A VARIABLE STRUCTURE OBSERVER FOR THE CONTROL OF ROBOT MANIPULATORS

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This paper deals with the application of a variable structure observer developed for a class of nonlinear systems to solve the trajectory tracking problem for rigid robot manipulators. The analyzed approach to observer design proposes a simple design methodology for systems having completely observable linear parts and bounded nonlinearities and/or uncertainties. This observer is basically the conventional Luenberger observer with an additional switching term that is used to guarantee robustness against modeling errors and system uncertainties. To solve the tracking problem, we use a control law developed for robot manipulators in the full information case. The closed loop system is shown to be globally asymptotically stable based on Lyapunov arguments. Simulation results on a 3-DOF robot manipulator show the asymptotic convergence of the vectors of observation and tracking errors.

Keywords: variable structure observers, switching-type observers, rigid robot manipulators, exponential stability, tracking control

1. Introduction

The control problem for rigid robot manipulators has been solved using several efficient classical and robust methods, and it has been shown that each control strategy ensures the stability of the trajectory tracking error in some suitable sense. One basic assumption in these methods is that full state information is available for feedback. In fact, for robotic systems, state feedback control is based on the exact knowledge of both the position and velocity vectors. Unfortunately, the velocity vector cannot generally be available for feedback for several reasons. A solution to this is the design of nonlinear observers that give the reconstruction of the missing velocity signal. Due to the nonlinear and coupled structure of the robot dynamical model, the problem of designing observers for robots is a very complex one.

For nonlinear systems, several approaches have been presented in the literature (Khelfi *et al.*, 1998) to solve the nonlinear observer design problem. The first possibility consists in transforming a nonlinear problem into a linear one by the extended linearization technique (Baumann and Rogh, 1986) or by the pseudo-linearization method (Lawrence, 1992; Walcott and Żak, 1987a), which yields constant eigenvalues of the reconstruction error dynamics when linearized about any fixed equilibrium point. We also have the exact linearization technique (Krener and Respondek, 1985), which consists in transforming the nonlinear system into a linear system with an output injection to apply linear observation theory. A second possibility consists in designing an observer with the nonlinear observation error dynamics. In this context, some techniques were established in the initial state coordinates (Hammami, 1993), and others in the observable canonical form (Bornard and Hammouri, 1991; Gauthier and Bornard, 1981; Gauthier *et al.*, 1991). All these methods are available for nonlinear systems without uncertainties or disturbances in their dynamic equations (for a survey on nonlinear observers, we refer the reader to (Khelfi *et al.*, 1998; Misawa and Hedrick, 1989; Tsinias, 1989; Walcott *et al.*, 1987b)).

Motivated by the above developments, the control problem of robots using partial knowledge of the state variable (only joint measurements) has attracted increasing interest. A straightforward approach to this problem goes along a two-step design: first, construct a nonlinear observer driven by the available inputs and outputs, which reconstructs the lacking velocity signal. Second, design a state feedback controller and replace the actual velocity with the one reconstructed from the observer. Indeed, based on this procedure, a number of conceptually different methods for both regulation and tracking control of robots equipped with only position sensors have been developed (Berghuis, 1993a; Berghuis and Nijmeijer, 1993b; Khelfi *et al.*, 1996; Nicosia and Tomei, 1990). These observers guarantee the exponential and asymptotic stability of the observation error, but do not take into consideration system uncertainties, even though several studies have shown that under suitable conditions some of them present robustness properties, especially those based on the passivity approach (Abdessameud and Khelfi 2003; Berghuis, 1993a). A solution to this issue is the design of robust observers.

The design of observers that take into consideration system uncertainties have taken the interest of many researchers (Berghuis, 1993a; Canudas *et al.*, 1990; Dawson *et al.*, 1992; Misawa and Hedrick, 1989; Slotine *et al.*, 1986; 1987; Walcott *et al.*, 1987b).

Walcott *et al.* (1987a) presented a variable structure observer for a class of nonlinear systems. They propose a simple design methodology for systems having completely observable linear parts and bounded nonlinearities or uncertainties. A minimum estimate for the rate of convergence of the observer error was also given. This observer is basically the conventional Luenberger observer with an additional switching term that is used to guarantee robustness against modeling errors and system uncertainties. Due to this supplementary switching term, this observer suffers from chattering, usually associated with variable structure systems. To deal with this problem, the original observer is modified and a boundary layer approach is considered. However, with this modification, the asymptotic stability aspect of the observation error dynamics is lost, and only the global uniform ultimate boundedness stability of the observation error is obtained. In (Dawson *et al.*, 1992), an extension to the above variable structure scheme was proposed, and a continuous observer was used to ensure the global exponential stability of the observation error system.

In this paper, we apply the variable structure observer, as proposed by Walcott *et al.* (1987b), to the system of n-DOF robot manipulators to solve the tracking control problem with only position measurements. The exponential stability of the observation error is shown under the condition that system nonlinearities and uncertainties can be bounded, which is generally guaranteed for this class of systems. The main difference between the proposed observer and other solutions is that in most nonlinear observer designs, the system dynamics and their estimates are entirely considered in the observer structure with a correction term designed differently. Simultaneously, the proposed observer structure is mainly the Luenberger observer with an additional switching term used to cope with system nonlinearities and guarantee robustness against system uncertainties and disturbances. The main drawback of the additional switching term is the occurrence of chattering. To deal with this situation, a boundary Layer approach can be used to eliminate chattering, and the observation error is ensured to be globally uniformly ultimately bounded.

The estimated velocity vector is used in the trajectory tracking control law proposed by Paden and Panja (1988), which guarantees the global asymptotic stability of the tracking error for the manipulator control system. Keeping in mind that no separation principle exists for nonlinear systems, the study of the closed loop stability is performed using a Lyapunov function that contains two terms, one for the tracking error and the other for the estimation error. The asymptotic stability of the closed loop system is shown under a suitable choice of the observer and controller gains.

This paper is organized as follows: We first review the literature on the variable structure observer design method. Then, we apply this observer to the class of rigid robot manipulators and show that under some assumptions, the exponential convergence of the observation error is guaranteed. Section 4 is devoted to closed loop control, where we use Lyapunov arguments to prove the closed loop stability. Finally, simulation results of the proposed scheme are illustrated on a 3-DOF robot manipulator (the first three joints of the 6-DOF robot manipulator given by Yoshikawa (1990)).

2. Variable Structure Observer

Consider the following nonlinear system:

$$\begin{cases} \dot{x}(t) = A\,x(t) + f(x,u,t), \\ y(t) = C\,x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^p$ is the output vector and $u(t) \in \mathbb{R}^m$ is the control input. The vector $f(\cdot,\cdot,\cdot)$, assumed to be continuous in $x(t)$, is used to represent nonlinearities and/or uncertainties in the plant. The problem is to design an observer with inputs $y(t)$ and $u(t)$, whose output $\hat{x}(t)$ is the estimated state that is ensured to converge in finite time to the real state. Before we give the observer structure, the following assumptions should be made:

Assumption 1. The pair (A, C) is detectable, i.e., there exists a matrix L of appropriate dimensions such that the spectrum of $A_o = A - L\,C$ is completely contained in the open left half-plane.

Assumption 2. There exist a positive definite matrix $Q \in \mathbb{R}^{n\times n}$ and a function h where $h(\cdot,\cdot) : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^p$, such that the following matching conditions hold:

$$f(t,x) = P^{-1}C^T\,h(t,x), \tag{2}$$

where P is the unique positive definite solution to the Lyapunov equation

$$A_o^T P + P A_o = -Q. \tag{3}$$

Assumption 3. There exists a nonnegative function ρ, where $\rho(\cdot,\cdot) : \mathbb{R}_+ \times \mathbb{R}^m \to \mathbb{R}_+$, such that

$$\| h(t,x,u) \| \le \rho(t,u), \tag{4}$$

$\forall x \in \mathbb{R}^n,\ \forall u \in \mathbb{R}^m$ and $t \in \mathbb{R}_+$.

If Assumptions 1–3 are satisfied, then the proposed observer is described by the following differential equations:

$$\dot{\hat{x}} = A\,\hat{x} + L\,(y - C\,\hat{x}) + \nu_0(t,\hat{x},y), \tag{5}$$

where

$$\nu_0(t,\hat{x},y) = \begin{cases} \dfrac{-P^{-1}C^T C\,e}{\|C\,e\|}\rho\,(t,u), & \forall \|C\,e\| \neq 0, \\ 0, & \forall \|C\,e\| = 0, \end{cases} \tag{6}$$

and L is a positive diagonal design matrix.

Let the observation error be defined as $e = \hat{x} - x$. The observation error system will then be described by

$$\dot{e} = A_o e + \nu_0(t,\hat{x},y) - f(t,x,u). \tag{7}$$

The exponential convergence of the estimation error is stated by the following theorem:

Theorem 1. *Given the nonlinear system described by (1) and the observer governed by (5) and (6), if Assumptions 1–3 are satisfied, then the observation error $e = \hat{x} - x$ is globally exponentially stable.*

The proof of this theorem can be found in (Walcott *et al.*, 1987b). It can be seen that this observer is the conventional Luenberger observer with the additional switching term $\nu_0(t,\hat{x},y)$, which ensures robustness against system nonlinearities. Unfortunately, this discontinuous term will cause the undesirable phenomenon of "chattering". Hence, it is advantageous to design a gain law that is continuous in the error and ensures that the estimated state will converge at least asymptotically to some arbitrary small neighborhood of the real state.

To satisfy these requirements, a boundary layer strategy that offers a continuous gain function is proposed in (Walcott *et al.*, 1987b). This is done by replacing the discontinuous term given by (6) by the continuous term

$$\bar{\nu}_0(t,\hat{x},y) = \begin{cases} \dfrac{-P^{-1}C^T C\,e}{\|C\,e\rho\|}\rho^2 & \text{if } \|C\,e\rho\| > \varepsilon, \\ \dfrac{-P^{-1}C^T C\,e}{\varepsilon}\rho^2 & \text{if } \|C\,e\rho\| \le \varepsilon, \end{cases} \tag{8}$$

with $\varepsilon > 0$. With the observer (5)–(8), the error system satisfies

$$\dot{e} = A_o\,e + \bar{\nu}_0(t,\hat{x},y) - P^{-1}C^T h\,(t,x,u). \tag{9}$$

It can easily be shown that the error signal is globally uniformly ultimately bounded.

3. Application to Robot Manipulators

In order to apply the above variable structure observer to robot manipulators, we consider the dynamics of an n-DOF robot manipulator given by Yoshikawa (1990), written in the following state space representation:

$$\begin{cases} \dot{x} = A\,x + f(x,u,t) + B\,\eta_d(t), \\ y = C\,x, \end{cases} \tag{10}$$

with

$$x = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}, \quad A = \begin{pmatrix} 0 & I_n \\ 0 & 0 \end{pmatrix}, \tag{11a}$$

$$B = \begin{pmatrix} 0 \\ I_n \end{pmatrix}, \quad C = \begin{pmatrix} I_n & 0 \end{pmatrix}, \tag{11b}$$

and

$$f(x,u,t) = f_1(x,t)\,u + f_2(x,t), \tag{11c}$$

with

$$f_1(x,t) = \begin{pmatrix} 0 \\ M^{-1}(q) \end{pmatrix},$$

$$f_2(x,t) = \begin{pmatrix} 0 \\ -M^{-1}(q)\ (C(q,\dot{q})\,\dot{q} + G(q)) \end{pmatrix},$$

$$u = \tau, \tag{11d}$$

where $q \in \mathbb{R}^n$ is the vector of joint angular positions, $M(q) \in \mathbb{R}^{n\times n}$ is the positive definite inertia matrix, $C(q,\dot{q})\dot{q} \in \mathbb{R}^n$ is the Coriolis and centrifugal torque vector, $G(q) \in \mathbb{R}^n$ is the gravity vector, and $\tau \in \mathbb{R}^n$ is the vector of applied joint torques. Here $\eta_d(t)$ is the vector representing external disturbances and friction terms.

At this point, it is important to present some of the important structural properties of the inertia matrix and the Coriolis vector, which will be used to derive robot control schemes (Berghuis *et al.*, 1993a):

P1: For some strictly positive constants M_1 and M_2, we have

$$M_1\,I_n \le M(q) \le M_2\,I_n. \tag{12a}$$

P2: For all $x \in \mathbb{R}^n$ (for a revolute joint robot), we have

$$\| C(q,x)\, x \| \leq C_M \| x \|^2. \tag{12b}$$

P3: The matrix $C(q,\dot{q})$ satisfies the following relation:

$$C(q,x)y = C(q,y)x \tag{12c}$$

for all $x,\ y \in \mathbb{R}^n$.

P4: The gravity vector is bounded as follows:

$$G(q) \leq G_M, \tag{12d}$$

where $\|\cdot\|$ denotes the Euclidean vector norm.

The first step to be considered in the design of the variable structure observer for robot manipulators is to satisfy Assumptions 1–3. From the expressions (11), Assumption 1 can always be satisfied since the matrix $A_0 = A - LC$ can be selected to be a stable matrix for any positive gain matrix L, and hence $P = P^T > 0$ is the unique solution to the Lyapunov equation given in (3).

In addition, by exploiting the structural properties of rigid robot manipulators given in (12), we can always verify that for every $x \in \mathbb{R}^{2n}$ we have

$$f(x,u,t) = P^{-1}C^T\, h(x,u,t), \tag{13}$$

$$B\,\eta_d(t) = P^{-1}C^T w\,(t), \tag{14}$$

where

$$\|\, h(x,u,t) + w(t)\, \| = \|\, \zeta(x,u,t)\, \| \leq \rho\,(t), \tag{15}$$

and $w(t)$ is a parameterization of the disturbance vector $\eta_d(t)$.

The procedure to determine this nonlinearity bound $\rho(t)$ is similar to that used with saturating type controllers. The observer is given by

$$\dot{\hat{x}} = A\,\hat{x} + L\,(y - C\,\hat{x}) + \nu_0(t,\,\hat{x},\,y), \tag{16}$$

with $\nu_0(t,x,y)$ defined as in (6). The observation error system is obtained as

$$\dot{e} = A_o e + \nu_0(t,\hat{x},y) - f(t,x) - B\,\eta_d(t), \tag{17}$$

with $e = \hat{x} - x = \begin{pmatrix} e_1 & e_2 \end{pmatrix}^T$ being the observation error. We can see from (17) that the additional switching term $\nu_0(t,\hat{x},y)$ is used in the observer structure to cope with the effects of nonlinearities and/or uncertainties in the plant model and input disturbances.

To show the exponential convergence of the observation error, we consider the following Lyapunov function candidate:

$$V = \frac{1}{2} e^T P\, e, \tag{18}$$

whose time derivative evaluated along the error dynamics (17) is

$$\dot{V} = -\frac{1}{2} e^T Q\, e + e^T P \Big(\nu_0(t,\hat{x},y) - P^{-1}C^T\big(h(t,x) + w(t)\big)\Big). \tag{19}$$

If we consider Assumptions 1–3 together with Eqns. (6), (13) and (14), the last expression can be bounded as

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q)\,\|\, e\,\|^2 - \|\, Ce\,\|\ \rho - e^T C^T\, \zeta\,(t,x), \tag{20}$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of its argument. Using (15), we can finally write

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q)\|\, e\,\|^2 < 0. \tag{21}$$

Therefore, the time derivative of the Lyapunov function candidate is negative definite, which implies that the error converges exponentially to zero. Furthermore, from bounds on the Lyapunov function, we can write

$$\frac{1}{2}\lambda_{\min}(P)\,\|e\|^2 \leq V(e) \leq \frac{1}{2}\lambda_{\max}(P)\,\|e\|^2 \tag{22}$$

and

$$\dot{V} \leq -\frac{1}{2}\lambda_{\min}(Q)\,\|e\|^2, \tag{23}$$

where $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote the minimum and maximum eigenvalues of the matrix P, respectively. Then we can write

$$\frac{\dot{V}(e)}{V(e)} \leq -\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} = \varepsilon_1 \tag{24}$$

or

$$V\,(e(t)) \leq V\,(e(0))\; e^{-\varepsilon_1 t}. \tag{25}$$

Hence, the rate at which the error converges to zero is determined as

$$\|e(t)\|^2 \leq \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\,\|e(0)\|^2\, e^{-\varepsilon_1 t}. \tag{26}$$

This shows the exponential convergence of the observation error and the rate of convergence. Again, the switching term in the observer will cause “chattering”.

Remark 1. With the boundary layer approach we avoid the occurrence of the chattering phenomena, but we lose the asymptotic stability aspect of the observation error. In (Dawson *et al.*, 1992), the author extended the above result, and the global exponential stability was derived by a modification of the additional observer gain $\nu_0(t,\hat{x},y)$

given in (6). The modified observer law is given by (5) with $\nu_0(t, \hat{x}, y)$ replaced by

$$\nu_0'(t, \hat{x}, y) = -\frac{P^{-1} C^T C\, e\, \rho^2}{\|C\, e\| \rho + \varepsilon\, e^{-\beta t}}. \tag{27}$$

We can notice from this that the additional term is not discontinuous. Hence, the modified observer is not of a variable structure type. The stability result can be easily shown by taking the same Lyapunov function candidate (18). It can be verified that the time derivative of the Lyapunov function using (27) evaluated along the error dynamics (17) can be bounded as

$$\dot{V} \le -\frac{1}{2} \lambda_{\min}(Q) \,\|\, e \,\|^2 + \varepsilon\, e^{-\beta t}, \tag{28}$$

from which the global exponential stability of the observation error there results. It can be seen that the bounds on the observation error performance from (28) relate the transient response of the given observation error to the observer parameters ε, β and L. We can therefore calculate the transient response of the observer, from the initial observation error to zero, given specific choices of the observer parameters.

4. Closed Loop Control

In order to use the above observer for the tracking problem of robot manipulators, we consider the trajectory tracking controller proposed by Paden and Panja (1988), with the real velocity state vector replaced with the estimated one. We have the control law given by

$$\begin{aligned} \tau = {} & M(q)\, \ddot{q}_d + C(q, \dot{\hat{q}})\, \dot{q}_d + G(q) \\ & - K_v(\dot{\hat{q}} - \dot{q}_d) - K_p \tilde{q}, \end{aligned} \tag{29}$$

where $\tilde{q} = q - q_d$ defines the position tracking error, and K_p and K_v are positive design controller gains. We should make the assumption that the desired velocity vector is bounded as $\|\dot{q}_d\| \le V_P$, which is reasonable from the implementation point of view. Using the robot dynamics, the closed loop system is governed by

$$\begin{aligned} M(q)\, \ddot{\tilde{q}} + C(q, \dot{q})\, \dot{q} - C(q, \dot{\hat{q}})\, \dot{q}_d & \\ = -K_p \tilde{q} - K_v(\dot{\tilde{q}} + e_2), & \end{aligned} \tag{30}$$

where $\dot{\tilde{q}} = \dot{q} - \dot{q}_d$ is the velocity tracking error and $e_2 = \dot{\hat{q}} - \dot{q}$ is the velocity observation error.

Using the structural properties of the Coriolis and centrifugal torque vector (Yoshikawa, 1990), we can write

$$C(q, \dot{q})\, \dot{q} - C(q, \dot{\hat{q}})\, \dot{q}_d = C(q, \dot{q})\, \dot{\tilde{q}} - C(q, \dot{q}_d)\, e_2. \tag{31}$$

Accordingly, consider the following result (Abdessameud and Khelfi, 2005):

Main Result: *Given the control law stated in (29) and the observer (16) with (6), if Assumption 1 and the relations (13)–(15) are satisfied, then the closed loop system described by (17) and (30) is globally asymptotically stable.*

To investigate the stability of the closed loop dynamics, consider the Lyapunov function candidate

$$V(e, \tilde{q}, \dot{\tilde{q}}, t) = \frac{1}{2} e^T P\, e + \frac{1}{2} \dot{\tilde{q}}^T M(q)\, \dot{\tilde{q}} + \frac{1}{2} \tilde{q}^T K_p \tilde{q}. \tag{32}$$

The time derivative of this Lyapunov function evaluated along the trajectories of the error dynamics (17) and (30) and using the relations (13)–(15) is obtained directly as

$$\begin{aligned} \dot{V} = {} & -\frac{1}{2} e^T Q\, e + e^T P \left(\nu_0 - P^{-1} C^T \zeta\right) \\ & - \dot{\tilde{q}}^T K_v \dot{\tilde{q}} - \dot{\tilde{q}}^T K_v e_2 + \dot{\tilde{q}}^T C(q, \dot{q}_d)\, e_2. \end{aligned} \tag{33}$$

This can be bounded as, using the structural properties of the Coriolis and centrifugal torque vector (Yoshikawa, 1990)

$$\begin{aligned} \dot{V} \le {} & -\frac{1}{2} \lambda_{\min}(Q) \|e\|^2 - K_{v,m} \|\dot{\tilde{q}}\|^2 \\ & + \|\dot{\tilde{q}}\| \,\|\, e_2 \| \,(K_{v,M} + C_M V_P), \end{aligned} \tag{34}$$

with $K_{v,m}$ and $K_{v,M}$ denoting the minimum and maximum eigenvalues of the matrix K_v respectively. Knowing that $\|e_2\| \le \|e\|$, we can write

$$\begin{aligned} \dot{V} \le {} & - \begin{pmatrix} \|\dot{\tilde{q}}\| \\ \|e\| \end{pmatrix}^T \\ & \times \begin{pmatrix} K_{v,m} & -\frac{1}{2}(K_{v,M} + C_M V_P) \\ -\frac{1}{2}(K_{v,M} + C_M V_P) & \frac{1}{2} \lambda_{\min}(Q) \end{pmatrix} \\ & \times \begin{pmatrix} \|\dot{\tilde{q}}\| \\ \|\, e \,\| \end{pmatrix}. \end{aligned} \tag{35}$$

The matrix on the right-hand side of the above inequality is positive if

$$\lambda_{\min}(Q) > \frac{(K_{v,M} + C_M V_P)^2}{2 K_{v,m}}. \tag{36}$$

Then, using Barballat's lemma, we can conclude the asymptotic stability of the equilibrium point $(\tilde{q}, \dot{\tilde{q}}, e_1, e_2) = (0, 0, 0, 0)$.

Note that the above stability condition can always be satisfied if the matrices K_v and Q are properly selected. In all cases, Q should be maximized. Unfortunately, if the observer gain matrix L is fixed, increasing Q will give large solutions for P in (3), which will cause a high gain switching term and high chattering. On the other hand, if P is fixed, large values for Q will lead to high observer gains, L. In both situations, the system will be more sensitive to measurement noise and high frequency-unmodeled dynamics.

Remark 2. The design of the switching term in the observer structure mainly depends on Eqns. (13) and (14), and the bound on system nonlinearities and disturbances (15). In order to apply this technique, the bound should be computed as much accurately as possible.

5. Simulation Results

In order to test the validity of our design, we have considered a 3-DOF robot manipulator (the first three joints of the 6-DOF robot manipulator given by Yoshikawa (1990)). The objective of our simulation work was to show that the tracking objective is achieved when a robustly estimated velocity vector is used in the tracking control law.

In order to implement the robust observer, we first have to determine an upper bound on system nonlinearities and uncertainties, where we have considered a randomly additive term to the inertia matrix. To obtain this bound, we conduct simulations in the full information case, with the control law considered, and we take the maximal norm of the nonlinearity and disturbance vectors along a trajectory. We repeat this work with different trajectories and we take the worst case as our upper bound to be used in the switching term of the variable structure observer.

Then, due to the complexity of the control system, the control system gains should be carefully selected. The controller gains are selected to be high enough such that the tracking controller ensures the asymptotic convergence of the tracking error in the case of full state information. We encountered several problems during observer gains tuning and noticed that if the gain matrix L is fixed, increasing the matrix Q will give large solutions for the matrix P, which will cause a high gain switching term, and if the matrix P is fixed, increasing the matrix Q will lead to a high observer gain matrix L. In both situations, the system will be more sensitive to measurement noise and high frequency unmodeled dynamics. Moreover, the observer gains should be selected according to the condition (36).

The results obtained from the MATLAB simulation of the proposed scheme with a 3 DOF robot manipulator along a trajectory of order 5 and the upper bound on system nonlinearities estimated at 32.5 are shown below. Figures 1–3 show the velocity observation errors of the three

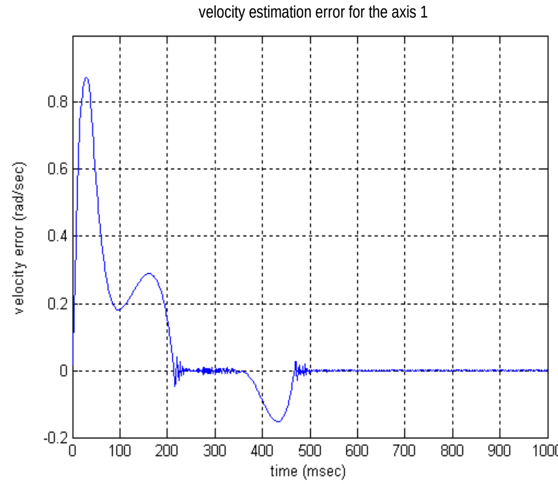

Fig. 1. Velocity estimation error of the axis 1.

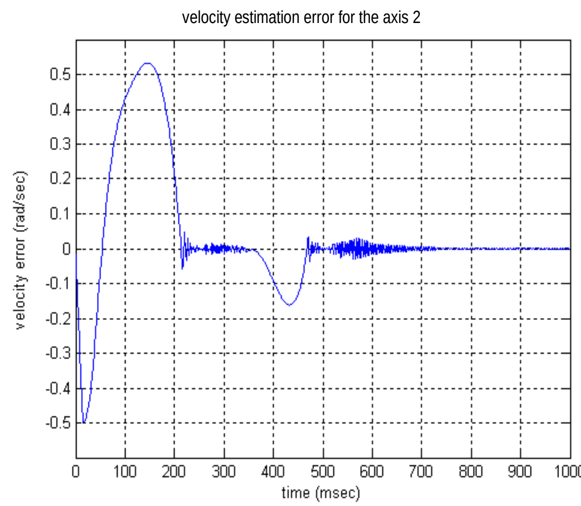

Fig. 2. Velocity estimation error of the axis 2.

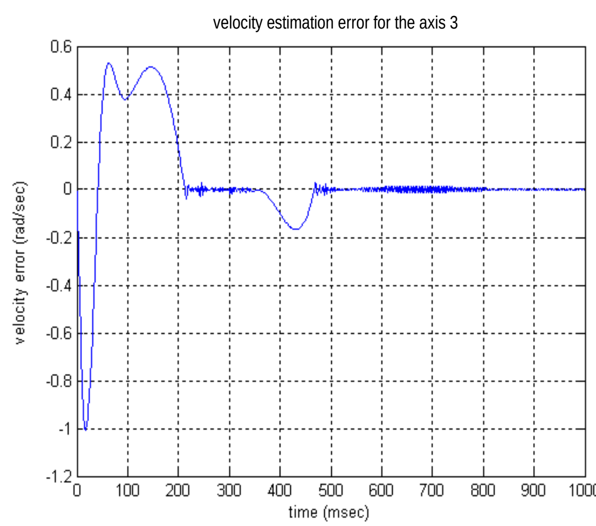

Fig. 3. Velocity estimation error of the axis 3.

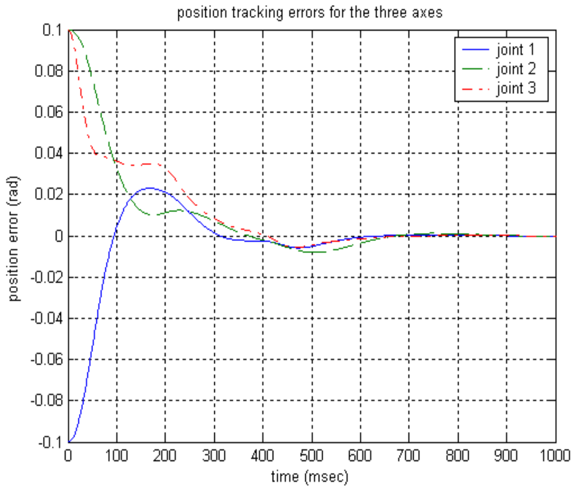

Fig. 4. Position tracking errors of the three axes.

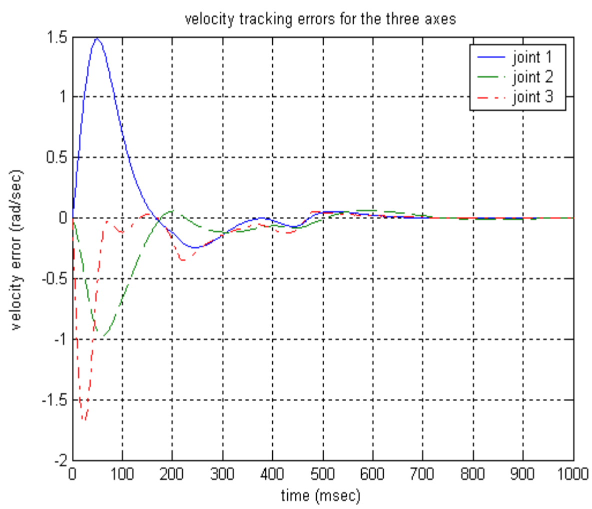

Fig. 5. Velocity tracking errors of the three axes.

axes, where we can see the convergence of the error signals, high frequency oscillations caused by the switching term and the high gain values used. To solve this problem, we can consider the boundary layer approach to eliminate chattering. Figures 4 and 5 show the position and velocity tracking errors of the three axes, respectively, when the robustly estimated velocity vector is used in the tracking control law (29) and asymptotic convergence is guaranteed.

6. Conclusion

In this paper, we have presented the application of a variable structure observer, found in the literature, to the class of rigid robot manipulators. The observer considered is basically the Luenberger observer with an additive switching term used to cope with system nonlinearities and/or uncertainties. The design of the robust observer is based on the assumption that the linear part of the nonlinear system is completely observable, and the system nonlinearities and uncertainties are upper bounded and satisfy some matching conditions. One drawback of this design is that the presence of the switching term causes "chattering". To solve this problem, the use of a boundary layer is a solution. Another solution is to use a continuous term that guarantees the global exponential stability of the observation error just as is done by Dawson *et al.* (1992).

The robustly estimated states are then used in a control loop with a trajectory tracking control law, which ensures the global asymptotic stability of the system in the full information case, that is, both the velocity and position vectors are available for feedback. Under the assumption that the desired velocity vector is bounded, the extended error vector is proved to be globally asymptotically stable under the condition that the desired velocity vector is bounded. Through simulations, we illustrated the feasibility of the designed control system.

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