ON SOME PROPERTIES OF GROUNDING NONUNIFORM SETS OF MODAL CONJUNCTIONS

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A language grounding problem is considered for nonuniform sets of modal conjunctions consisting of conjunctions extended with more than one modal operator of knowledge, belief or possibility. The grounding is considered in the context of semiotic triangles built from language symbols, communicative cognitive agents and external objects. The communicative cognitive agents are assumed to be able to observe external worlds and store the results of observations in internal knowledge bases. It is assumed that the only meaning accessible to these agents and assigned to modal conjunctions can be extracted from these internal knowledge bases. Commonsense requirements are discussed for the phenomenon of grounding nonuniform sets of modal conjunctions and confronted with an original idea of epistemic satisfaction relation used to define proper conditions for language grounding. Theorems are formulated and proved to show that the communicative cognitive agents based on the proposed model of grounding fulfill some commonsense requirements given for processing sets of nonuniform modal conjunctions. The main result is that the communicative cognitive agents considered can be constructed in a way that makes them rational and intentional as regards the processing of modal conjunctions and from the human point of view.

Keywords: cognitive agent, semantic communication, language grounding, conjunction, modality, artificial cognition

1. Introduction

The grounding of modal conjunctions is a sub-case of a broader phenomenon known as symbol grounding. This phenomenon has to be considered in a richer context of other research carried out in the field of artificial intelligence and artificial cognition. In these two fields, in the 1990s a new paradigm was widely accepted according to which systems need material bodies to produce intelligent behavior. This approach to intelligent behavior was at first developed in robotics, where an influential group of researchers rejected the necessity to define intelligence on a symbolic level and assumed it to be an emergent property resulting from an autonomous activity of reactive modules constituting the bodies of robots (Brooks 1991a; 1991b). Soon this change in fundamental theoretical assumptions resulted in the bulk of new ideas known under the term "embodied artificial intelligence" (Chrisley, 2003). In the meantime, a similar evolution was realized in cognitive sciences, where previous assumptions of symbolic and computational nature of cognition (Newell, 1990) have been replaced by new ideas underlying the embodiment of cognitive processes in natural cognitive systems (Lakoff and Johnson, 1999). This assumed property of cognition had originally been mainly attributed to biological cognitive systems. However, it was soon accepted by many research groups involved in the development of artificial cognition (Tomasello, 2000). In the last two decades, the theoretical similarity between embodied artificial intelligence and embodied artificial cognition (Anderson, 2003; Chrisley, 2003) has led to publishing many interesting papers, some of which were directly dedicated to the problem of grounding languages (Harnad, 1990; 1994; Ziemke, 1999).

Developing computer systems that could carry out meaningful conversations with human users has always been one of the main research issues studied in artificial intelligence and related fields of science and technology. However, the usual approach to modeling language production and comprehension assumed that the language is a product of mechanically realized manipulations on symbols and does not need material bodies to be created. New paradigms in artificial intelligence and artificial cognition forced researchers to change this assumption and to take into account the embodiment and grounding as necessary factors for the production of the language. In consequence, since the mid-1990s, the field of language comprehension and language production, which both belong to artificial intelligence and cognition, has seriously evolved. The grounding problem and the related anchoring problem (Tomasello, 2000; Vogt, 2002; 2003) have become intensively studied in modern computer science and technology (Roy and Ritter, 2005).

Perhaps the most popular and influential definition of language grounding accepted in artificial intelligence and cognitive sciences was formulated by Harnad (1990): "*How can the semantic interpretation of formal symbol system be made intrinsic to the system, rather than just parasitic on the meanings in our heads? How can the meanings of meaningless symbol tokens, manipulated solely on the basis of their (arbitrary) shapes, be grounded in anything but other meaningless symbols? The problem is analogous to trying to learn Chinese from a Chinese/Chinese dictionary alone.*" Unfortunately, this popular definition has been usually interpreted in a limited way. Namely, for quite a long period the language grounding problem was usually studied on the level at which the internal cognitive processes carried out by agents are related to empirical data incoming from the external world. This interpretation of grounding is based on the fact that many symbols of real languages for semantic communication consist of complicated symbols that cannot be grounded directly to the external world and require intermediate processing of meaning on various levels of embodied conceptualization. This simplified interpretation resulted in an undesirable and generally wrong impression that effective implementations of grounding (and anchoring) do not need to deal with multilevel systems of mental structures. A further result was that the majority of previous work on the grounding and anchoring of symbols was concentrated on relatively simple classes of languages (Vogt, 2002; 2003). (Which does not mean that these works are not valuable.) Latest attempts known from the literature have tried to fill this gap in theoretical models, and the grounding problem has become to be assumed to consist of a richer collection of problems. In particular, the study of a direct relation between simple communication languages and the external reality was extended with similar research carried out for more advanced and richer symbols. To be grounded, these symbols seem to require a consideration of higher-level structures. An interesting example of such an extension is given in the work (Roy, 2005), where the so-called semiotic schemas were considered in order to cover advanced cases of grounding. Another example is given in this work, where symbols to be grounded are complex syntactic structures built from less combined symbols for atom predicates, logic negation, logic conjunction, modal operators of belief, possibility, knowledge, etc. Moreover, the important feature is that in order to relate symbols of the language considered below, it is necessary to take into account the content of empirical experience stored in dedicated and embodied knowledge bases of communicative systems. Otherwise, the model of grounding would be incomplete on the theoretical level.

In what follows, the phenomenon of grounding is considered for communicative cognitive agents which are expected to produce intentional language behavior provided that the intentionality of behavior is understood as in the cognitive paradigm (Denett, 1996; Newell, 1990). The phenomenon of grounding is often considered in the context of the semiotic triangle (Eco, 1991) consisting of three elements: a symbol of a language, an agent which is able to produce, perceive and interpret this symbol, and an object that is external to the agent and pointed at by this symbol. The application of the semiotic triangle to studies of language grounding has been intensively used in many works (see, e.g., Roy, 2005; Vogt, 2002; 2003), and has been accepted in the approach proposed in this one and previous papers (see, e.g., Katarzyniak, 2005). In this research, modal conjunctions are language symbols used by communicative cognitive agents in order to communicate their state of knowledge about an external object. Each modal conjunction is produced in order to attract the attention of potential receivers towards this external object and represents a piece of subjective knowledge about this object. This piece of knowledge is always autonomously developed by the communicative cognitive agent.

According to the accepted definition of grounding, this original approach to study the phenomenon of language grounding developed in the previous works (Katarzyniak, 2004a, 2004c) assumes that there exists a subtle relation between each communicative cognitive agent and each language symbol produced by this agent in order to communicate its internal states of knowledge. In this approach, such a relation is treated as equivalent to the phenomenon of grounding known from the case of natural communicative cognitive agents, and due to its nature is called the epistemic satisfaction relation (Katarzyniak, 2005). The epistemic satisfaction relation is complementary to the classic satisfaction of formulas developed by Tarski in his well-known theory of truth, where the satisfaction relation always binds an external object with a language symbol instead of binding it with a subjective state of knowledge (Tarski, 1935).

The basic role of the epistemic satisfaction relation is to define internal states of communicative cognitive agents in which a proper binding (not contradictory to the commonsense of language production) between external language messages and internal knowledge states takes place. The appropriateness of a particular case of grounding is evaluated according to some criteria developed in the commonsense perspective of natural language discourse. Moreover, in the proposed approach it is expected that a particular modal conjunction would be used by a communicative cognitive agent as an external representation of knowledge in these, and only in these, situations in which a natural communicative cognitive agent would use the natural language counterpart for this conjunction to communicate the same content of knowledge. The latter assumption is in common with the intentional systems paradigm (Denett, 1996; Newell, 1990).

The detailed research target defined in this paper is to prove that the originally proposed definition of epistemic satisfaction of modal conjunctions makes it possible to generate language behavior exhibiting at least some of the basic commonsense properties of natural language semantics and pragmatics. The grounding is considered for a case of nonuniform sets of modal conjunctions in which conjunctions are extended with different modal operators of knowledge, belief or possibility.

The organization of the text is as follows: In Section 2, a brief overview of the basic formal concepts used in the model of grounding and related to the idea of semiotic triangle is presented. Section 2 consists of the original definition of epistemic grounding given for the case of modal conjunctions. In Section 3, some results from the previous analysis of the proposed definition of grounding uniform sets of modal conjunctions are briefly overviewed. These results are considered in detail in other works (Katarzyniak, 2005a; 2006). Section 4 contains two groups of results: a detailed discussion of commonsense requirements for the grounding of nonuniform sets of modal conjunctions, and a detailed presentation of theorems in which the most important properties of grounding modal conjunctions are covered. The final section contains comments on practical consequences resulting from this study.

2. Overview of a Model for Grounding Modal Conjunctions

Let the language L of modal conjunctions considered in this paper be given as in other works (Katarzyniak, 2005a; 2006). This language is a set of all formulas of the form 'α' or '$\Sigma(\alpha)$', where $\alpha \in \{p(o) \wedge q(o),\ p(o)\neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$, $p \neq q$, and $\Sigma \in \{\mathit{Know}, \mathit{Bel}, \mathit{Pos}\}$. It is assumed that $o \in O = \{o_1, o_2, \dots, o_M\}$ denotes an object from the external world and the symbols 'p' and 'q' are referred to the properties P and Q of the external object o, respectively. Each conjunction is assigned intentional (commonsense) meaning, e.g., the conjunction $p(o) \wedge q(o)$ represents the content that is communicated in the natural language by the sentence "Object o exhibits the property P and exhibits the property Q." The role and interpretation of other conjunctions are similar. The modal extensions *Pos*, *Bel*, *Know* are interpreted as usual and stand for possibility, belief and knowledge, respectively (Katarzyniak, 2005a; 2006).

In the actual language behavior in which formulas are used as interpreted, each modal conjunction belongs to an instantiation of the general idea of the semiotic triangle. The artificial communicative cognitive agent is assumed to embody a collection of observations of properties P and Q for an object o. These representations of observations differ from one to another as regards their content, and all of them can reflect the object o as exhibiting a particular distribution of both properties P and Q. These distributions reflect the observed states of the object o at time points when observations were taken by the agent. As was given in (Katarzyniak, 2004c; 2005; Katarzyniak and Nguyen, 2000), the content of each individual observation is stored in the so-called base profile:

$$BP(t) = \big\{\langle O, P_1^+(t), P_1^-(t), P_2^+(t), P_2^-(t), \dots, P_K^+(t), P_K^-(t)\rangle\big\}, \quad (1)$$

where $t \in T = \{0, 1, 2, \dots\}$ is a time point at which the observation was realized, $O = \{o_1, \dots, o_M\}$ is the set of all individual objects known to the communicative cognitive agents, $P_1, P_2, \dots, P_K$ are properties that can be exhibited by objects from O, and for each $i = 1, 2, \dots, K$ and $o \in O$:

- $P_i^+(t) \subseteq O$ and $P_i^-(t) \subseteq O$ hold,
- $o \in P_i^+(t)$ holds if and only if o was observed as exhibiting P_i at the time point t,
- $o \in P_i^-(t)$ holds if and only if o was observed as nonexhibiting P_i at the time point t.

Obviously, in this case for each $i = 1, 2, \dots, K$, the condition $P_i^+(t) \cap P_i^-(t) = \emptyset$ has to be fulfilled, and at each time point $t \in T$ the overall state of empirical knowledge is given by the following temporally ordered set of base profiles:

$$\mathit{KnowledgeState}(t) = \big\{BP(t_n) : t_n \in T \ \text{ and } \ t_n \leq^{\mathrm{TM}} t\big\}, \quad (2)$$

where the symbol $\leq^{\mathrm{TM}}$ denotes temporal precedence. As was already explained in other works (Katarzyniak, 2004a; 2004c; 2005; 2005a), each group of base profiles in which observations with the same distribution of the properties P and Q in the object o are stored induces a related mental model for nonmodal conjunction from L. In this approach, the role of mental models is understood similarly as it is assumed for the case of natural cognitive systems, namely, each mental model represents the meaning of assigned nonmodal conjunction (Johnsons-Laird, 1983). Following the approach suggested in (Katarzyniak, 2004c), individual symbols for particular mental models are given:

- m_1^c representing the mental model for the nonmodal conjunction $p(o) \wedge q(o)$,
- m_2^c representing the mental model for the nonmodal conjunction $p(o) \wedge \neg q(o)$,
- m_3^c representing the mental model for the nonmodal conjunction $\neg p(o) \wedge q(o)$,
- m_4^c representing the mental model for the nonmodal conjunction $\neg p(o) \wedge \neg q(o)$.

It is assumed that at each time point t, all mental models m_c^1, m_2^c, m_3^c and m_c^4 are extracted (induced) from the related subsets of base profiles defined below (Katarzyniak, 2004c):

$$C^1(t) = \Big\{BP(t_n) : t_n \leq^{\mathrm{TM}} t \text{ and } BP(t_n) \in \textit{KnowledgeState}(t) \text{ and } o \in P^+(t_n) \text{ and } o \in Q^+(t_n)\Big\}, \quad (3)$$

$$C^2(t) = \Big\{BP(t_n) : t_n \leq^{\mathrm{TM}} t \text{ and } BP(t_n) \in \textit{KnowledgeState}(t) \text{ and } o \in P^+(t_n) \text{ and } o \in Q^-(t_n)\Big\}, \quad (4)$$

$$C^3(t) = \Big\{BP(t_n) : t_n \leq^{\mathrm{TM}} t \text{ and } BP(t_n) \in \textit{KnowledgeState}(t) \text{ and } o \in P^-(t_n) \text{ and } o \in Q^+(t_n)\Big\}, \quad (5)$$

$$C^4(t) = \Big\{BP(t_n) : t_n \leq^{\mathrm{TM}} t \text{ and } BP(t_n) \in \textit{KnowledgeState}(t) \text{ and } o \in P^-(t_n) \text{ and } o \in Q^-(t_n)\Big\}, \quad (6)$$

where each $C^i(t)$ corresponds to $m_i^c,\ i = 1, 2, 3, 4$. In other works (Katarzyniak, 2004a; 2004c; 2005a; 2006), it was assumed that mental models are more or less strong (comparing one to another), and these mental models are relatively stronger, which is induced (extracted) from the relatively richer subsets $C^1(t), C^2(t), C^3(t)$ or $C^4(t)$. The so-called relative grounding values are used to measure this level of intensity (Katarzyniak, 2006):

$$\lambda\big(t, p(o) \wedge q(o)\big) = \frac{\mathrm{card}(\mathrm{C}^1(t))}{\sum_{j=1}^{4} (\mathrm{card}(\mathrm{C}^j(t)))}, \quad (7)$$

$$\lambda\big(t, p(o) \wedge \neg q(o)\big) = \frac{\mathrm{card}(\mathrm{C}^2(t))}{\sum_{j=1}^{4} (\mathrm{card}(\mathrm{C}^j(t)))}, \quad (8)$$

$$\lambda\big(t, \neg p(o) \wedge q(o)\big) = \frac{\mathrm{card}(\mathrm{C}^3(t))}{\sum_{j=1}^{4} (\mathrm{card}(\mathrm{C}^j(t)))}, \quad (9)$$

$$\lambda\big(t, \neg p(o) \wedge \neg q(o)\big) = \frac{\mathrm{card}(\mathrm{C}^4(t))}{\sum_{j=1}^{4} (\mathrm{card}(\mathrm{C}^j(t)))}. \quad (10)$$

For each communicative cognitive agent, the related grounding values are evaluated against the so-called system of modality thresholds. The latter system consists of four numbers, $0 < \lambda_{\min \textit{Pos}} < \lambda_{\max \textit{Pos}} < \lambda_{\min \textit{Bel}} < \lambda_{\max \textit{Bel}} < 1$ which define the basic intervals of grounding intensity $[\lambda_{\min \textit{Pos}}, \lambda_{\max \textit{Pos}}]$, $[\lambda_{\min \textit{Bel}}, \lambda_{\max \textit{Bel}}]$ and $[1, 1]$, correlating with the assigned modal operators of possibility, belief and knowledge, respectively. The concept of systems of similarity thresholds is discussed in other works (Katarzyniak, 2006). Similar ideas were defined for the language of modal literals called simple modalities (Katarzyniak, 2005b) and suggested for other binary logic connectives (Katarzyniak, 2004a).

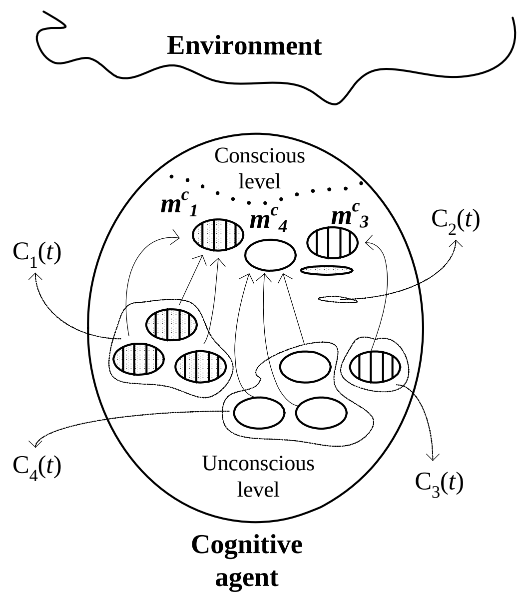

Fig. 1. Induction of mental models for nonmodal conjunctions.

Let us now illustrate the above concepts. In Fig. 1, the sets $C^1(t)$, $C^3(t)$ and $C^4(t)$ are nonempty. This means that the agent has already experienced all related distributions of properties P and Q in the object o at least once for each distribution. The only empty set is $C^2(t)$. In consequence, the mental model m_c2 is treated as nonactivated and in this sense inaccessible to the agent's internal cognitive processes. In other words, the commonsense meaning assigned to the symbol $p(o) \wedge \neg q(o)$ is not rational from the agent's point of view.

Another element of the model presented in Fig. 1 is the division of the internal cognitive space of the agent into the so-called conscious and nonconscious levels of cognition. The rationale for the integration of this property of natural cognition with the artificial cognition considered in this work was given in (Katarzyniak, 2004a; 2005) and results from theoretical assumptions known from cognitive linguistics (Freeman, 2000; Lakoff and Johnson, 1999; Paivio, 1986). It is assumed below that at each time point $t \in T$, the content of the empirical knowledge base should be treated as distributed over two interconnected but different levels of knowledge processing. The result is the following partition:

$$\textit{CognitionState}(t) = \{CM(t), NM(t)\}, \qquad (11)$$

called "the state of cognition", in which $CM(t)$ states for this part of experience that is located in the conscious cognitive subspace, $NM(t)$ is the remaining empirical material located on nonconscious levels of knowledge representation, $CM(t) \cup NM(t) = \textit{KnowledgeState}(t)$ and $CM(t) \cap NM(t) = \emptyset$. Obviously, this division can be applied to the grounding sets given by Eqns. (3)–(6), too, and leads to the following distribution of base profiles (Katarzyniak, 2005):

$$DC(t) = \Big\{RC^1(t), TC^1(t), RC^2(t), TC^2(t), RC^3(t), TC^3(t), RC^4(t), TC^4(t)\Big\}, \qquad (12)$$

where for each $i = 1, 2, 3, 4$ the following is assumed:

$$RC^i(t) = CM(t) \cap C^i(t),$$

$$TC^i(t) = NM(t) \cap C^i(t),$$

$$RC^i(t) \cap TC^i(t) = \emptyset,$$

$$RC^i(t) \cup TC^i(t) = C^i(t).$$

The partition into conscious and unconscious levels of knowledge representations makes the model of internal organization more complex. However, it is needed to take into account the actual nature of cognitive processes which always have to be perceived as highly integrated ones. In particular, no piece of data located on the conscious level of processing can be treated as a separated one if there are other copies of this piece located under the level of consciousness. In fact, this other part of the whole base profiles' collection influences participates indirectly in processes realized on the conscious level.

In the situation presented in Fig. 1, the set $CM(t)$ is empty. This means that the communicative cognitive agent has not collected any empirical experience that could relate the mental model m_2^c to the actual world. In this sense, this model is presented as a potential and ungrounded structure.

The above concepts cover the required set of elements that are used to define conditions for the proper grounding of modal conjunctions in the stored empirical knowledge, provided that the expected semantics of these modal conjunctions is given as in the commonsense language discourse. These requirements are given in the form of the epistemic satisfaction relation (Katarzyniak, 2004c; 2005a; 2006):

Definition 1. (*Epistemic satisfaction of modal extensions of $p(o) \wedge q(o)$*) Let the time point $t \in T$, the state of knowledge *KnowledgeState*(t) with the distribution

$$DC(t) = \Big\{RC^1(t), TC^1(t), RC^2(t), TC^2(t), RC^3(t), TC^3(t), RC^4(t), TC^4(t)\Big\},$$

and the system of modality thresholds $0 < \lambda_{\min \textit{Pos}} < \lambda_{\max \textit{Pos}} < \lambda_{\min \textit{Bel}} < \lambda_{\max \textit{Bel}} < 1$ be given. For each $P, Q \in \{P_1, \dots, P_K\}$ such that $P \neq Q$ and for each object o, the following epistemic satisfaction relation is defined:

The epistemic satisfaction
KnowledgeState$(t) \models_G$ *Pos*$(p(o) \wedge q(o))$
holds if and only if all requirements
$o \in O \backslash (P^+(t) \cup P^-(t))$, (13)
$o \in O \backslash (Q^+(t) \cup Q^-(t))$,
$RC^1(t) \neq \emptyset$
and $\lambda_{\min \textit{Pos}} \leq \lambda(t, p(o) \wedge q(o)) \leq \lambda_{\max \textit{Pos}}$ hold.

The epistemic satisfaction relation
KnowledgeState$(t) \models_G$ *Bel*$(p(o) \wedge q(o))$
holds if and only if all requirements
$o \in O \backslash (P^+(t) \cup P^-(t))$, (14)
$o \in O \backslash (Q^+(t) \cup Q^-(t))$,
$RC^1(t) \neq \emptyset$
and $\lambda_{\min \textit{Bel}} \leq \lambda(t, p(o) \wedge q(o)) \leq \lambda_{\max \textit{Bel}}$ hold.

The epistemic satisfaction relations
KnowledgeState$(t) \models_G$ *Know*$(p(o) \wedge q(o))$
and *KnowledgeState*$(t) \models_G p(o) \wedge q(o)$
hold if and only if either the conjunction
of requirements $o \in P^+(t)$ and $o \in Q^+(t)$ (15)
holds or the conjunction of requirements
$o \in O \backslash (P^+(t) \cup P^-(t))$,
$o \in O \backslash (Q^+(t) \cup Q^-(t))$,
$RC^1(t) \neq \emptyset$ and $\lambda(t, p(o) \wedge q(o)) = 1$ holds.

The epistemic satisfaction relations for all modal extensions of the remaining three conjunctions $p(o) \wedge \neg q(o)$, $\neg p(o) \wedge q(o)$ and $\neg p(o) \wedge \neg q(o)$ are similarly defined. A more detailed discussion of the rationale underlying this set of definitions can be found in other works (see, e.g., Katarzyniak, 2004c; 2005a).

A simple extension of this idea is the epistemic satisfaction of sets of conjunctions. This concept can be given in the following way:

Definition 2. (*Epistemic satisfaction of the set of modal conjunctions*) Let $S \subseteq L$, where L is the assumed language of modal conjunctions. The generalized epistemic satisfaction relation *KnowledgeState*$(t) \models_G S$ holds if and only if for each formula $\alpha \in S$ the relation *KnowledgeState*$(t) \models_G \alpha$ holds.

3. Previous Research into the Properties of Grounding Modal Conjunctions

An important research issue related to the epistemic satisfaction relation (and grounding) is to prove that it makes it possible for a communicative cognitive agent to produce rational language behavior. A subdimension of this research issue covers the problem of possibility to ground sets of modal conjunctions acceptable in the natural language discourse, and the opposite problem of permanent rejection of grounding these sets that are not acceptable at all. This issue has already been partly discussed for the case of uniform sets of modal conjunctions in which conjunctions extended with the same modal operator are included (Katarzyniak, 2005a; 2006). For four mutually different conjunctions $\alpha, \beta, \delta, \phi \in \{p(o) \wedge q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$, the list of possible uniform sets can be given as follows:

$$S_1 = \{\mathit{Know}(\alpha), \mathit{Know}(\beta), \mathit{Know}(\delta), \mathit{Know}(\phi)\},$$

$$S_2 = \{\mathit{Know}(\alpha), \mathit{Know}(\beta), \mathit{Know}(\delta)\},$$

$$S_3 = \{\mathit{Know}(\alpha), \mathit{Know}(\beta)\},$$

$$S_4 = \{\mathit{Bel}(\alpha), \mathit{Bel}(\beta), \mathit{Bel}(\delta), \mathit{Bel}(\phi)\},$$

$$S_5 = \{\mathit{Bel}(\alpha), \mathit{Bel}(\beta), \mathit{Bel}(\delta)\},$$

$$S_6 = \{\mathit{Bel}(\alpha), \mathit{Bel}(\beta)\},$$

$$S_7 = \{\mathit{Pos}(\alpha), \mathit{Pos}(\beta), \mathit{Pos}(\delta), \mathit{Pos}(\phi)\},$$

$$S_8 = \{\mathit{Pos}(\alpha), \mathit{Pos}(\beta), \mathit{Pos}(\delta)\},$$

$$S_9 = \{\mathit{Pos}(\alpha), \mathit{Pos}(\beta)\}.$$

The following requirements have to be fulfilled in order to implement (commonsense) rationality into artificial language behavior:

- The sets S_1–S_3 cannot be satisfied in the epistemic sense for the same state of cognition. This constraint follows from the natural expectation that if the agent knows that a certain distribution of the properties P and Q is realized in an object o, then it is not possible for this agent to know that anything opposite holds.

- The sets S_4–S_6 cannot be satisfied in the epistemic sense for the same state of cognition. This constraint would be similar to the pragmatic rule known from the natural language discourse where the use of the belief operator with one conjunction excludes this operator as an extension of another conjunction.

- The sets S_7–S_9 cannot be forbidden for the communicative cognitive agent because they are acceptable in the natural language discourse as external reflections of internal knowledge.

An additional assumption for the interpretation of the above-given sets is that each modal conjunction excluded from a set $S_i, i = 1, 2, \ldots, 9$ is treated as unsatisfied in the epistemic sense. For instance, if we say that the set $S_8 = \mathit{Pos}(\alpha), \mathit{Pos}(\beta), \mathit{Pos}(\delta)$ is satisfied in the epistemic sense, this also means that the modal conjunction $\mathit{Pos}(\phi)$ is not satisfied in the epistemic sense.

In (Katarzyniak, 2005a; 2006), several theorems were already proved to show that the accepted definition of the epistemic satisfaction of modal conjunctions makes the above-mentioned and desired properties of grounding possible. In this particular sense, the desirable rationality of artificial language behavior and its correspondence to natural language behavior is said to be proved. Results covered by these theorems can be divided into two groups. Namely, there are some theorems which say that some desirable properties of grounding follow directly from the accepted definitions of the epistemic satisfaction of conjunctions. The other group of results says that, in order to achieve other desirable properties of language grounding, it is necessary to implement appropriate systems of modality thresholds. Detailed proofs of these properties of epistemic satisfaction can be found in (Katarzyniak, 2005a; 2006). A brief overview of these theorems is given as follows:

Theorem 1. *The definition of the epistemic satisfaction relation of modal conjunctions ensures that it is not possible to ground more than one modal conjunction Know*$(p(o) \wedge q(o))$, *Know*$(p(o) \wedge \neg q(o))$, *Know*$(\neg p(o) \wedge q(o))$ *or Know*$(\neg p(o) \wedge \neg q(o))$ *in the same state of cognition KnowledgeState*(t).

Theorem 2. *Let four mutually different conjunctions* $\alpha, \beta, \gamma, \chi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ *be given. In order to make the simultaneous grounding of more than two formulas from the set* $\{\mathit{Bel}(\alpha),\ \mathit{Bel}(\beta),\ \mathit{Bel}(\gamma),\ \mathit{Bel}(\chi))\}$ *impossible for each state of cognition KnowledgeState*(t)*, the cognitive communicative agent has to be equipped with a system of modality thresholds* $(\lambda_{\min \mathit{Pos}}, \lambda_{\max \mathit{Pos}}, \lambda_{\min \mathit{Bel}}, \lambda_{\max \mathit{Bel}})$ *in which the condition* $1/2 < \lambda_{\min \mathit{Bel}}$ *or* $\lambda_{\max \mathit{Bel}} < 1/2$ *is fulfilled.*

Theorem 3. *Let four mutually different conjunctions* $\alpha, \beta, \gamma, \chi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ *be given. In order to make the simultaneous grounding of all modal conjunctions possible,* $\mathit{Pos}(\alpha), \mathit{Pos}(\beta), \mathit{Pos}(\gamma) and \mathit{Pos}(\chi)$*, the communicative cognitive agent has to be equipped with a system of modality thresholds* $(\lambda_{\min \mathit{Pos}}, \lambda_{\max \mathit{Pos}}, \lambda_{\min \mathit{Bel}}, \lambda_{\max \mathit{Bel}})$ *in which the condition* $\lambda_{\min \mathit{Pos}} \leq 1/4 \leq \lambda_{\max \mathit{Pos}}$ *is fulfilled.*

Theorem 4. *Let four mutually different conjunctions* $\alpha, \beta, \gamma, \chi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),$

$\neg p(o) \wedge \neg q(o)\}$ and a system of modality thresholds $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ in which the conditions $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\max Pos} < 1$ are fulfilled be given. It makes it possible to ground the sets of modal conjunctions $\{Pos(\alpha),\ Pos(\beta),\ Pos(\delta)\}$ and $\{Pos(\alpha),\ Pos(\beta)\}$ provided that an appropriate empirical content has been collected by the communicative cognitive agent up to the time point t.

Theorem 5. *Let a system of modality thresholds $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ be given in which the conditions $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ are fulfilled. The communicative cognitive agent equipped with this system of modality thresholds makes it permanently impossible to ground the sets S_1–S_6 and makes it possible to ground the sets S_7–S_9 provided that the latter can happen if the appropriate empirical content has been developed and stored in this agent.*

4. Commonsense Requirements for Nonuniform Sets of Modal Conjunctions

In the works (Katarzyniak, 2005a; 2006), no attention was paid to other interesting sets of modal conjunctions. These sets will be further called nonuniform sets and are grouped into two classes.

The so-called first type nonuniform sets of modal conjunctions are defined each for a particular conjunction $\varphi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o), \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$. These sets can be enumerated as follows:

$$\begin{aligned} S_{10} &= \{Know(\varphi), Bel(\varphi)\},\\ S_{11} &= \{\varphi, Bel(\varphi)\},\\ S_{12} &= \{Know(\varphi), Pos(\varphi)\},\\ S_{13} &= \{\varphi, Pos(\varphi)\},\\ S_{14} &= \{Bel(\varphi), Pos(\varphi)\}. \end{aligned}$$

A natural requirement for the acceptability of the simultaneous grounding of the sets S_{10}–S_{14} of modal conjunctions is obvious and can be stated in the following way: in the natural language discourse, the above sets of modal conjunctions are never used to communicate an individual agent's opinion on the same state of the properties P and Q in an external object o. Such a requirement should be fulfilled by the definition for epistemic satisfaction. Below some research into this issue is presented and the related results are covered by the appropriate theorems:

Theorem 6. *Let $\varphi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o), \neg p(o) \wedge \neg q(o)\}$.*

1. *If the relation KnowledgeState(t) $\models_G$ Pos(φ) holds then the relation KnowledgeState(t) $\models_G$Bel(φ) does not hold.*
2. *If the relation KnowledgeState(t) $\models_G$Bel(φ) holds, then the relation KnowledgeState(t) $\models_G$Pos(φ) does not hold.*

Proof. For any conjunction φ, this property follows directly from the assumption that the system of modality thresholds $0 < \lambda_{\min Pos} < \lambda_{\max Pos} < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ is implemented in the communicative cognitive agent. ■

Theorem 7. *Let $\varphi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$.*

1. *If the relation KnowledgeState(t) $\models_G$ Pos(φ) holds, then the relations KnowledgeState(t) $\models_G$ Know$(\varphi)$$and$KnowledgeState$(t)$ $\models_G$ φ do not hold.*
2. *If the relations KnowledgeState(t) $\models_G$Know(φ) and KnowledgeState(t) $\models_G$ φ hold, then the relation KnowledgeState(t) $\models_G$Pos(φ) does not hold.*

Proof. Let us consider the case of the conjunction $p(o) \wedge q(o)$. If the relation *KnowledgeState*(t) $\models_G$ *Pos*$(p(o) \wedge q(o))$ holds, then from Definition 1 it follows that the statements $o \in O\backslash(P^+(t) \cup P^-(t)), o \in O\backslash(Q^+(t) \cup Q^-(t))$ and $\lambda_{\min Pos} \leq \lambda(t, p(o) \wedge q(o)) \leq \lambda_{\max Pos}$ are true. From this fact and the additional assumption that the system of modality thresholds $0 < \lambda_{\min Pos} < \lambda_{\max Pos} < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ is implemented, it can be deduced that the three statements $o \neq\in P^+(t)$, $o \neq\in Q^+(t)$ and $\lambda(t, p(o) \wedge q(o)) \neq 1$ are true. In consequence, none of the two alternative requirements given in the definition of the epistemic satisfaction of *Know*$(p(o) \wedge q(o))$ and $p(o) \wedge q(o)$ is fulfilled.

The proofs for the remaining conjunctions $p(o) \wedge \neg q(o), \neg p(o) \wedge q(o)$ and $\neg p(o) \wedge \neg q(o)$ are similar. This completes the proof. ■

Theorem 8. *Let $\varphi \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o), \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$.*

1. *If the relation KnowledgeState(t) $\models_G$ Bel(φ) holds, then the relations KnowledgeState(t) $\models_G$ Know$(\varphi)$$and$KnowledgeState$(t)$ $\models_G\varphi$ do not hold.*
2. *If the relations KnowledgeState(t) $\models_G$Know(φ) and KnowledgeState(t) $\models_G$ φ hold, then the relation KnowledgeState(t) $\models_G$Bel(φ) does not hold.*

The proof is similar to that of Theorem 6.

The second type of the nonuniformity of sets consisting of modal conjunctions is considered for all sets of mutually different conjunctions $\alpha, \beta, \delta, \phi \in \{p(o) \wedge$

$q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$. The cases considered are as follows:

$$S_{15} = \{Know(\alpha), \beta, \delta, \phi\},$$

$$S_{16} = \{Know(\alpha), \beta, \delta\},$$

$$S_{17} = \{Know(\alpha), \beta\},$$

$$S_{18} = \{Know(\alpha), Bel(\beta), Bel(\delta), Bel(\phi)\},$$

$$S_{19} = \{Know(\alpha), Bel(\beta), Bel(\delta)\},$$

$$S_{20} = \{Know(\alpha), Bel(\beta)\},$$

$$S_{21} = \{Know(\alpha), Pos(\beta), Pos(\delta), Pos(\phi)\},$$

$$S_{22} = \{Know(\alpha), Pos(\beta), Pos(\delta)\},$$

$$S_{23} = \{Know(\alpha), Pos(\beta)\},$$

$$S_{24} = \{Bel(\alpha), Pos(\beta), Pos(\delta), Pos(\phi)\},$$

$$S_{25} = \{Bel(\alpha), Pos(\beta), Pos(\delta)\},$$

$$S_{26} = \{Bel(\alpha), Pos(\beta)\},$$

$$S_{27} = \{Bel(\alpha)\}.$$

For these sets, the following commonsense requirements need to be fulfilled in order to achieve the rationality of the language behavior produced by artificial communicative cognitive agents:

- The sets S_{15}–S_{23} cannot be satisfied in the epistemic sense in the same state of cognition. For each set, the reason is that any conjunction extended with the modal operator of knowledge excludes the possibility to ground modal extensions of the remaining conjunctions from the same set.
- The sets S_{24}–S_{27} cannot be forbidden and should be sometimes used by artificial communicative cognitive agents as appropriate (adequate) descriptions of their opinions, provided that these agents have collected necessary sets of empirical data.

Similarly to the case of the sets $S_i,\ i = 1, 2, \ldots, 9$, it is assumed that all modal conjunctions not included in a certain set S_{15}–S_{27} are treated as unsatisfied in the epistemic sense. For instance, for the mutually different conjunctions $\alpha, \beta, \delta, \phi \in \{p(o) \wedge q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$, if we say that the set S_{15} is satisfied in the epistemic sense, this means that ϕ, $Know(\phi)$, $Bel(\phi)$ and $Pos(\phi)$ are assumed to be unsatisfied in the epistemic sense.

Let us now analyze the properties of the epistemic satisfaction relation related to the above commonsense requirements for the unacceptability of the sets S_{15}–S_{23}. This issue is covered by Theorems 9 and 10:

Theorem 9. *Let* $\alpha, \beta \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ *and* $\alpha \neq \beta$. *If the relation KnowledgeState*$(t) \models_G$ *Know*(α) *holds, then the relation KnowledgeState*$(t) \models_G \beta$ *does not hold.*

Proof. Let us consider a general case in which α represents the conjunctions $p(o) \wedge q(o)$, and β represents another conjunction from $\{p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$. If the epistemic satisfaction relation *KnowledgeState*$(t) \models_G$ *Know*$(p(o) \wedge q(o))$ holds, then from Definition 1 it follows that one of the following two cases can happen:

(a) $o \in P^+(t)$ and $o \in Q^+(t)$,

(b) $o \in O\backslash(P^+(t) \cup P^-(t)),\quad o \in O\backslash(Q^+(t) \cup Q^-(t)),\ RC1(t) \neq \emptyset,\ \lambda(t, p(o) \wedge q(o)) = 1.$

Case (a): In this situation, it is true that $o \neq\in Q^-(t)$ and $o \neq\in O\backslash(Q^+(t) \cup Q^-(t))$. Therefore, none of the conjunctions given below is true:

- $o \in P^+(t)$ and $o \in Q^-(t)$,
- $o \in P^-(t)$ and $o \in Q^+(t)$,
- $o \in P^-(t)$ and $o \in Q^-(t)$,
- $o \in O\backslash(P^+(t) \cup P^-(t))$ and $o \in O\backslash(Q^+(t) \cup Q^-(t))$.

This means that if the epistemic satisfaction of *Know*$(p(o) \wedge q(o))$ holds because both $o \in P^+(t)$ and $o \in Q^+(t)$ are satisfied, then the requirements given in Eqns. (14) and (15) are not true for any conjunction from $\{p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$. This completes the proof for Case (a).

Case (b): From the fact that $o \in O\backslash(P^+(t) \cup P^-(t))$ and $o \in O\backslash(Q^+(t) \cup Q^-(t))$, it can be deduced that none of the conjunctions given below is true:

- $o \in P^+(t)$ and $o \in Q^-(t)$,
- $o \in P^-(t)$ and $o \in Q^+(t)$,
- $o \in P^-(t)$ and $o \in Q^-(t)$.

This means that the definition requirements given in Eqn. (14) are not fulfilled for any conjunction from $\{p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$. At the same time, from the equality $\lambda(t, p(o) \wedge q(o)) = 1$ and Eqns. (7)–(10), the following set of requirements has to be simultaneously considered:

$$\lambda\big(t, p(o) \wedge q(o)\big) = 1,$$

$$\lambda\big(t, p(o) \wedge q(o)\big) + \lambda\big(t, p(o) \wedge \neg q(o)\big) + \lambda(t, \neg p(o) \wedge q(o)) + \lambda\big(t, \neg p(o) \wedge \neg q(o)\big) = 1,$$

$$\lambda\big(t, p(o) \wedge q(o)\big) \geq 0,$$

$$\lambda\big(t, p(o) \wedge \neg q(o)\big) \geq 0,$$

$$\lambda\big(t, \neg p(o) \wedge q(o)\big) \geq 0,$$

$$\lambda\big(t, \neg p(o) \wedge \neg q(o)\big) \geq 0.$$

It is quite obvious that this may happen if and only if $\lambda(t, p(o) \wedge \neg q(o)) = \lambda(t, \neg p(o) \wedge q(o)) = \lambda(t, \neg p(o) \wedge \neg q(o)) = 0$. In consequence, the definitional requirements given in Eqn. (15) are not fulfilled for any conjunction from $\{p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$. The theorem has thus been proved for α equal to $p(o) \wedge q(o)$. The proofs for α equal to the remaining conjunctions $p(o) \wedge \neg q(o)\ \neg p(o) \wedge q(o)$ or $\neg p(o) \wedge \neg q(o)\}$ are similar. ■

Theorem 9 covers the case of the commonsense unacceptability of S_{17}. It has, however, to be stressed that the above proof for Theorem 9 should be the same as the proof for the required permanent lack of the epistemic satisfaction of S_3. This fact follows from Definition 1 and the following result:

Theorem 10. *Let $\alpha, \beta \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ and $\alpha \neq \beta$. If the relation KnowledgeState$(t) \models_G$Know(α) holds, then the relation KnowledgeState$(t) \models_G$Know(β) does not hold.*

It is enough to notice that Theorem 10 follows directly from Theorem 1, and the proof for Theorem 9 has been given in detail to show the proof methodology used in the forthcoming sections.

Theorem 11. *Let $\alpha, \beta, \delta, \phi \in \{p(o) \wedge q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$ be mutually different. The sets S_{15} and S_{16} cannot be satisfied in the sense of Definition 2.*

The proof follows directly from Theorem 9.

Theorem 12. *Let $\alpha, \beta \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ and $\alpha \neq \beta$. If the relation KnowledgeState$(t) \models_G$Know(α) holds, then the relation KnowledgeState$(t) \models_G$Bel(β) does not hold.*

Proof. As in the proof of Theorem 9, let us start with α denoting the conjunction $p(o) \wedge q(o)$ and β denoting any of the other conjunctions $p(o) \wedge \neg q(o), \neg p(o) \wedge q(o), \neg p(o) \wedge \neg q(o)$. It is necessary to consider the following two general cases:

(a) $o \in P^+(t)$ and $o \in Q+(t)$,

(b) $o \in O\backslash(P^+(t) \cup P^-(t)),\quad o \in O\backslash(Q^+(t) \cup Q^-(t)),\ RC^1(t) \neq \emptyset,\ \lambda(t, p(o) \wedge q(o)) = 1$.

It can be easily proved that in both cases it is not possible to ground any of the modal conjunctions *Bel*$(p(o) \wedge \neg q(o))$, *Bel*$(\neg p(o) \wedge q(o))$, *Bel*$(\neg p(o) \wedge \neg q(o))$.

Case (a): It is true that both $o \in O\backslash(P^+(t) \cup P^-(t))$ and $o \in O\backslash(Q^+(t) \cup Q^-(t))$ are not true. In consequence, the set of definitional requirements given in Eqn. (14) cannot be fulfilled for the conjunctions $p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o)$ and $\neg p(o) \wedge \neg q(o)$. In this case, the epistemic satisfaction relation *KnowledgeState*$(t) \models_G$ *Bel*(β) does not hold.

Case (b): It follows from these assumptions that $\lambda(t, p(o) \wedge q(o)) = 1$. Now, based on Eqns. (7)–(10) it can be deduced that $\lambda(t, p(o) \wedge \neg q(o)) = \lambda(t, \neg p(o) \wedge q(o)) = \lambda(t, \neg p(o) \wedge \neg q(o)) = 0$. This means that for each conjunction $p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o)$, and $\neg p(o) \wedge \neg q(o)$, its grounding values do not belong to the interval $[\lambda_{\min Bel}, \lambda_{\max Bel}]$. This means that the epistemic satisfaction relation *KnowledgeState*$(t) \models_G$*Bel*(β) does not hold, either. The proofs for each $\alpha \in \{p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ are similar. ■

Theorem 12 covers the desired unacceptability of S_{20}.

Theorem 13. *Let $\alpha, \beta, \delta, \phi \in \{p(o) \wedge q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$ be mutually different. The sets S_{18} and S_{19} cannot be satisfied in the sense of Definition 2.*

The proof follows directly from Theorem 12.

Theorem 14. *Let $\alpha, \beta \in \{p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge q(o),\ \neg p(o) \wedge \neg q(o)\}$ and $\alpha \neq \beta$. If the relation KnowledgeState$(t) \models_G$Know(α) holds, then the relation KnowledgeState$(t) \models_G$Pos(β) does not hold.*

The proof is similar to that of Theorem 12, provided that instead of $[\lambda_{\min Bel}, \lambda_{\max Bel}]$, the interval $[\lambda_{\min Pos}, \lambda_{\max Pos}]$ is considered.

Theorem 15. *Let $\alpha, \beta, \delta, \phi \in \{p(o) \wedge q(o),\ \neg p(o) \wedge q(o),\ p(o) \wedge \neg q(o),\ \neg p(o) \wedge \neg q(o)\}$ be mutually different. The sets S_{21} and S_{22} cannot be satisfied in the epistemic sense.*

The proof follows directly from Theorem 14.

Let us now consider all situations for which it is required that exactly one modal conjunction *Bel*(α) and three modal conjunctions *Pos*(β), *Pos*(δ) and *Pos*(ϕ) be satisfied in the epistemic sense (see set S_{24}). It is possible to prove that such a situation is not excluded and may happen, provided that appropriate empirical experience is collected and the mechanism of epistemic satisfaction given by Definition 1 is implemented in the artificial communicative cognitive agent. The following theorem covers this case of grounding:

Theorem 16. *There exists a system of modality thresholds that ensures the epistemic satisfaction of the set S_{24} provided that an appropriate collection of base profiles has been developed and stored in the communicative cognitive agent.*

Proof. At first, from Theorem 5 it follows that implementing the system of modality thresholds in which the condition $1/2 < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ holds makes it possible to ground at most one modal conjunction *Bel*(α) in the same state of cognition. Let us assume that the system of modality thresholds fulfils this condition.

Now, let us determine what conditions are needed for the pair of modality thresholds $\lambda_{\min Pos}$ and $\lambda_{\max Pos}$ in order to make the remaining three modal conjunctions simultaneously satisfied in the same state of cognition. Let the following symbols be introduced:

$$g_1 = \lambda(t,\alpha), \quad g_2 = \lambda(t,\beta),$$
$$g_3 = \lambda(t,\gamma), \quad g_4 = \lambda(t,\chi).$$

It is possible to determine the highest value of $\lambda_{\min Pos}$, which makes it possible to ground simultaneously the above three possibility conjunctions. The explanation is as follows: From the definition of relative grounding values it follows that $g_1 = 1 - g_2 - g_3 - g_4$. This means that the value g_1 can be treated as a decreasing function of three values, g_2, g_3 and g_4. It can be seen now that the highest value acceptable for g_2, g_3 and g_4 equals 1/6. Namely, if g_1 describes the relative grounding value for the modal conjunction *Bel*(α), then $1/2 < \lambda_{\min Bel} < g_1$. Let K represent the highest possible value for all measures g_2, g_3 and g_4. Now, in the border case it follows that the inequality $g_1 = 1 - 3K > 1/2$ holds, which leads to the inequality $K < 1/6$. This means, too, that the epistemic satisfaction of the modal conjunctions *Pos*(β), *Pos*(δ) and *Pos*(ϕ) with the simultaneous epistemic satisfaction of the modal conjunction *Bel*(α) takes place if and only if $g_2, g_3, g_4 \in [\lambda_{\min Pos}, K]$, provided that $K < 1/6$ and $\lambda_{\min Pos} \neq K$. A practical consequence is that in order to design a communicative cognitive agent with intentional language behavior in which the epistemic satisfaction of the set S_{24} is achievable, one needs to implement a system of modality thresholds for which the requirement $\lambda_{\min Pos} < 1/6$ is fulfilled.

The requirement given in the proof of Theorem 16 is not contradictory with the previously assumed properties of the system of modality thresholds, namely, with the system of inequalities $0 < \lambda_{\min Pos} < 1/2 < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$. This means that, to complete the proof, it is enough to implement a system of modality thresholds in which the conditions $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ are satisfied. ■

Let us notice now that the set of requirements $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ accepted in Theorem 16 for the system of modality thresholds $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ does not contradict the assumptions given in Theorems 1–5. However, an additional comment is needed to explain why the system of modality thresholds accepted for Theorem 4 consists of an additional inequality $1/2 < \lambda_{\max Pos}$, which is neither mentioned in the previous theorems nor follows from any of them. The main premise for including this inequality into the set of assumptions is a commonsense requirement that the shift from each situation in which the possibility operator is used to another situation in which the modal operator of belief has to be used should be smooth to ensure the continuity of modality usage. Obviously, it can be realized by setting the value of $\lambda_{\max Pos}$ very close to the value of $\lambda_{\min Bel}$ provided that the demand for their difference is actually needed and rational. Since the length of $[1/2, \lambda_{\min Bel}]$ is not equal to 0, the modality threshold $\lambda_{\max Pos}$ can always be located between 1/2 and $\lambda_{\min Bel}$, which can be described in a direct way by two inequalities, $1/2 < \lambda_{\max Pos} < \lambda_{\min Bel}$, accepted in Theorem 16. However, it has to be stressed that the existence of such an nonempty interval is an additional research issue which is not discussed in this paper in detail. Two solutions to this problem are possible: the complete reduction of $[\lambda_{\max Pos}, \lambda_{\min Bel}]$ by assuming the equality $\lambda_{\max Pos} = \lambda_{\min Bel}$, or the introduction of additional language symbols to capture these relative grounding values, which are related neither to the operator of possibility nor to the modal operator of belief. However, the choice of a proper solution should be based on a deeper analysis of rules used in the natural language discourse, where additional language symbols may sometimes be used to cover the naturally fuzzy border between possibility- and belief-related cognitive states.

A remaining research issue is now to develop further requirements for the upper modality thresholds $\lambda_{\max Pos}, \lambda_{\min Bel}$, and $\lambda_{\max Bel}$. The forthcoming analysis will show that in order to make the grounding of the sets S_{25}–S_{27} possible, the communicative cognitive agent needs to use a system of modality thresholds in which the lower belief threshold $\lambda_{\min Bel}$ depends on the lower possibility threshold $\lambda_{\min Pos}$. Let us take into account the following discussion:

An important property of the intentional language behavior studied in this paper is that there exists a certain commonsense correlation between changes observable in relative grounding values and the agent's intention to move from the epistemic satisfaction of one set of modal conjunctions from the sets S_{24}–S_{27} to the others. For instance, let us assume that the set S_7 is satisfied in the epistemic sense. In this situation, the constant increase in the grounding value of α should transform this state of cognition to a state in which the set S_{24} is grounded. Further changes should lead to other situations in which the list of grounded modal conjunctions *Pos*(β), *Pos*(δ), *Pos*(ϕ) is subsequently reduced. Namely, at the next step an increase in the relative grounding value for α should result in the grounding of the set S_{25}, then the grounding of the set S_{26} and, finally, the grounding of the set S_{27}. This logic of the desirable shift from the set S_7 to the set S_{27} leads to a conclusion that in each communicative cognitive agent a particular threshold K should exist which belongs to the interval $[\lambda_{\min Pos}, 1/6]$ and divides this interval into two nonempty intervals, $[\lambda_{\min Pos}, K]$ and $[K, 1/6]$. The role of this threshold is as follows: if all relative grounding values related to the conjunctions β, δ, ϕ are moved from $[K, 1/6]$ to $[\lambda_{\min Pos}, K]$, then the modal operator of possibility assigned to the conjunction α is replaced with the modal operator of belief. This remark will help us while proving the next result.

Theorem 17. *Let a system of modality thresholds* $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ *be given in which the set of requirements* $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\max Pos} \le \lambda_{\min Bel} < \lambda_{\max Bel} < 1$ *is fulfilled. It is possible to implement two modality thresholds,* $\lambda_{\max Pos}$ *and* $\lambda_{\min Bel}$*, which ensure the possibility of grounding the sets* S_7 *and* S_{24}*, provided that an appropriate empirical content has been collected.*

Proof. Let a number $K \in (\lambda_{\min Pos}, 1/6)$ be given which defines the threshold of balanced relative grounding values for the conjunctions β, δ, ϕ such that if these values are lower than K, then the conjunction α is extended with the modal operator of belief. This means that if these relative grounding values decrease to the level K, then the relative grounding value assigned to α increases to the level above which the operator *Bel* has to be applied (instead of the operator *Pos*). The following assumptions for $\lambda_{\max Pos}$ and $\lambda_{\min Bel}$ are compatible with the above remarks:

$$\begin{aligned}&\lambda_{\max Pos} = 1 - 3K - 3\Delta,\\&\Delta \in (0, 1/6 - K) \text{ and } \Delta \to 0,\\&\lambda_{\min Bel} = 1 - 3K,\\&1 - 3\lambda_{\min Pos} < \lambda_{\max Bel}.\end{aligned}$$

Case (a). It can be easily proved that the numbers g_1, g_2, g_3 and g_4 which fulfill the following requirements:

$$\begin{aligned}&g_1 = 1 - (g_2 + g_3 + g_4),\\&K + \Delta < g_2 < 1/6,\\&K + \Delta < g_3 < 1/6,\\&K + \Delta < g_4 < 1/6\end{aligned}$$

can be relative grounding values for the modal conjunctions *Pos*(α), *Pos*(β), *Pos*(δ) and *Pos*(ϕ), respectively.

Namely, having summed up the above inequalities, the following can be obtained:

$$\begin{aligned}&3K + 3\Delta \le g_2 + g_3 + g_4 < 1/2,\\&1 - 3K - 3\Delta < 1 - (g_2 + g_3 + g_4) > 1/2,\\&1 - 3K - 3\Delta < g_1 > 1/2.\end{aligned}$$

This means that the following inequalities hold:

$$\begin{aligned}&\lambda_{\min Pos} \le g_1 < \lambda_{\max Pos},\\&\lambda_{\min Pos} \le g_2 < \lambda_{\max Pos},\\&\lambda_{\min Pos} \le g_3 < \lambda_{\max Pos},\\&\lambda_{\min Pos} \le g_4 < \lambda_{\max Pos},\\&1 = g_1 + g_2 + g_3 + g_4.\end{aligned}$$

This means that a state of cognition is possible in which the stored empirical experience makes four modal conjunctions, *Pos*(α), *Pos*(β), *Pos*(δ) and *Pos*(ϕ), well grounded (satisfied in the epistemic sense) because all conditions given in Definition 1 are fulfilled as regards the case of relations

$$\begin{aligned}&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\alpha),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\beta),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\delta),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\phi).\end{aligned}$$

Case (b). For the case of the set S_{24} the following assumptions can be accepted:

$$\begin{aligned}&g_1 = 1 - (g_2 + g_3 + g_4),\\&\lambda_{\min Pos} \le g_2 < K,\\&\lambda_{\min Pos} \le g_3 < K,\\&\lambda_{\min Pos} \le g_4 < K.\end{aligned}$$

From the above it follows that

$$\begin{aligned}&3\lambda_{\min Pos} \le g_2 + g_3 + g_4 < 3K,\\&1 - 3\lambda_{\min Pos} \ge 1 - (g_2 + g_3 + g_4) > 1 - 3K,\\&1 - 3\lambda_{\min Pos} > g_1 \ge 1 - 3K.\end{aligned}$$

Having considered all the assumptions made above, we obtain the following result:

$$\begin{aligned}&1 - 3\lambda_{\min Pos} > g_1 \ge 1 - 3K = \lambda_{\min Bel},\\&\lambda_{\min Pos} \le g_2 < \lambda_{\max Pos}.\end{aligned}$$

Accordingly, the inequalities

$$\begin{aligned}&\lambda_{\min Bel} \le g_1 < \lambda_{\max Bel},\\&\lambda_{\min Pos} \le g_2 < \lambda_{\max Pos},\\&\lambda_{\min Pos} \le g_3 < \lambda_{\max Pos},\\&\lambda_{\min Pos} \le g_4 < \lambda_{\max Pos},\end{aligned}$$

and the equality $1 = g_1 + g_2 + g_3 + g_4$ are true.

The conclusion is that in the situation which is characterized by the accepted assumptions, the following relations hold:

$$\begin{aligned}&\mathit{KnowledgeState}(t) \models_G \mathit{Bel}(\alpha),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\beta),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\delta),\\&\mathit{KnowledgeState}(t) \models_G \mathit{Pos}(\phi).\end{aligned}$$

The general conclusion is that, in order to make the grounding of the sets S_7 and S_{24}, the system of modality thresholds $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ needs to satisfy the set of inequalities $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\max Pos} \leq \lambda_{\min Bel} < 1 - 3\lambda_{\min Pos} < \lambda_{\max Bel} < 1$. Obviously, even if such a system is implemented in the communicative cognitive agent, it is still required that the appropriate content of empirical experience be embodied (stored) in the agent, namely, the content described by the assumed relative grounding values. This completes the proof. ■

The above result has to be further evaluated in order to prove that, with this system of modality thresholds, the communicative cognitive agent can ground the acceptable sets of modal conjunctions S_{25}–S_{27}. Theorem 18 covers these cases.

Theorem 18. *Let a system of modality thresholds* $(\lambda_{\min Pos}, \lambda_{\max Pos}, \lambda_{\min Bel}, \lambda_{\max Bel})$ *be given in which the inequalities* $0 < \lambda_{\min Pos} < 1/6 < 1/2 < \lambda_{\max Pos} \leq \lambda_{\min Bel} < 1 - 3\lambda_{\min Pos} < \lambda_{\max Bel} < 1$ *are satisfied. This system of modality thresholds ensures the possibility to satisfy in the epistemic sense the sets* S_{25}–S_{27}*, provided that appropriate empirical experience has been collected and stored by the communicative cognitive agent.*

Proof. Let us take into account a number $K \in (\lambda_{\min Pos}, 1/6)$. Let the following assumptions be made for the modality thresholds $\lambda_{\max Pos}$ and $\lambda_{\min Bel}$:

$$\lambda_{\max Pos} = 1 - 3K - 3\Delta,$$
$$\Delta \in (0, 1/6 - K) \text{ and } \Delta \to 0,$$
$$\lambda_{\min Bel} = 1 - 3K,$$
$$1 - 3\lambda_{\min Pos} < \lambda_{\max Bel} < 1, \ \lambda_{\max Bel} \to 1.$$

It follows that there always exists ε such that $\lambda_{\max Bel} = 1 - \varepsilon$, where ε satisfies $\lambda_{\min Pos} > 1/3, \ \varepsilon > 0$.

Let us take into account four numbers, g_1, g_2, g_3, and g_4, fulfilling the requirements

$$g_1 = 1 - (g_2 + g_3 + g_4),$$
$$0 < g_2 < K,$$
$$0 < g_3 < K,$$
$$0 < g_4 < K.$$

This means that $1 > 1 - (g_2 + g_3 + g_4) > 1 - 3K$ and, in consequence, $1 > g_1 > 1 - 3K$. Let us now consider three sets of additional assumptions, each for an individual set S_{25}–S_{27}.

The assumptions for the set S_{25} are as follows:

$$g_1 = 1 - (g_2 + g_3 + g_4),$$
$$\lambda_{\min Pos} \leq g_2 < K,$$
$$\lambda_{\min Pos} \leq g_3 < K,$$
$$1/3\varepsilon < g_4 < \lambda_{\min Pos}.$$

Consequently, we get

$$1/3\varepsilon \leq g_2 < K,$$
$$1/3\varepsilon \leq g_3 < K,$$

which yields

$$\varepsilon < g_2 + g_3 + g_4 < 2K + \lambda_{\min Pos},$$
$$1 - \varepsilon > g_1 > 1 - 2K - \lambda_{\min Pos},$$
$$\lambda_{\max Bel} = 1 - \varepsilon > g_1 > 1 - 2K - \lambda_{\min Pos} > 1 - 3K = \lambda_{\min Bel}.$$

This means that if the empirical content stored in the agent's knowledge base is represented by the four numbers g_1, g_2, g_3, and g_4, then all conditions proposed in Definition 1 are satisfied for the relations

$$\textit{KnowledgeState}(t) \models_G \textit{Bel}(\alpha),$$
$$\textit{KnowledgeState}(t) \models_G \textit{Pos}(\beta),$$
$$\textit{KnowledgeState}(t) \models_G \textit{Pos}(\delta).$$

In consequence, this means that the set consisting of modal conjunctions $\textit{Bel}(\alpha), \textit{Pos}(\beta), \textit{Pos}(\delta)$ is satisfied in the epistemic sense and the remaining conjunction is not satisfied.

The assumptions for the set S_{26} are

$$g_1 = 1 - (g_2 + g_3 + g_4),$$
$$\lambda_{\min Pos} \leq g_2 < K,$$
$$1/3\varepsilon \leq g_3 < \lambda_{\min Pos},$$
$$1/3\varepsilon < g_4 < \lambda_{\min Pos}.$$

Similar reasoning leads to the conclusion that this collection of relative grounding values g_1, g_2, g_3, and g_4 results in the satisfaction of $\{\textit{Bel}(\alpha), \ \textit{Pos}(\beta)\}$.

The assumptions for the set S_{27} are

$$g_1 = 1 - (g_2 + g_3 + g_4),$$
$$1/3\varepsilon \leq g_2 < \lambda_{\min Pos},$$
$$1/3\varepsilon \leq g_3 < \lambda_{\min Pos},$$
$$1/3\varepsilon < g_4 < \lambda_{\min Pos}.$$

This can be used to prove that the only modal conjunction that is satisfied in the epistemic sense is $\textit{Bel}(\alpha)$. This completes the proof. ■

5. Conclusions

The above results have important practical consequences. Namely, they suggest in a systematic way a certain organization of artificial cognition for the class of artificial communicative cognitive agents considered in this paper. It has been proved that, as regards the processing of nonuniform sets of modal conjunctions, communicative cognitive agents based on the idea of epistemic satisfaction fulfil important commonsense requirements known from the natural language discourse. In particular, they are not allowed to produce these nonuniform sets of modal conjunctions that are not acceptable in the natural language discourse and are able to produce sets of modal conjunctions that are acceptable in the context of this discourse. In other words, if an agent uses epistemic satisfaction to choose grounded formulas as acceptable language messages to be "spoken", then unacceptable sets of modal conjunctions are not produced externally for any distribution of stored empirical experiences, and acceptable sets of modal conjunctions can happen if the internal distribution of stored empirical experience fulfils the requirements given in the definition for epistemic satisfaction.

The proposed model of grounding based on the idea of the epistemic satisfaction relation can be applied in various situations. Obviously, the most advanced implementation would be realized in a humanoid cognitive agent. However, in such a case the main technical problem is to develop an adequate knowledge base in which huge collections of individual observations could be stored. Another interesting application is to use this model in software modules filled up with data stored in organizational resources and producing linguistic representations of particular pieces of the collected knowledge resources. This idea was already partly presented in (Katarzyniak, 2004b). A simplified model of grounding was also applied in order to describe artificial cognitive processes of producing responses to queries about objects' states. These queries are directed to an artificial cognitive agent that is organized according to the assumptions given for artificial cognitive agents in this paper (Katarzyniak and Pieczyńska-Kuchtiak, 2004). Other simplified implementations are artificial cognitive processes of producing modal messages including modal conjunctions (Katarzyniak and Pieczyńska-Kuchtiak, 2002; Pieczyńska-Kuchtiak 2004). All these implementations cover collections of formulas extended with modal exclusive and inclusive alternatives. However, they are simplified in the sense that they do not apply systems of modality thresholds with properties developed by the research given in this paper and the previous works (Katarzyniak, 2005b; 2006). The general idea underlying the above implementations is that artificial cognitive agents are asked questions about the states of properties in particular objects. If these states of properties can be observed by the agents directly, an answer is chosen which is properly grounded in the latest results of the observation of these objects. However, if these objects are not accessible, the agents need to compensate for the lack of knowledge by applying empirical experience stored in internal knowledge bases. This kind of knowledge forms the actual relation between the language symbol (an appropriate modal formula) representing a relevant answer and the actual world with the objects pointed at in the question. In this sense, properly chosen answers are formulas which are satisfied in the epistemic sense.

To conclude, it is necessary to mention that similar research into the nature of grounding has already been carried out for other classes of modal formulas, and related results from this research have been published for the case of uniform and nonuniform sets of simple modalities where simple modalities are understood as modal extensions of positive literals $p(o)$ and modal extensions of negative literals $\neg p(o)$ (Katarzyniak, 2005b). All these results are complementary to and consistent with results developed for the case of uniform and nonuniform sets of modal conjunctions. The remaining classes of formulas are modal inclusive alternatives and modal exclusive alternatives (Katarzyniak, 2002), modal implications and modal equivalences (Katarzyniak, 2004a). It has already been discovered that for some of these classes, additional constraints need to be introduced for systems of modality thresholds and some minor extensions are needed to reformulate definitions of the epistemic satisfaction of modal alternatives (Katarzyniak, 2005a).

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Received: 18 January 2005
Revised: 25 July 2006