems, only one subsystem is fired at each sampling time, which decreases the number of rules to be actually considered.

- Although the proposed techniques are dedicated to fuzzy *TS* systems with singleton conclusions, they can be directly applied to Mamdani fuzzy systems subject to the constraints of the product-sum gravity inference method (Mizumoto, 1993). Furthermore, the developed strategy can be easily extended to any fuzzy system which is piecewise affine in the inversion variable.
- The proposed control structures can also be integrated into conventional adaptive and model reference control schemes to deal with plants with varying parameters. Indeed, the fuzzy model and the inverse model controller parameters can be determined on-line using closed-loop input-output data (Boukezzoula *et al.*, 2001; Boukezzoula *et al.*, 2004). In this context, the advantage of using decomposable fuzzy systems is that they can be viewed as collections of affine-in-control systems and permit the application of the adaptive control methodology in a local way.

References

- Babuska R. (1998): *Fuzzy Modeling for Control*. — Dordrecht: Kluwer Academic Publishers.
- Baoming G., Jingping J., Pengsheng S. and Xiangheng W. (2002): *Nonlinear internal-model control for switched reluctance drives*. — IEEE Trans. Power Electron., Vol. 17, No. 3, pp. 379–388.
- Baranyi P., Bavelaar I., Babuska R., Koczy L.T., Titli A. and Verbruggen H.B. (1998): *A method to invert a linguistic fuzzy model*. — Int. J. Systems Sci., Vol. 29, No. 7, pp. 711–721.
- Boukezzoula R., Galichet S. and Foulloy L. (2001): *Fuzzy nonlinear adaptive internal model control (FNAIMC)*. — Europ. J. Contr., Vol. 7, No. 5, pp. 523–540.
- Boukezzoula R., Galichet S. and Foulloy L. (2003): *Nonlinear internal model control: Application of inverse model based fuzzy control*. — IEEE Trans. Fuzzy Syst., Vol. 11, No. 6, pp. 814–829.
- Boukezzoula R., Galichet S. and Foulloy L. (2004): *Observer-based fuzzy adaptive control for a class of nonlinear systems: Real-time implementation for a robot wrist*. — IEEE Trans. Contr. Syst. Technol., Vol. 12, No. 3, pp. 340–351.
- Boukezzoula R., Foulloy L. and Galichet S. (2006): *Inverse controller design for interval fuzzy systems*. — IEEE Trans. Fuzzy Syst., Vol. 14, No. 1, pp. 111–124.
- Cabrera J.B.D and Narendra K.S. (1999): *Issues in the application of neural networks for tracking based on inverse control*. — IEEE Trans. Automat. Contr., Vol. 44, No. 11, pp. 2007–2027.
- Chen F.C. and Khalil H.K. (1995): *Adaptive control of a class of nonlinear discrete-time systems using neural networks*. — IEEE Trans. Automat. Contr., Vol. 40, No. 5, pp. 791–801.
- Devanathan R., Rahman F. and Kuanyi Z. (2000): *Neural network approach for linearizing control of nonlinear plants*. — IEEE Trans. Ind. Electron., Vol. 47, No. 2, pp. 470–477.
- Economou C.G., Morari M. and Palsson B.O. (1986): *Internal model control 5. Extension to nonlinear systems*. — Ind. Eng. Chem. Proc. Des. Dev., Vol. 25, No. 5, pp. 403–409.
- Fang W. and Rad A. B. (2000): *Fuzzy adaptive internal model control*. — IEEE Trans. Ind. Electron., Vol. 47, No. 1, pp. 193–202.
- Foulloy L., Boukezzoula R. and Galichet S. (2006): *An educational tool for fuzzy control*. — IEEE Trans. Fuzzy Syst., Vol. 14, No. 2, pp. 217–221.
- Galichet S., Boukezzoula R. and Foulloy L. (2004): *Explicit analytical formulation and exact inversion of decomposable fuzzy systems with singleton consequents*. — Fuzzy Sets Syst., Vol. 146, No. 3, pp. 421–436.
- Hunt K.J. and Sbarbaro D. (1991): *Neural Networks for Nonlinear Internal Model Control*. — IEE Proc. D, Vol. 138, No. 5, pp. 431–438.
- Jagannathan S. (1998): *Adaptive fuzzy logic control of feedback linearizable discrete-time dynamical systems under persistence excitation*. — Automat., Vol. 34, No. 11, pp. 1295–1310.
- Jagannathan S. (1999): *Discrete-time CMAC NN control of feedback linearizable nonlinear systems under persistence of excitation*. — IEEE Trans. Neural Netw., Vol. 10, No. 1, pp. 128–137.
- Johansen T.A. and Foss B.A. (1995): *Identification of nonlinear system structure and parameters using regime decomposition*. — Automat., Vol. 31, No. 2, pp. 321–326.
- Kambhampati C., Mason J. and Warwick K. (2000): *A stable one-step-ahead predictive control of non-linear systems*. — Automat., Vol. 36, No. 4, pp. 485–495.
- Kang H.J., Kwon C., Lee H. and Park M. (1998): *Robust stability analysis and design method for the fuzzy feedback linearization regulator*. — IEEE Trans. Fuzzy Syst., Vol. 6, No. 4, pp. 464–472.
- Knospe C.R. and Lindlau J.D. (2000): *Feedback linearization of an active bearing with voltage control*. — IEEE Trans. Contr. Syst. Technol., Vol. 10, No. 1, pp. 21–31.
- Kwanghee N. (1999): *Stabilization of feedback linearizable systems using a radial basis function network*. — IEEE Trans. Automat. Contr., Vol. 44, No. 5, pp. 1026–1031.
- Leland R.P. (1998): *Feedback linearization control design for systems with fuzzy uncertainty*. — IEEE Trans. Fuzzy Syst., Vol. 6, No. 4, pp. 492–503.
- Li H-X and Deng H. (2006): *An approximate internal model-based neural control for unknown nonlinear discrete processes*. — IEEE Trans. Neural Netw., Vol. 17, No. 3, pp. 659–670.

- Liao H.E. and Chen W.S. (1997): *Determination of nonlinear delay elements within NARMA models using dispersion functions*. — IEEE Trans. Instrum. Meas., Vol. 46, No. 4, pp. 868–872.
- Lightbody G. and Irwin G.W. (2000): *Nonlinear control structures based on embedded neural networks*. — IEEE Trans. Neural Netw., Vol. 11, No. 3, pp. 553–567.
- Morari M. and Zafiriou E. (1989): *Robust Process Control*. — Englewood Cliffs, NJ: Prentice-Hall.
- Mizumoto M. (1993): *Fuzzy control under product-sum gravity methods and new fuzzy control methods*, In: *Fuzzy Control Systems* (A. Kandel and G. Langholz, Eds.). — Boca Raton, FL: CRC Press, pp. 276–294.
- Nahas E.P., Henson M.A. and Seborg D.E. (1992): *Nonlinear internal model control strategy for neural networks*. — Comput. Chem. Eng., Vol. 16, No. 12, pp. 1039–1057.
- Nakoula Y., Galichet S. and Foulloy L. (1997): *A learning method for structure and parameter identification of fuzzy linguistic models*, In: *Selected Approaches for Fuzzy Model Identification* (H. Hellendoorn and D. Driankov, Eds.). — Berlin: Springer Verlag, pp. 282–319.
- Narendra K.S. and Mukhopadhyay S. (1997): *Adaptive control using neural networks and approximate models*. — IEEE Trans. Neural Netw., Vol. 8, No. 3, pp. 475–485.
- Narendra K.S. and Parthasarathy K. (1990): *Identification and control of dynamical systems using neural networks*. — IEEE Trans. Neural Netw., Vol. 1, No. 1, pp. 4–27.
- Park S. and Han T. (2000): *Iterative inversion of fuzzified neural networks*. — IEEE Trans. Fuzzy Syst., Vol. 8, No. 3, pp. 266–280.
- Rivals I. and Personnaz L. (2000): *Nonlinear internal model control using neural networks: Application to processes with delay and design issues*. — IEEE Trans. Neural Netw., Vol. 11, No. 1, pp. 80–90.
- Rovatti R. (1998): *Fuzzy piecewise multilinear and piecewise linear systems as universal approximators in Sobolev norms*. — IEEE Trans. Fuzzy Syst., Vol. 6, No. 2, pp. 235–249.
- Slotine J.J. and Li W. (1991): *Applied Nonlinear Control*. — Englewood Cliffs, NJ: Prentice-Hall.
- Sugeno M. (1999): *On stability of fuzzy systems expressed by fuzzy rules with singleton consequents*. — IEEE Trans. Fuzzy Syst., Vol. 7, No. 2, pp. 201–224.
- Wang L.X. (1993): *Stable adaptive fuzzy control of nonlinear systems*. — IEEE Trans. Fuzzy Syst., Vol. 1, No. 2, pp. 146–155.
- Wang L.X. (1994): *Adaptive Fuzzy Systems and Control. Design and Stability Analysis*. — Englewood Cliffs, NJ: Prentice Hall.
- Ying H. and Chen G. (1997): *Necessary conditions for some typical fuzzy systems as universal approximators*. — Automat., Vol. 33, No. 7, pp. 1333–1338.
- Ying H. (1999): *Analytical analysis and feedback linearization tracking control of the general Takagi-Sugeno fuzzy dynamic systems*. — IEEE Trans. Syst. Man Cybern., Part C: Applic. Rev., Vol. 29, No. 1, pp. 290–298.
- Zeng X.J. and Singh M.G. (1996a): *Decomposition property of fuzzy systems and its applications*. — IEEE Trans. Fuzzy Syst., Vol. 4, No. 2, pp. 149–165.
- Zeng X. J. and Singh M.G. (1996b): *Approximation accuracy analysis of fuzzy systems as function approximators*. — IEEE Trans. Fuzzy Syst., Vol. 4, No. 1, pp. 44–63.

Received: 3 November 2006

Revised: 22 February 2006