

## FAST LEAK DETECTION AND LOCATION OF GAS PIPELINES BASED ON AN ADAPTIVE PARTICLE FILTER

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Leak detection and location play an important role in the management of a pipeline system. Some model-based methods, such as those based on the extended Kalman filter (EKF) or based on the strong tracking filter (STF), have been presented to solve this problem. But these methods need the nonlinear pipeline model to be linearized. Unfortunately, linearized transformations are only reliable if error propagation can be well approximated by a linear function, and this condition does not hold for a gas pipeline model. This will deteriorate the speed and accuracy of the detection and location. Particle filters are sequential Monte Carlo methods based on point mass (or “particle”) representations of probability densities, which can be applied to estimate states in nonlinear and non-Gaussian systems without linearization. Parameter estimation methods are widely used in fault detection and diagnosis (FDD), and have been applied to pipeline leak detection and location. However, the standard particle filter algorithm is not applicable to time-varying parameter estimation. To solve this problem, artificial noise has to be added to the parameters, but its variance is difficult to determine. In this paper, we propose an adaptive particle filter algorithm, in which the variance of the artificial noise can be adjusted adaptively. This method is applied to leak detection and location of gas pipelines. Simulation results show that fast and accurate leak detection and location can be achieved using this improved particle filter.

**Keywords:** gas pipeline, leak detection and location, particle filter

### 1. Introduction

Pipelines are principal devices for natural gas transportation, and lots of large scale pipeline networks have been constructed in many countries in the past 40 years. However, leaks, which are the main faults of gas pipelines, can cause serious problems related not only to the environment but also to economy. Therefore, many methods and techniques for leak detection with various applicability and restrictions have been proposed to prevent further loss and danger (Muhlbauer, 2004). The primary methods include acoustic monitoring, optical monitoring, gas sampling, soil monitoring, flow monitoring, magnetic flux leakage, and dynamic model-based methods.

Acoustic monitoring techniques utilize acoustic detectors to detect the wave of noise which will be generated as the gas escapes from the pipeline (Brodetsky and Savic, 1993; Hough, 1988; Klein, 1993). This kind of methods is simple and accurate, and can detect relatively small leaks. However, a large number of acoustic sensors along the pipeline are required, which will increase the cost. If the leaks are too small to produce acoustic signals

at levels substantially higher than the background noise, this technology will be useless (Sivathau, 2003).

Optical monitoring methods can be classified as either passive or active (Reichardt *et al.*, 1999). Active methods involve the illumination of the area above the pipeline with a radiation source, usually a laser or a broad band source. Then the absorption or scattering caused by gas molecules above the surface is monitored using an array of sensors at specific wavelengths (Ikuta *et al.*, 1999; Iseki *et al.*, 2000; Spaeth and O’Brien, 2003). In contrast to active methods, passive methods do not require a source. They detect the radiation emitted by the natural gas, or the background radiation serves directly. Optical monitoring techniques can be used for moving vehicles, aircraft, portable systems, or on site, and are able to monitor the pipeline over an extended range. Moreover, they have high spatial resolution and sensitivity under specific conditions. But for most of these optical methods, the implementation cost is high. A high false alarm rate is another disadvantage.

Sampling methods are mostly used to detect hydrocarbon gas leaks (Sperl, 1991). The sampling can be done

by carrying a flame ionization detector along a pipeline or using a sensor tube buried in parallel to the pipeline. Very tiny leaks can be detected using these methods. But the response time is usually from several hours to days, and the cost of monitoring long pipelines is very high.

A small amount of a tracer chemical is injected into the pipeline using soil monitoring methods. If a leak occurs, this tracer chemical will seep out of the pipe, which can be detected by dragging an instrument along the surface above the pipeline (Tracer Research Corporation, 2003). The sensitivity of this method is high, while the false alarm rate is very low. However, the method does not function for exposed pipelines, and it is very expensive because the trace chemical needs to be added to the gas continuously.

Flow monitoring methods rely on pressure and/or flow signals at different sections of a pipeline, mostly only the extremes. When the pipeline operates normally, there are some steady relationships among these signals. Changes in these relationships will indicate the occurrence of leaks. Volume balance is the most straightforward flow monitoring method. A leak alarm will be generated when the difference between upstream and downstream flow measurements changes by more than an established tolerance (Ellul, 1989; Furness, 1985). But because of the inherent flow dynamics and the superimposed noise, only relatively large leaks, which exceed about 10% for gas pipelines, can be detected. Fukuda and Mitsuoka (1983), and Wang *et al.* (1993), respectively, formulated the pressure gradients by using the autoregressive (AR) model, and then they used the AIC and Kullback information to detect leaks. Chernick and Wincelberg (1985) applied the autoregressive moving-average (ARMA) model with the “variate difference method” to the pressure. Using this model, some improvement in the leak detection capability could be achieved. Considering the fact that the inlet flow rate measurements are unavailable and the conventional mass balance techniques cannot be used, Dinis *et al.* (1999) presented a statistical method to detect leaks in subsea liquid pipelines. But this method has not been tested in gas pipelines.

The magnetic flux leakage method periodically inspects the pipeline for damages using the device called ‘pig’, which is launched at the inlet and retrieved at the outlet. The pig is a magnetizer-sensor assembly. It employs the magnetic flux leakage (MFL) technique for assessing the condition of the pipe (Afzal and Udpa, 2002). This method can detect pits as shallow as 5–10% of the pipe wall thickness. The leak location is also very accurate using this method. But it is functional only for a seamless gas pipeline, and the pipeline cannot be detected continuously. Moreover, because a pig is needed to put into the pipeline, jams may occur.

Dynamic model-based methods attempt to mathematically model the gas flow within a pipeline. Using this model, flow parameters are calculated at different sections of the pipeline, and these parameters are measured as well. Then leaks can be detected by comparing the calculated and measured parameters. Billmann and Isermann (1987) used a nonlinear state observer and a special correlation technique for leak detection and location. Shields *et al.* (2001) further considered the disturbance (noise and modeling errors) in a pipeline model. Based on this model, a disturbance decoupled nonlinear fault detection observer was proposed. By discretizing the pipeline model with nonuniform regions along the line, Verde (2005) proposed an accommodation scheme to tackle the multi-leak detection and location problem. But this method cannot estimate the leak size. Benkherouf and Allidina (1988) presented a pipeline model with a leak, and used the EKF to estimate the leak parameters as augmented state variables. Based on the same model, Zhao and Zhou (2001) used an STF (Zhou and Frank, 1996) to detect and locate leaks, and the detection speed was faster. By considering a thermal model, Tiang (1997) presented a more accurate leak location method.

The particle filter (PF), based on a Monte-Carlo technique, was first proposed by Gordon (Gordon *et al.*, 1993). Thereafter, a number of alternative PF algorithms have been proposed. The PF uses sequential Monte-Carlo methods to approximate the optimal filtering by representing the probability density function (PDF) with a swarm of particles. Because the PF is able to handle any functional nonlinearity and system or measurement noise of any probability distribution, it has attracted much attention in the nonlinear non-Gaussian state estimation field (Bolvikén *et al.*, 2001; Doucet *et al.*, 2000; Kitagawa, 1996).

Leak detection and location are a significant industrial application of FDD (Kowalczyk and Gunawickrama, 2004), and have been an active research area over the last two decades (Gertler, 1998; Korbicz *et al.*, 2004; Patton *et al.*, 2000). Analytical model based FDD is a class of the most important approaches. In these approaches, filters are widely used, such as the Kalman Filter (KF), the EKF, and the STF. Based on the state estimation ability of the PF, Kadiramanathan *et al.* (2000) first used it in FDD. In order to estimate a fault, they introduced random walk noise to the fault parameters treated as augmented states. But because the noise signals are not adaptive, the estimation is not very accurate. Then Li and Kadiramanathan (2001) proposed a PF based likelihood ratio approach to fault diagnosis. This approach can detect faults in time, and identify them correctly. However, it cannot estimate their sizes.

In this paper, a new model based approach to leak detection and location of gas pipelines with the use of an

adaptive particle filter (APF) is proposed. For the nonlinearity of a gas pipeline system, the PF is more efficient than the EKF. In order to estimate the leakage, the state augmentation technique is used, and artificial dynamics of the leak parameters are introduced to track the leak dynamics. Then if the augmented system is observable, the parameter estimation problem has a solution. However, the variances of the artificial dynamic disturbances will closely relate to the detection speed and accuracy, and a satisfying estimation result cannot be obtained by using invariant variances. Thus, a variance-adaptive method based PF is proposed to achieve fast and accurate leak detection and location. Computer simulation results illustrate the effectiveness of the proposed approach.

## 2. Mathematical Model of a Gas Pipeline

Neglecting viscous and turbulent effects of the flow, and assuming that the temperature changes within the gas and heat exchanges with the surroundings of the pipeline are negligible, a one-dimensional isothermal gas pipeline model can be derived as follows (Billmann, 1982):

$$\frac{\partial p}{\partial t} + \frac{c^2}{A} \frac{\partial q}{\partial x} = 0, \quad (1)$$

$$\frac{\partial q}{\partial t} + A \frac{\partial p}{\partial x} + \frac{\lambda c^2}{2DA} \frac{q|q|}{p} = 0, \quad (2)$$

where  $p$  [Pascal] is the pressure,  $q$  [Kg/s] is the mass flow rate,  $p$  and  $q$  are functions of time  $t$  [s] and distance  $x$  [m],  $c$  [m/s] is the isothermal speed of sound in gases,  $A$  [m<sup>2</sup>] is the pipeline cross-section,  $D$  [m] is the pipeline diameter,  $\lambda$  [-] is the friction coefficient.

This model is composed of two differential equations, and they form a nonlinear distributed parameter system of the hyperbolic type. Suitable boundary and initial conditions of this system can be chosen as follows:

$$\begin{cases} p(0, t) = f_p(t), \\ q(L, t) = f_q(t), \end{cases} \quad (3)$$

and

$$\begin{cases} p(x, 0) = p_0(x), \\ q(x, 0) = q_0(x), \end{cases} \quad (4)$$

where  $L$  [m] is the pipe length. If a leak  $K$  [kg/s] occurs at  $x = x_K$ , Eqns. (1) and (2) are still valid for all  $x \in [0, x_K) \cup (x_K, L]$ . Owing to mass conservation, we can get the following equation at  $x = x_K$ :

$$q(x_K^-, t) - q(x_K^+, t) = K. \quad (5)$$

We assume that the leak introduces a negligible momentum in the  $x$  direction, so that Eqn. (2) is unaffected

for  $x = x_K$ . Hence, Eqn. (1) for  $x \neq x_K$ , and Eqns. (2)–(5) represent an approximate model for the leaking gas pipeline (Benkherouf and Allidina, 1988).

## 3. Adaptive Particle Filter Based Leak Detection and Location Scheme

### 3.1. Adaptive Particle Filter

Consider a class of discrete-time nonlinear systems of the form

$$x_{k+1} = f(x_k, u_k, \theta_k) + w_k, \quad (6)$$

$$y_{k+1} = h(x_{k+1}) + v_{k+1}, \quad (7)$$

where  $x_k \in \mathbb{R}^n$  is the state,  $u_k \in \mathbb{R}^p$  stands for the input,  $y_k \in \mathbb{R}^m$  means the output,  $\theta_k \in \mathbb{R}^l$  signifies the fault parameter vector,  $f : \mathbb{R}^p \times \mathbb{R}^n \rightarrow \mathbb{R}^n$  is the state transition function, and  $h : \mathbb{R}^n \rightarrow \mathbb{R}^m$  is the measurement function. The process noise  $w_k \in \mathbb{R}^n$  is zero mean, and independent of past and current states. The measurement noise  $v_k \in \mathbb{R}^m$  is zero mean and independent of the past and current states and the process noise. Here  $\theta_k$  is the unknown fault parameter vector to be estimated. Using the state augmentation technique, a new state vector can be defined:

$$z_k := \begin{bmatrix} x_k \\ \theta_k \end{bmatrix}.$$

Because  $\{\theta_k\}$  is not ergodic,  $\theta_k$  cannot be tracked in the PF algorithm. Therefore, in order to track the dynamics of  $\theta_k$ , an artificial dynamic noise vector is added to the model of the unknown parameter  $\theta_k$ :

$$\theta_{k+1} = \theta_k + w_k^\theta, \quad (8)$$

where  $w_k^\theta$  is the parameter noise. Then the augmented system and measurement functions are respectively defined as

$$z_{k+1} = f_e(z_k, u_k) + \tilde{w}_k,$$

$$y_{k+1} = h_e(z_{k+1}) + v_{k+1},$$

where  $\tilde{w}_k = [w_k^T (w_k^\theta)^T]^T$ . Because  $w_k^\theta$  is artificial, its statistical properties need to be determined. In this paper, we assume that  $w_k^\theta$  is a zero-mean Gaussian white noise process, so only its variance needs to be determined. Obviously, if the variance of  $w_k^\theta$  is too large, the estimation of  $\theta_k$  will be inaccurate; if the variance is too small, abrupt parameter changes cannot be tracked (this is illustrated by a simulation in Section 4). So we have to have the variance determined adaptively. Zhou *et al.* (1991) presented a strong tracking filter (STF) by adaptively adjusting the predicted state error covariance according to

the covariance of residuals. Similarly, we propose an adaptive approach to determine the variance of  $w_k^\theta$  according to the covariance of residuals, ensuring that both the estimation accuracy and the parameter tracking speed are improved.

First, a vector of artificial zero-mean Gaussian white noise signals  $w_k^\theta$  is selected, whose variances are small and invariant. If we replace  $w_k^\theta$  in (8) by  $\lambda_k w_k^\theta$ , then only  $\lambda_k$  has to be determined. In the STF, a suboptimal fading factor is introduced to track the time-varying states or parameters. This factor is adjusted according to the covariance of residuals. When the accuracy of state prediction is reduced, the factor will become larger. This property is just what we want for  $\lambda_k$ . So, similarly, we adjust  $\lambda_k$  as follows:

$$\lambda_0 = \frac{\text{tr}[V_{k+1}]}{\text{tr}[M_{k+1}]}, \quad (9)$$

$$\lambda_k = \begin{cases} \lambda_0 & \text{if } \lambda_0 > 1, \\ 1 & \text{if } \lambda_0 \leq 1, \end{cases} \quad (10)$$

where  $V_{k+1}$  is the covariance of the actual residual, and  $M_{k+1}$  is the covariance of the theoretical residual.  $V_{k+1}$  is unknown. However, it can be roughly approximated by

$$V_{k+1} = E[\gamma_{k+1}^i (\gamma_{k+1}^i)^T] = \begin{cases} \frac{1}{N_s} \sum_{j=1}^{N_s} \gamma_1^j (\gamma_1^j)^T & \text{if } k = 0, \\ \frac{[\rho V_k + \frac{1}{N_s} \sum_{j=1}^{N_s} \gamma_{k+1}^j (\gamma_{k+1}^j)^T]}{1 + \rho} & \text{if } k \geq 1, \end{cases} \quad (11)$$

$$\gamma_{k+1}^i = y_{k+1} - y_{k+1|k}^i, \quad (12)$$

where  $N_s$  is the number of particles, the superscripts  $i$  or  $j$  denote respectively the  $i$ -th or  $j$ -th particles,  $\gamma_{k+1}$  is the residual vector,  $y_{k+1|k}$  is the predicted output vector,  $y_{k+1}$  is the real output vector, and  $0 \leq \rho \leq 1$  is a forgetting factor. Here  $\rho = 0$  implies that  $V_{k+1}$  has nothing to do with  $V_k$ . One often selects  $\rho = 0.95$ . We can use the residual data in faultless conditions to get the experiential residual covariance as  $M_{k+1}$ . If we have  $S$  steps of the faultless regime,  $M_{k+1}$  can be determined via

$$M_{k+1} = M_0 + \frac{1}{SN_s} \sum_{j=0}^{S-1} \sum_{i=1}^{N_s} (\bar{y}_{j+1|j} - y_{j+1|j}^i) (\bar{y}_{j+1|j} - y_{j+1|j}^i)^T, \quad (13)$$

$$\bar{y}_{j+1|j} = \frac{1}{N_s} \sum_{i=1}^{N_s} y_{j+1|j}^i, \quad (14)$$

where  $\bar{y}_{j+1|j}$  is the mean value of the predicted output vector. If the augmented measurement function is linear, i.e.,  $y_{k+1} = H_e z_{k+1} + v_{k+1}$ ,  $H_e \in \mathbb{R}^{m \times n}$ , and the measurement noise is Gaussian with the covariance  $R$ ,  $M_{k+1}$  can be calculated using the following formulae:

$$M_{k+1} = H_e P_{k+1|k} H_e^T + R, \quad (15)$$

$$P_{k+1|k} = \frac{1}{N_s} \sum_{i=1}^{N_s} (z_{k+1|k}^i - \bar{z}_{k+1|k}) (z_{k+1|k}^i - \bar{z}_{k+1|k})^T, \quad (16)$$

$$\bar{z}_{k+1|k} = \frac{1}{N_s} \sum_{i=1}^{N_s} z_{k+1|k}^i, \quad (17)$$

where  $P_{k+1|k}$  is the covariance of the state prediction error,  $z_{k+1|k}$  is the predicted state vector,  $\bar{z}_{k+1|k}$  is the mean value of the predicted state vector.

Let  $D_k$  denote the available measurement information at the time  $k$ :

$$D_k = \{y_i | i = 1, \dots, k\}.$$

The APF algorithm is detailed as follows:

### Step 1: Initialization

Augment the state vector with the unknown fault parameters:

$$z_k := \begin{bmatrix} x_k \\ \theta_k \end{bmatrix}.$$

The system and measurement functions of the augmented system are  $f_e$  and  $h_e$ , respectively. Sample  $N_s$  particles  $\{z_0^i, i = 1, \dots, N_s\}$  from the supposed PDF  $p(z_0)$ .

### Step 2: Prediction

Sample  $N_s$  values  $\{w_k^i, i = 1, \dots, N_s\}$  and  $\{w_k^{\theta i}, i = 1, \dots, N_s\}$  from the PDFs of  $w_k$  and  $w_k^\theta$ , respectively. Here  $w_k^\theta$  is an artificial zero-mean Gaussian white noise process, the variance of which is set to be invariant and small. Then compute

$$x_{k+1|k}^i = f(x_k^i, u_k) + w_k^i,$$

$$\theta_{k+1|k}^i = \theta_k^i + w_k^{\theta i},$$

$$y_{k+1|k}^i = h_e(z_{k+1|k}^i).$$

### Step 3: Update

If the measurement function is nonlinear,  $M_{k+1}$  can be calculated via (13) and (14). If the measurement function is linear and the measurement noise is Gaussian with the covariance  $R$ , we can get  $M_{k+1}$  by (15)–(17).

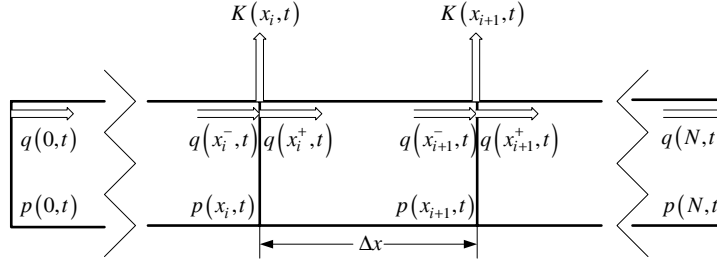


Fig. 1. Nodal representation of a pipeline.

Obtain  $\lambda_k$  by (9)–(12), and determine

$$\tilde{\theta}_{k+1|k}^i = \theta_k^i + \lambda_k w_k^{\theta^i},$$

$$\tilde{z}_{k+1|k}^i = \begin{bmatrix} x_{k+1|k}^i \\ \tilde{\theta}_{k+1|k}^i \end{bmatrix},$$

$$\begin{aligned} \omega^i &= \frac{p(y_{k+1} | \tilde{z}_{k+1|k}^i)}{\sum_{j=1}^{N_s} p(y_{k+1} | \tilde{z}_{k+1|k}^j)} \\ &= \frac{p_v(y_{k+1} - h_e(\tilde{z}_{k+1|k}^i))}{\sum_{j=1}^{N_s} p_v(y_{k+1} - h_e(\tilde{z}_{k+1|k}^j))}. \end{aligned}$$

We have

$$\tilde{p}(z_{k+1} | D_{k+1}) = \sum_{i=1}^{N_s} \omega^i \delta(z_{k+1} - \tilde{z}_{k+1|k}^i),$$

where  $\delta$  is the Dirac-delta function.

#### Step 4: Resampling

Resample independently  $N_s$  times from the above discrete distribution. The resulting particles  $\{z_{k+1}^i, i = 1, \dots, N_s\}$  satisfy  $\Pr\{z_{k+1}^i = \tilde{z}_{k+1|k}^i\} = \omega^i, j = 1, \dots, N_s$ . Then the PDF becomes

$$p(z_{k+1} | D_{k+1}) = \frac{1}{N_s} \sum_{i=1}^{N_s} \delta(z_{k+1} - z_{k+1}^i).$$

Then compute

$$\hat{z}_{k+1} = E[z_{k+1}^i] = \frac{1}{N_s} \sum_{i=1}^{N_s} z_{k+1}^i.$$

**Step 5:** The prediction, update and resample steps form a single iteration and are recursively applied at each time  $k$ .

### 3.2. Leak Detection and Location Scheme

The pipeline model described in Section 2 is a Distributed Parameter System (DPS). In order to use the APF to estimate the leakage  $K$  and its location  $x_K$ , a discrete-time/discrete-space (DTDS) model of the pipeline is required. Consequently, the pipeline is first divided into  $N$

sections with  $N + 1$  nodes as shown in Fig. 1. Then the DTDS model can be obtained using the method of characteristics (Wylie and Streeter, 1993):

$$\begin{aligned} (p_{i,j} - p_{i-1,j-1}) + \frac{c}{A}(q_{i,j}^- - q_{i-1,j-1}^+) \\ + \frac{\lambda c^3 \Delta t}{4DA^2} \left( \frac{q_{i,j}^- |q_{i,j}^-|}{p_{i,j}} + \frac{q_{i-1,j-1}^+ |q_{i-1,j-1}^+|}{p_{i-1,j-1}} \right) = 0, \end{aligned} \quad (18)$$

$$\begin{aligned} (p_{i,j} - p_{i+1,j-1}) - \frac{c}{A}(q_{i,j}^+ - q_{i+1,j-1}^-) \\ - \frac{\lambda c^3 \Delta t}{4DA^2} \left( \frac{q_{i,j}^+ |q_{i,j}^+|}{p_{i,j}} + \frac{q_{i+1,j-1}^- |q_{i+1,j-1}^-|}{p_{i+1,j-1}} \right) = 0, \end{aligned} \quad (19)$$

$$q_{i,j}^- - q_{i,j}^+ = K_{i,j}, \quad (20)$$

where  $p_{i,j} = p(x_i, t_j)$ ,  $q_{i,j}^- = q(x_i^-, t_j)$ ,  $q_{i,j}^+ = q(x_i^+, t_j)$ ,  $K_{i,j} = K(x_i, t_j)$ ,  $x_i = (i-1)\Delta x$ ,  $t_j = j\Delta t$ . Here  $\Delta x$  is the length of one section and  $\Delta t$  is the sampling interval, satisfying  $\Delta x/\Delta t = c$ . The upstream boundary condition is  $p_{1,j} = f_p(t_j)$ , and the downstream boundary condition has the form  $q_{N,j} = f_q(t_j)$ . Moreover,  $K_{i,j}$  are modelled leaks, which are assumed to be constant, so that the model is augmented by

$$K_{i,j} = K_{i,j-1}. \quad (21)$$

The relationship between the actual leak ( $K(j), x_K(j)$ ) at the time step  $j$  and the modelled leaks ( $K_{i,j}, x_i$ ) are as follows (Benkherouf and Allidina, 1988):

$$K(j) = \sum_{i=2}^{N-1} K_{i,j}, \quad (22)$$

$$x_K(j) = \sum_{i=2}^{N-1} \frac{K_{i,j} x_i}{K(j)}. \quad (23)$$

To apply the APF, we select the state variables as follows:

$$x_j = \left[ p_{2,j}, p_{3,j}, \dots, p_{N,j}, q_{1,j}^+, q_{2,j}^+, \dots, q_{N-1,j}^+, K_{2,j}, K_{3,j}, \dots, K_{N-1,j} \right]^T.$$

This state vector has been augmented by  $K_{i,j}$ . Adding process and measurement noise vectors to Eqns. (18)–(21), we get

$$g(x_{j+1}, x_j, u_j, w_j) = 0, \tag{24}$$

$$y_{j+1} = H_e x_{j+1} + v_{j+1}, \tag{25}$$

where

$$u_j = [f_p(j) f_q(j)]^T = [f_p(t_j) f_q(t_j)]^T,$$

and  $H_e$  is a matrix with elements being 0 or 1, to provide the available measurements (in the simulation study in Section 4 we assume that pressure measurements at  $m$  discrete points along the pipeline are available). Note that the dimension of the state vector is  $n = 3N - 4$ , and  $g(\cdot)$  consists of  $3N - 4$  equations. Furthermore,

$$w_j = [w_{1,j}, w_{2,j}, \dots, w_{3N-4,j}]^T$$

is the process noise vector, and

$$v_{j+1} = [v_{1,j+1}, v_{2,j+1}, \dots, v_{m,j+1}]^T$$

is the measurement noise vector. Then, based on (24) and (25), we can use the APF to estimate  $x_j$ , and according to (22) the leak size can be estimated. Select a threshold  $\varepsilon$  for fault detection. When the leak estimate  $\hat{K}(j) > \varepsilon$ , calculate the leak location via (23).

### 3.3. Evaluation of Leak Location Capability

A space discretization is introduced in the leak detection and location schemes mentioned above. Since all methods with a space discretization have a leak position error, the sensitivity of our method has to be evaluated.

Since there are only two modelled leaks  $(K_1, x_{K_1})$  and  $(K_2, x_{K_2})$ , the following equations can be derived in a steady state (Benkherouf and Allidina, 1988):

$$K = K_1 + K_2, \tag{26}$$

$$\begin{aligned} & 2 \frac{Kx_K}{f_q} + \left(\frac{K}{f_q}\right)^2 x_K \\ &= 2 \frac{K_1x_{K_1} + K_2x_{K_2}}{f_q} + \left(\frac{K_1}{f_q}\right)^2 x_{K_1} \\ & \quad + \left(\frac{K_2}{f_q}\right)^2 x_{K_2} + 2 \frac{K_1K_2}{f_q^2} x_{K_1}, \end{aligned} \tag{27}$$

where  $f_q$  is the outlet mass flow rate. The leak is usually small compared with  $f_q$ , and therefore the second-order terms  $(K/f_q)^2$ ,  $(K_1/f_q)^2$ ,  $(K_2/f_q)^2$ ,  $(K_1K_2/f_q)^2$  are small compared with  $K/f_q$  and  $(K_1x_{K_1} + K_2x_{K_2})/f_q$ . From (27) it follows that

$$Kx_K \approx K_1x_{K_1} + K_2x_{K_2}. \tag{28}$$

Equation (23) can be derived from (28) (Benkherouf and Allidina, 1988). Consequently, we can evaluate the location capability of this method by discussing the location accuracy of (28). If  $\hat{x}_K$  is the located leak position using (28), then we have the following result:

**Theorem 1.** *If there are two modelled leaks  $(K_1, x_{K_1})$  and  $(K_2, x_{K_2})$ , the upper bound of the location error is*

$$\frac{1}{8} \left(\frac{K}{f_q}\right) |x_{K_2} - x_{K_1}|.$$

*Proof.* From (27) we see that

$$\begin{aligned} x_K &= \frac{2K_1f_qx_{K_1} + 2K_2f_qx_{K_2} + K_1^2x_{K_1}}{2Kf_q + K^2} \\ & \quad + \frac{K_2^2x_{K_2} + 2K_1K_2x_{K_1}}{2Kf_q + K^2}. \end{aligned} \tag{29}$$

From (28) it follows that

$$\hat{x}_K = \frac{K_1x_{K_1} + K_2x_{K_2}}{K}. \tag{30}$$

Then, from (29) and (30) we have

$$\begin{aligned} |x_K - \hat{x}_K| &= \frac{K_1^2x_{K_1} + K_2^2x_{K_2} + 2K_1K_2x_{K_1}}{2Kf_q + K^2} \\ & \quad - \frac{K_1Kx_{K_1} - K_2Kx_{K_2}}{2Kf_q + K^2}. \end{aligned}$$

Applying (26) in the numerator, we obtain

$$|x_K - \hat{x}_K| = \frac{K_1K_2}{2Kf_q + K^2} |x_{K_1} - x_{K_2}|.$$

Because  $K_1K_2 \leq (K_1 + K_2)^2/4 = K^2/4$ , we get

$$\begin{aligned} |x_K - \hat{x}_K| &\leq \frac{K^2}{8Kf_q + 4K^2} |x_{K_1} - x_{K_2}| \\ &< \frac{1}{8} \left(\frac{K}{f_q}\right) |x_{K_1} - x_{K_2}|, \end{aligned} \tag{31}$$

which is the desired claim. ■

**Remark 1.** Theorem 1 shows that the upper bound of the location error is affected by two factors,  $K/f_q$  and

$|x_{K_1} - x_{K_2}|$ . As they become smaller, so does the upper bound, and the location becomes more accurate. The conclusion is reasonable. First, the smallness of  $K/f_q$  constitutes the basis for deriving (28), and for a leak which is not too large, this assumption is reasonable. Secondly,  $|x_{K_1} - x_{K_2}|$  is the distance between two neighboring sensors. Obviously, the closer the sensors, the smaller the leak location error. Moreover, this conclusion can be easily extended to  $n$  modelled leaks, that is, the leak location will be more accurate as  $K/f_q$  is smaller or there are more sensors.

#### 4. Simulation Results

A pipeline simulator is used to model a noise-corrupted gas flow in a pipeline whose specification is (Benkherouf and Allidina, 1988)

$$L = 90 \text{ km}, \quad D = 0.875 \text{ m}, \quad c = 300 \text{ m/s}, \quad \lambda = 0.02.$$

Because the leak size estimate (22) and the leak location equation (23) are for the fluid in a steady state, we do not change the operation conditions. However, when the fluid is in a transient state, the method can also detect the leak, while its size and location cannot be estimated.

The data used in the measurement simulator are as follows:

$$\Delta x = 10 \text{ km},$$

i.e., the pipe is divided into nine sections, boundary conditions being constant,

$$P(0, t) = 100 \text{ bar} = 10^7 \text{ Pa}, \quad Q(L, t) = 200 \text{ kg/s}.$$

The data used in the APF are as follows:

$$\Delta x = 30 \text{ km},$$

i.e., the pipe is divided into three sections,

$$\Delta t = x/c = 100 \text{ s},$$

$$x_j = [p_{2,j}, p_{3,j}, p_{4,j}, q_{1,j}^+, q_{2,j}^+, q_{3,j}^+, K_{2,j}, K_{3,j}]^T,$$

i.e., there are two modelled leaks,  $k_{2,j}$  and  $k_{3,j}$ , at 30 and 60 km, respectively.

Three pressure measurements at 30, 60, 90 km are generated for the filter (i.e.,  $m = 3$ ). Then the measurement function is linear,

$$H_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

The boundary conditions are assumed to be known exactly (100 bar and 200 kg/s).

The initial conditions are

$$\hat{x}_0 = [\hat{p}_{2,0}, \hat{p}_{3,0}, \hat{p}_{4,0}, \hat{q}_{1,0}, \hat{q}_{2,0}, \hat{q}_{3,0}, \hat{k}_{2,0}, \hat{k}_{3,0}]^T,$$

$$\hat{p}_{2,0} = 94 \text{ bar}, \quad \hat{p}_{3,0} = 87 \text{ bar}, \quad \hat{p}_{4,0} = 80 \text{ bar},$$

$$\hat{q}_{1,0} = \hat{q}_{2,0} = \hat{q}_{3,0} = 200 \text{ kg/s}, \quad \hat{k}_{1,0} = \hat{k}_{2,0} = 0 \text{ kg/s}.$$

The initial artificial noise is  $w_j^\theta = [w_{j1}^\theta, w_{j2}^\theta]^T$ ,  $w_{j1}^\theta \sim N(0, 0.05^2)$ ,  $w_{j2}^\theta \sim N(0, 0.05^2)$ . The forgetting factor is  $\rho = 0.95$ . The threshold for leak detection is  $\varepsilon = 0.5 \text{ kg/s}$ .

**Case 1.** The measurement and process noise signals are both set to be Gaussian with covariance matrices

$$R = \sigma_m^2 I_3, \quad \tilde{Q} = \begin{bmatrix} \sigma_p^2 I_3 & 0 \\ 0 & \sigma_q^2 I_3 \end{bmatrix},$$

where  $\sigma_m^2 = 10^6 \text{ Pa}^2$ ,  $\sigma_p^2 = 10^4 \text{ Pa}^2$ ,  $\sigma_q^2 = 1 \text{ (kg/s)}^2$ .

At the time  $t = 100 \text{ min}$  a leakage of 3% (6 kg/s) at 50 km from the upstream end of the pipeline is suddenly introduced. The leak size and location are calculated using (22) and (23), respectively. Simulation results with the APF are presented in Fig. 2. For comparison, we also present simulation results with the EKF in Fig. 3, and with a general PF in Fig. 4.

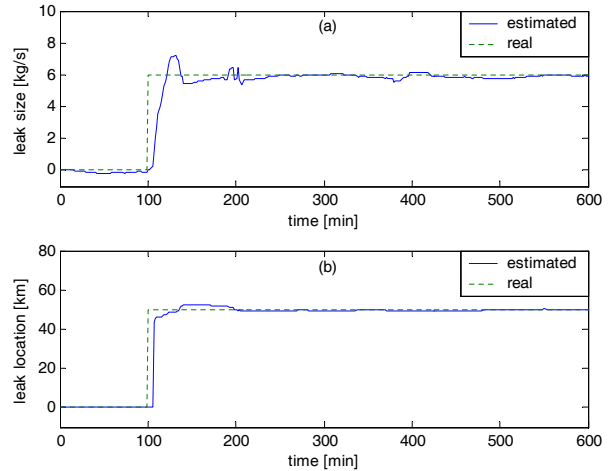


Fig. 2. Simulation results with the APF in Case 1: (a) real leak size and its estimate, (b) real leak location and its estimate.

From Figs. 2 and 3, it is obvious that the tracking of the APF is much faster than that of the EKF. The leak is detected by the APF at 110 min, only 10 minutes after the leak occurs. The average estimated leak location using the APF is  $x = 49.7936 \text{ km}$  with a relative error of 0.41%. Obviously, using the EKF, the detection time is much longer, and the location error is much larger.

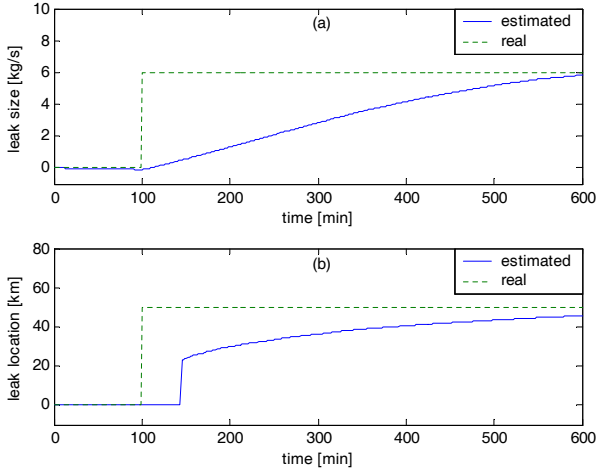


Fig. 3. Simulation results with the EKF in Case 1: (a) real leak size and its estimate, (b) real leak location and its estimate.

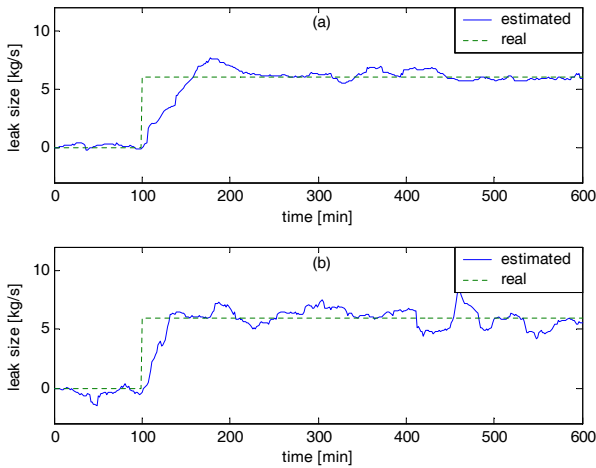


Fig. 4. Real leak size and its estimates with the PF in Case 1: (a) the variance of the artificial zero-mean Gaussian white noise is  $0.2^2$ , (b) the variance of the artificial zero-mean Gaussian white noise is  $0.4^2$ .

Figure 4 shows leak size estimates with a general PF, in which the artificial noise signals are not adaptive. The variances of the artificial zero-mean Gaussian white noise processes are set to be  $0.2^2$  and  $0.4^2$ , respectively. We can see that when the variance is relatively small, the PF cannot track the abrupt change quickly. When the variance is relatively large, the estimation accuracy deteriorates. Because the variance can be adjusted adaptively, the APF has good performance in both estimation speed and accuracy.

**Case 2.** In this case, the simulation is performed with the non-Gaussian system noise  $w_j$  and the non-Gaussian

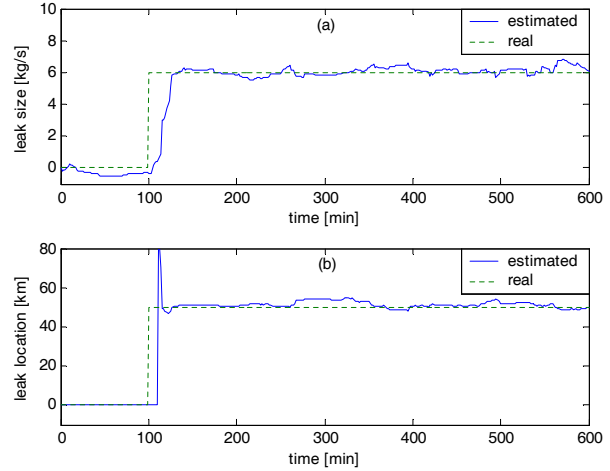


Fig. 5. Simulation results in Case 2: (a) real leak size and its estimate, (b) real leak location and its estimate.

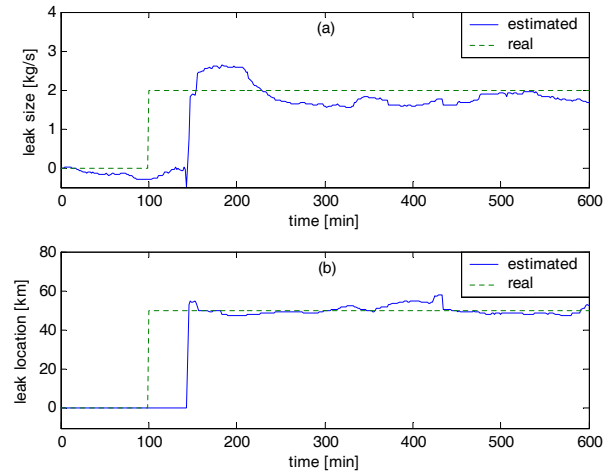


Fig. 6. Simulation results in Case 3: (a) real leak size and its estimate, (b) real leak location and its estimate.

measurement noise  $v_j$ , which have the following PDFs:

$$f_{w_j}(x) = 0.5 \begin{bmatrix} e^{-|x/80|} & e^{-|x/80|} & e^{-|x/80|} \\ e^{-|x/0.8|} & e^{-|x/0.8|} & e^{-|x/0.8|} \end{bmatrix}^T,$$

$$f_{v_j} = 0.5 \begin{bmatrix} e^{-|x/800|} & e^{-|x/800|} & e^{-|x/800|} \end{bmatrix}^T.$$

The other parameters are the same as in Case 1. The simulation results with the APF are shown in Fig. 5. They illustrate that the APF method is still valid in the presence of non-Gaussian noise.

**Case 3.** A leakage of size 1% (2 kg/s) is generated in this case. Other simulation parameters are the same as in Case 1. The simulation results are illustrated in Fig. 6, from which we see that for a small leak it will take more



time to detect it, and the estimates of the leak size and location are not accurate. Anyhow, the results are still satisfactory.

## 5. Conclusions

The speed and accuracy are very important performance factors of an FDD method. For a gas pipeline, fast leak detection and accurate location can reduce the loss of leakage. This work presents an adaptive particle filter to tackle the leak detection and location problem in gas pipelines. Because there is no need to linearize the nonlinear pipeline model in the APF algorithm, its performance for parameter estimation is good. Additionally, the location capability of this method is evaluated, and the upper bound of the location error is derived. The main advantages of our algorithm are as follows: (i) the leak size and location can be estimated accurately and quickly, (ii) it is applicable to both Gaussian or non-Gaussian noise processes, and (iii) it can detect and locate relatively small leaks. Future work is to extend the proposed approach to leak detection and location in transient states.

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