Int. J. Appl. Math. Comput. Sci., 2005, Vol. 15, No. 1, 141–150

A NOVEL CONTINUOUS MODEL TO APPROXIMATE TIME PETRI NETS: MODELLING AND ANALYSIS

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In order to approximate discrete-event systems in which there exist considerable states and events, David and Alla define a continuous Petri net (CPN). So far, CPNs have been a useful tool not only for approximating discrete-event systems but also for modelling continuous processes. Due to different ways of calculating instantaneous firing speeds of transitions, various continuous Petri net models, such as the CCPN (constant speed CPN), VCPN (variable speed CPN) and the ACPN (asymptotic CPN), have been proposed, where the continuous flow is specified uniquely by maximal firing speeds. However, in applications such as chemical processes there exist situations where the continuous flow must be above some minimal speed or in the range of minimal and maximal speeds. In this paper, from the point of view of approximating a time Petri net, the CPN is augmented with maximal and minimal firing speeds, and a novel continuous model, i.e., the Interval speed CPN (ICPN) is defined. The enabling and firing semantics of transitions of the ICPN are discussed, and the facilitating of continuous transitions is classified into three levels: 0-level, 1-level and 2-level. Some policies to resolve the conflicts and algorithms to undertake the behavioural analysis for the ICPN are developed. In addition, a chemical process example is presented.

Keywords: continuous Petri nets, hybrid systems, discrete event systems.

1. Introduction

Petri nets (PN), as graphical and mathematical tools, provide a powerful and uniform environment for the modelling, analysis, and control of discrete event systems, such as computer systems, discrete manufacturing systems, and communication systems (Murata, 1989). In order to handle time, classical Petri nets have been extended, resulting in two basic models—timed Petri nets (Ramchandani, 1974) and time Petri nets (Merlin and Farber, 1976). The timed Petri nets are derived by associating a finite time duration with a transition or place, and the classic firing rule is modified to account for time. In a time Petri net, the transition or place has one time interval with two bounds of time. When the time interval is associated with transitions, the first bound denotes the minimal time that must elapse, starting from the time at which the transition is enabled until this transition can fire. The second bound represents the maximum time during which that the transition can be enabled, and before which the transition must fire. The time Petri nets are more general than the timed ones since a timed Petri net can be modelled using a time Petri net, but the converse is not true. Time Petri nets have been proved very convenient for expressing most temporal constraints while some of these constraints are difficult to express only in terms of time duration.

In order to efficiently handle discrete event systems in which there exist considerable states and events, David and Alla defined a continuous Petri net (CPN) (David and Alla, 1987). The main differences between the CPN and the classic Petri nets are nonnegative real number markings (or tokens) of places and continuous firing of transitions at some speed. Instantaneous firing speeds (IFSs) of transitions play an important role in the evolution of a CPN, which is specified uniquely by either a maximal constant speed or a maximal variable speed (a maximal speed function in time) (Dubois *et al.* 1994). Due to different ways of calculating IFSs of transitions, various continuous Petri net models, such as the CCPN (the constant speed CPN), the VCPN (the variable speed CPN) and the ACPN (the asymptotic CPN) (Bail Le *et al.*, 1992; David and Alla, 2001) were developed. Balduzzi *et al.* (2000) presented a first-order hybrid Petri net (FOHPN), where instantaneous firing speeds are specified by minimal and maximal speeds, and calculated iteratively by linear programming. However, the minimal speed is always supposed to be zero. Gu and Parisa discussed one typical application of sugar milling systems, where the continuous flow is specified by minimal and maximal speeds, and developed a hybrid time Petri net model (Gu and Bahri, 2002; Gu *et al.* 2002).

Owing to the fact that the dynamics of time Petri nets can also be approximated continuously, a general CPN formalism, i.e., the Interval speed CPN (ICPN) is defined here. Due to constraints on maximal and minimal firing speeds, the ICPN requires more subtle and complicated semantics for enabling and firing transitions. In an ICPN, enabling of transitions is then classified into three levels: 0-level, 1-level and 2-level. In order to analyze the dynamic behaviour of ICPNs, conflicts are resolved by either priority or proportional rules. In addition to that, as a novel tool for modelling and analyzing discrete or continuous systems, several illustrative examples are given.

The remainder of the paper is organized as follows: In Section 2, the formalism of ICPNs is presented and the semantics of firing and enabling are discussed. Section 3 deals with the computation of IFSs and conflict resolution policies, and two algorithms to undertake ICPN behavioural analysis are developed. The illustrative example of a chemical process is given in Section 4. The paper concludes in Section 5.

2. Interval Speed Continuous Petri Net

2.1. Intuitive Examples

Time Petri nets constitute general models for time dependent systems. In a time Petri net, time intervals can be associated with places (called the *time places*) or transitions (called the *time transitions*). A time Petri net with time transitions can be transformed into a time Petri net with time places, and vice versa. In the following, time transitions are assumed to exist in a time Petri net. Normally, a time interval $d_{j\min}, d_{j\max}$ is specified by two bounds of time: a maximum time $d_{j\max}$ and a minimal time $d_{j\min}$ $(d_{j\min} \leq d_{j\max})$. When the maximum times $d_{j\max}$ of time transitions are set to $+\infty$, the time Petri net reduces to a timed Petri net.

Consider the time Petri net in Fig. 1(a). Time intervals (David and Alla, 1987; Murata, 1989; Ramchandani, 1974) are associated with transitions t_1 and t_2, respectively, and the marking in Fig. 1(a) corresponds to time $\tau = 0$. Obviously, transition t_1 is enabled at time $\tau = 0$, and one token is reserved in place p_1 in order to fire transition t_1. From the semantics of time Petri nets, transition t_1 can be fired after time $\tau = 1$, and must be fired before time $\tau = 4$. Suppose that each transition works in the earliest firing mode, i.e., each enabled transition is fired as soon as its minimal time elapses. Then, at time $\tau = 1$ transition t_1 is fired, and the reserved token in p_1 is taken away, and a nonreserved token is put into p_2. At time $\tau = 1$, transitions t_1 and t_2 are enabled. At time $\tau = 2$ transition t_1 is fired again, and one reserved token in p_1 is taken away, and one nonreserved token is put into p_2. At time $\tau = 3$, transition t_2 is fired, and one reserved token in p_2 is taken away, and one nonreserved token is put into p_1. At time $\tau = 3$, transitions t_1 and t_2 are enabled again, and they are fired at times $\tau = 4$ and $\tau = 5$, respectively, and so on. The corresponding markings m_1 and m_2 are illustrated by dashed lines in Fig. 1(c). At time $\tau = 1$, periodical behaviour with period $\mu = 2$ is reached.

Similarly, the markings m_1 and m_2 for different firing modes of transitions t_1 and t_2 can be derived, respectively. Under the latest firing modes of both transition t_1 and transition t_2 (i.e., the enabled transition is fired when its maximum time elapses), at time $\tau = 4$ periodical behaviour is reached, as shown in Fig. 1(d). For the earliest firing mode of transition t_1 and the latest firing mode of transition t_2, at time $\tau = 1$ periodical behaviour with period $\mu = 4$ in Fig. 1(e) is reached. Figure 1(f) is the dynamic evolution of markings under the latest firing mode of transition t_1 and the earliest firing mode of transition t_2, where periodical behaviour with period $\mu = 4$ is reached at time $\tau = 1$.

From the time Petri net of Fig. 1(a), it is possible to construct a continuous model by replacing time values $d_{j\min}$ and $d_{j\max}$ with maximum firing speeds $V_{j\max} = 1/d_{j\min}$ and minimal firing speeds $V_{j\min} = 1/d_{j\max}$, respectively. This results in the continuous time Petri net of Fig. 1(b). In the continuous model, transitions t_1 and t_2 will be fired at IFSs $v_1(\tau)$ and $v_2(\tau)$, respectively, after time τ. At initial time $\tau = 0$, transition t_1 is strongly enabled, and transition t_2 is weakly enabled (Balduzzi *et al.*, 2000; David and Alla, 2001). The evolution of the markings in two places is governed by the following equations in interval $(0, \tau_1)$ (in this interval, $v_1(\tau) = 1$ and $v_2(\tau) = 0.5$):

$$\begin{cases} m_1(\tau) = 2 + (1 - 0.5)\tau, \\ m_2(\tau) = (1 - 0.5)\tau. \end{cases}$$

These equations remain true until place p_1 becomes empty at time $\tau_1 = 2/0.5 = 4$. During all times $\tau \geq \tau_1(= 4)$, transitions t_1 and t_2 are fired at speeds $v_1(\tau) = v_2(\tau) = 0.5$ since transition t_1 is weakly enabled and t_2 is strongly enabled. Thus, a steady state is reached at time $\tau = \tau_1$, and the evolution of markings is illustrated in Fig. 1(c). According to the firing and enabling semantics of time Perti nets, the continuous time Petri net might have different kinds of behaviour shown in Figs. 1(d)–(f), and so on.

If we change times specification of the time Petri net in Fig. 1(a), an unexpected situation occurs. Consider the time Petri net in Fig. 2(a), for which the corresponding

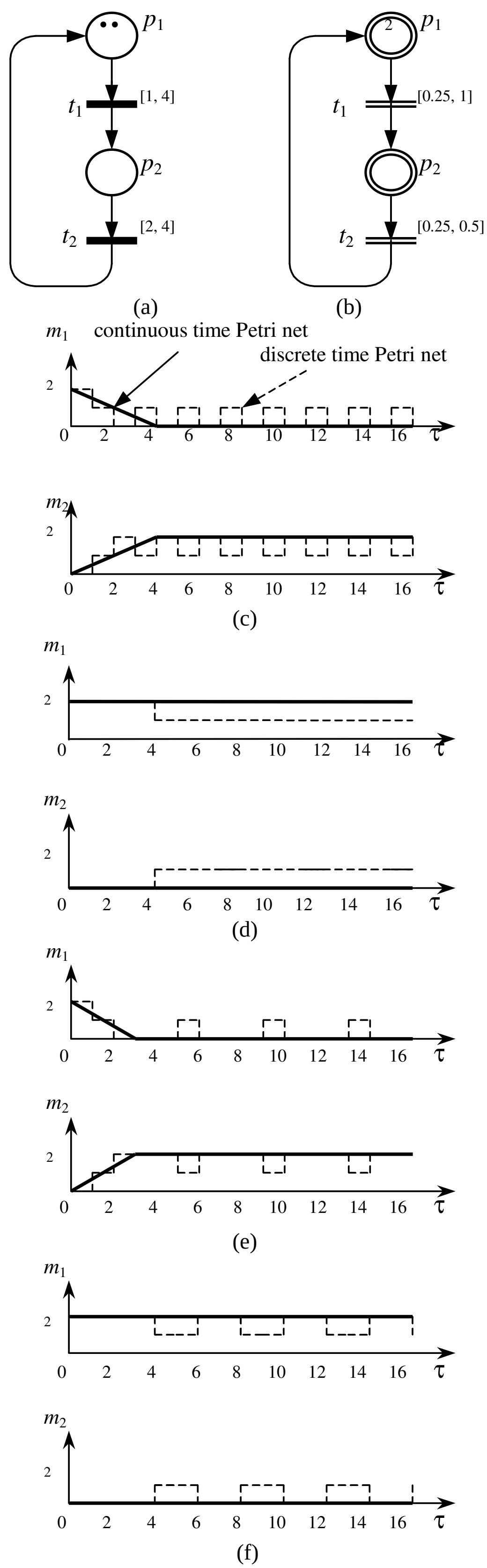


Fig. 1. Approximating time Petri net.

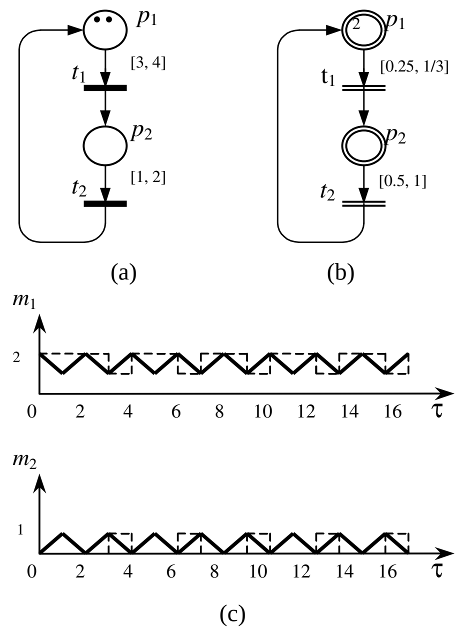


Fig. 2. An unexpected situation.

continuous time Petri net is shown in Fig. 2(b). The evolution of markings m_1 and m_2 of the time Petri net is illustrated by dashed lines in Fig. 2(c), while assumming that the earliest firing mode for transitions t_1 and t_2 is adopted. At time $\tau = 1$, periodical behaviour with the period $\mu = 3$ is reached.

In the continuous time Petri net, at initial time $\tau = 0$ transition t_1 is strongly enabled, and transition t_2 is weakly enabled (Balduzzi *et al.*, 2000; David and Alla, 2001). Clearly, we could not derive one feasible IFS $v_2(\tau)$ for transition t_2 because $v_2(\tau)$ must be in the interval $[0.5, 1]$, and its input place p_2 can receive the supply flow only at speed $v_1(\tau)$ = 1/3. However, when any time delay δ elapses, place p_2 will not be empty, and there will exist feasible IFS $v_2(\tau)$ for transition t_2. For this situation, we could assume that the weakly enabled transition t_2 has a time delay ($\delta = d_j = 1/V_{j\max}$). Then, at time $\tau_1 = \delta$, we can have a feasible IFS $v_2(\delta) = 1$. In this way, the evolution of markings in this continuous time Petri net can be continued, as shown in Fig. 2(c).

Remark 1. Obviously, the semantics of continuous Petri nets (David and Alla, 2001) is not sufficient to explain the situation, and neither is the FOHPN semantics (Balduzzi *et al.*, 2000). More reasonable semantics are required for continuous time Petri nets.

2.2. Formalism of Interval Speed Continuous PNs

This section formalizes some concepts which were presented intuitively in the previous section, and the interval speed continuous Petri net is defined here. The common formalism and notation of PNs and CPNs are adoped, and a comprehensive introduction can be found in (David and Alla, 2001; Merlin and Farber, 1976; Murata, 1989).

Definition 1. An interval speed continuous Petri net is the quintuple $N = (P, T, Pre, Post, F)$, where

$P = \{p_1, p_2, p_3, \dots, p_n\}$ is a set of continuous places,

$T = \{t_1, t_2, t_3, \dots, t_m\}$ is a set of continuous transitions,

$P \cap T = \emptyset$, i.e., the sets P and T are disjointed,

Pre: $T \times P \rightarrow \mathbb{R}^+$ (or $P \times T \rightarrow \mathbb{R}^+$) ($\mathbb{R}^+$ is a set of non-negative real number) is the transition (or place) input incidence mapping,

Post: $T \times P \rightarrow \mathbb{R}^+$ (or $P \times T \rightarrow \mathbb{R}^+$) is the transition (or place) output incidence mapping,

$F : T \rightarrow \mathbb{R}^+ \times (\mathbb{R}^+ \cup \infty)$ is the flow or speed interval function.

In an ICPN, the function F specifies firing speeds associated with continuous transitions. For any continuous transition $t_j \in T$, $F(t_i) = [V_{j\min}, V_{j\max}]$ with $V_{j\min} \leq V_{j\max}$, where $V_{j\min}$ represents the minimal speed and $V_{j\max}$ denotes the maximum speed. Here, the speed specification $F(t_i) = [V_{j\min}, V_{j\max}]$ of each continuous transition approximates the time specification $D(t_i) = [d_{j\min}, d_{j\max}]$ of its discrete version, by $V_{j\min} = 1/d_{j\max}$ and $V_{j\max} = 1/d_{j\min}$.

Definition 2. A marked ICPN is formalized as $(N, \boldsymbol{m}(\tau_0)) = (P, T, Pre, Post, F, \boldsymbol{m}(\tau_0))$, where the quantity $\boldsymbol{m}(\tau_0)$ represents the initial marking vector, and $\boldsymbol{m} : P \rightarrow \mathbb{R}^+$, is a marking function that assigns to each continuous place a nonnegative real token. For any place $p_i \in P$, its token at time τ is denoted by $m_i(\tau)$ or m_i.

Remark 2. An ICPN reduces to a CPN when $V_{j\min} = 0$ for all continuous transitions $t_j \in T$. Thus ICPNs can be considered as a general formalism of continuous Petri nets.

2.3. Enabling and Firing Semantics

The enabling of continuous transitions in ICPNs depends not only on the current marking, but also on the feeding flow of all its input places. We use the similar notation ${}^\bullet x$ ($x^\bullet$) to denote the input (resp. output) set of element $x \in P \cup T$.

Definition 3. A place $p_i \in P$ is supplied or fed if and only if there is at least one of its input transitions $t_j \in {}^\bullet p_i$ which is being fired at a positive speed $v_j(\tau)(> 0)$.

Definition 4. Transition $t_j \in T$ is enabled at time τ if all input places $p_i \in {}^\bullet t_j$ satisfy that either $m_i(\tau) > 0$ or p_i is supplied, and otherwise the transition is disabled.

Definition 5. An enabled transition $t_j \in T$ is called strongly enabled or 2-level enabled at time τ if all input places $p_i \in {}^\bullet t_j$ satisfy $m_i(\tau) > 0$.

Definition 6. An enabled transition $t_j \in T$ is called weakly enabled at time τ if at least one of its input places $p_i \in {}^\bullet t_j$ does not satisfy $m_i(\tau) > 0$.

Definition 7. A weakly enabled transition $t_j \in T$ is called 1-level enabled at time τ if all the supplied places $p_i \in {}^\bullet t_j$ satisfy the condition

$$\sum_k Pre(p_i, t_k) v_k(\tau) - \sum_{k \neq j} Post(p_i, t_k) v_k(\tau) \geq V_{j\min}.$$

Definition 8. A weakly enabled transition $t_j \in T$ is called 0-level enabled at time τ if one of the supplied places $p_i \in {}^\bullet t_j$ satisfies the condition

$$0 < \sum_k Pre(p_i, t_k) v_k(\tau) - \sum_{k \neq j} Post(p_i, t_k) v_k(\tau) < V_{j\min}.$$

It is clear that either a 2-level or a 1-level enabled continuous transition can be fired once it is enabled. However, 0-level enabled continuous transitions can be fired only after a time delay elapses, since there exists no feasible IFS at the moment.

Property 1. A 2-level enabled transition $t_j \in T$ can be fired at IFS $v_j(\tau) \in (V_{j\min}, V_{j\max})$.

Property 2. A 1-level enabled transition $t_j \in T$ can be fired at IFS $v_j(\tau)$ satisfying the relations

$$\begin{cases} V_{j\min} \leq v_j(\tau) \leq V_{j\max}, \\ v_j(\tau) \leq \sum_k Pre(p_i, t_k) v_k(\tau) \\ \qquad - \sum_{k \neq j} Post(p_i, t_k) v_k(\tau), \\ \qquad\qquad \text{for all places} \quad p_i \in {}^\bullet t_j. \end{cases}$$

Property 3. A 0-level enabled transition $t_j \in T$ can be fired after time delay $d_j = 1/V_{j\max}$ at IFS $v_j(\tau) \in [V_{j\min}, V_{j\max}]$, unless the transition is disabled before time $(\tau + d_j)$ elapses.

3. Behavioural Analysis of ICPNs

3.1. Conflict Resolution

In an ICPN, a conflict may arise if a unique place has to supply two or more transitions. When the unique place holds positive tokens or has proper feed flows, the conflict is not effective.

Definition 9. A conflict occurs when place $p_i \in P$ has at least two output transitions. We denote a conflict by $K = \langle p_i, t | t \in p_i^\bullet \rangle$. A conflict is effective if the following conditions are met:

$$\begin{cases} m_i(\tau) = 0, \\ \sum_j Post(p_i, t_j) V_{j\min} \le \sum_k Pre(p_i, t_k) v_k(\tau) \\ \qquad \le \sum_j Post(p_i, t_j) V_{j\max}. \end{cases}$$

Property 4. An effective conflict $K = \langle p_i, t | t \in p_i^\bullet \rangle$ can be resolved by one of the following policies:

Priority policy:

$$v_j(\tau) = \begin{cases} \min\Big(V_{j\max}, \Big(\sum_k Pre(p_i, t_k) v_k(\tau) - \sum_{r>j} Post(p_i, t_r) V_{r\max}\Big)\Big) \\ \text{if } \Big(\sum_k Pre(p_i, t_k) v_k(\tau) - \sum_{r>j} Post(p_i, t_r) V_{r\max}\Big) \ge V_{j\min}, \\ 0 \qquad \text{otherwise.} \end{cases}$$

Proportional policy:

$$v_j(\tau) = \min\Big(V_{j\max}, \max\Big(V_{j\min}, V_{j\max} \sum_k Pre(p_i, t_k) v_k(\tau) / \sum_r Post(p_i, t_r) V_{r\max}\Big)\Big).$$

(Here $r > j$ corresponds to all transitions that are in $p_i^\bullet$ and have priority over t_j).

Proposition 1. *If there exist feasible IFSs for an effective conflict* $K = \langle p_i, t | t \in p_i^\bullet \rangle$ *by the priority policy, then*

$$\sum_r Post(p_i, t_r) v_r(\tau) \le \sum_k Pre(p_i, t_k) v_k(\tau), \quad \forall t_k \in {}^\bullet p_i, \forall t_r \in p_i^\bullet.$$

Proposition 2. *If there exist feasible IFSs for an effective conflict* $K = \langle p_i, t | t \in pi^\bullet \rangle$ *by the proportional policy, then*

$$\sum_r Post(p_i, t_r) V_{r\min} \le \sum_r Post(p_i, t_r) v_r(\tau) \le \sum_k Pre(p_i, t_k) v_k(\tau), \quad \forall t_k \in {}^\bullet p_i, \ \forall t_r \in p_i^\bullet.$$

Proof. By the proportional policy, any feasible IFS $v_r(\tau)$ for every $t_r \in pi^\bullet$ must satisfy

$$v_r(\tau) \le \max\Big(V_{j\min}, V_{j\max} \sum_k Pre(p_i, t_k) v_k(\tau) / \sum_r Post(p_i, t_r) V_{r\max}\Big).$$

Thus

$$\begin{aligned} &\sum_r Post(p_i, t_r) v_r(\tau) \\ &\le \max\Big(\sum_j Post(p_i, t_j) V_{j\min}, \Big(\sum_j Post(p_i, t_j) V_{j\max}\Big) \times \sum_k Pre(p_i, t_k) v_k(\tau) / \sum_r Post(p_i, t_r) V_{r\max}\Big) \\ &= \max\Big(\sum_j Post(p_i, t_j) V_{j\min}, \sum_k Pre(p_i, t_k) v_k(\tau)\Big). \end{aligned}$$

From Definition 9, we have

$$\sum_j Post(p_i, t_j) V_{j\min} \le \sum_k Pre(p_i, t_k) v_k(\tau).$$

Hence

$$\sum_r Post(p_i, t_r) v_r(\tau) \le \sum_k Pre(p_i, t_k) v_k(\tau).$$

If

$$V_{r\min} \ge V_{r\max} \sum_k Pre(p_i, t_k) v_k(\tau) / \sum_r Post(p_i, t_r) V_{r\max},$$

then

$$v_r(\tau) = \min(V_{r\max}, V_{r\min}) = V_{r\min}.$$

Thus

$$\sum_r Post(p_i, t_r) v_r(\tau) = \sum_r Post(p_i, t_r) V_{r\min}.$$

If

$$V_{r\min} \le V_{r\max} \sum_k Pre(p_i, t_k) v_k(\tau) / \sum_r Post(p_i, t_r) V_{r\max},$$

then

$$v_r(\tau) = \min\Big(V_{r\max}, V_{r\max}\sum_k Pre(p_i, t_k)t_k(\tau) \\ / \sum_r Post(p_i, t_r)V_{r\max}\Big) \geq V_{r\min}.$$

Thus

$$\sum_r Post(p_i, t_r)v_r(\tau) \geq \sum_r Post(p_i, t_r)V_{r\min}.$$

■

Some typical conflicts are shown in Fig. 3. Figures 3(a), (b) and (c) illustrate conflicts, but not effective ones. Figures 3(d), (e) and (f) correspond to conflicts, and effective conflicts.

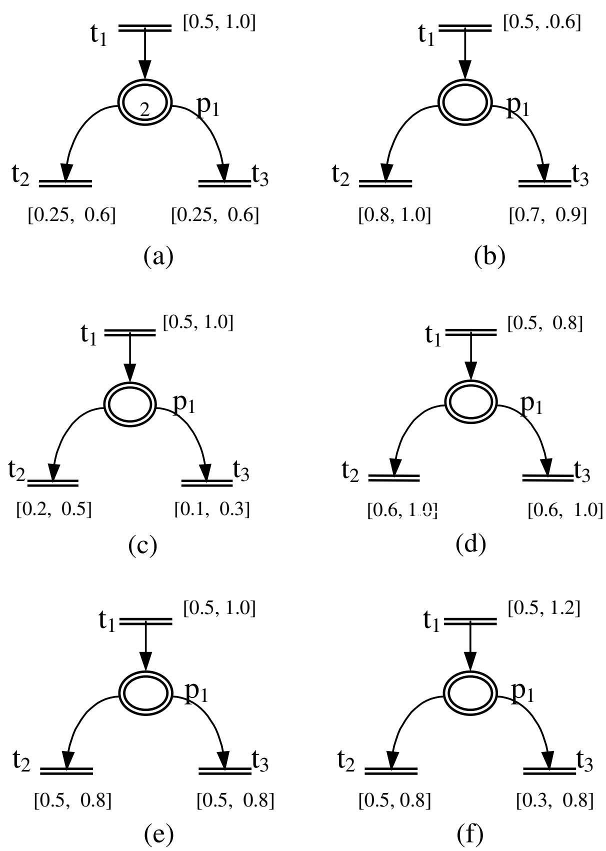


Fig. 3. Typical conflicts.

Transitions t_2 and t_3 in Figs. 3(a) and (c) can be fired in the maximal speed mode, i.e., their IFSs can be set as $v_2(\tau) = v_3(\tau) = 0.6$ and $v_2(\tau) = 0.5$, $v_3(\tau) = 0.3$, respectively. In Fig. 3(b), neither of transitions t_2 and t_3 can be fired at the moment, i.e., $v_2(\tau) = v_3(\tau) = 0$, since there do not exist feasible IFSs for them.

The conflict in Fig. 3(d) can be resolved only by the priority policy, and IFSs are $v_2(\tau) = 0.8$ and $v_3(\tau) = 0$ if transition t_2 has priority over transition t_3. If transition t_3 has priority over transition t_3, IFSs are $v_2(\tau) = 0$ and $v_3(\tau) = 0.8$.

In Figs. 3(e) and (f), either of resolution policies can be used, resulting in different IFSs. In terms of the priority policy, we have $v_2(\tau) = 0.8$ and $v_3(\tau) = 0.0$, $v_2(\tau) = 0.8$ and $v_3(\tau) = 0.4$, respectively, if transition t_2 has priority over transition t_3. When transition t_3 has priority over transition t_3, we have $v_2(\tau) = 0.0$ and $v_3(\tau) = 0.8$, $v_2(\tau) = 0$ and $v_3(\tau) = 0.8$, respectively. Using the proportional policy, IFSs are $v_2(\tau) = v_3(\tau) = 0.5$ and $v_2(\tau) = v_3(\tau) = 0.6$, respectively.

3.2. Enabled Transitions and Their IFS

The behavioural evolution of ICPNs depends on both enabled transitions and IFSs. Due to recursive definitions, it is not trivial to know whether or not a transition is enabled and to calculate IFSs. The algorithm to determine enabled transitions and IFSs is presented as follows. It is assumed that 2-level enabled transitions function in the maximal speed mode.

Algorithm 1. *(Calculation of enabled transitions and IFSs)*

1. Initialization: $ET_0 = ET_1 = ET_2 = \emptyset$; $\mathrm{time}(t_j) = 0$, $\forall t_j \in T$.

2. For $p_i \in P$ with $m_i(\tau) > 0$, find all 2-level enabled transitions $t_j \in p_i^{\bullet}$. Let $ET_2 := ET_2 \cap \{t_j\}$, $v_j(\tau) = V_{j\max}$, $\mathrm{time}(t_j) := \tau$ and $T := T - \{t_j\}$.

3. For $p_i \in P$ with $m_i(\tau) = 0$, find all disabled transitions $t_j \in T \cap p_i^{\bullet}$. Let $v_j(\tau) = 0$, $\mathrm{time}(t_j) := \tau$ and $T := T - \{t_j\}$.

4. Find all 0-level enabled transitions $t_j \in T$. Let $v_j(\tau) = 0$, $\mathrm{time}(t_j) := \tau + 1/V_{j\max}, ET_0 := ET_0 \cap \{t_j\}$ and $T := T - \{t_j\}$.

5. By conflict resolution policies, calculate the IFSs of transition $t_j \in T$. Let $\mathrm{time}(t_j) := \tau$, $ET_1 := ET_1 \cap \{t_j\}$ and $T := T - \{t_j\}$.

3.3. Behavioural Analysis

Similarly to CCPNs, the marking of a place in ICPNs is a time continuous function. A characteristic quantity of the dynamic evolution of ICPNs is the IFS vector, which remains constant in a regional state.

Definition 10. A regional state is defined as $(\boldsymbol{m}, V, [\tau_1, \tau_2])$, where $\boldsymbol{m}$ is the marking vector of all continuous places, and V is the IFS vector of all continuous transitions which remain unchanged in time interval $[\tau_1, \tau_2]$.

The behavioural evolution of ICPNs is driven by discrete events of empting continuous place. However, a regional state occurs when: (a) a continuous place becomes empty, or (b) a 0-level enabled transition is fired after some delay elapses. Thus the duration of time interval $[\tau_1, \tau_2]$ in a regional state is determined by the first place whose marking becomes zero, or the first 0-level enabled transition which will be fired, i.e., $\Delta k = \tau_k - \tau_{k-1}$ is given by

$$\Delta_k = \min\Big\{\min_i \Big\{m_i(\tau_{k-1})/\sum_r Post(p_i, t_r) \, v_r(\tau_{k-1})\Big\}, \min_j\{1/V_{j\max}|t_j \in ET_0\}\Big\}.$$

Algorithm 2. (*Behavioural analysis of ICPN*)

1. Initialization: $k = 1,\ \tau_k = 0,\ \boldsymbol{m}(\tau_k)$.

2. If $V(\tau_{k+1}) = V(\tau_k)$, then STOP. Otherwise, using Algorithm 1, calculate $ET_0,\ ET_1,\ ET_2$, vector V, and time vector time.

3. Calculate $\Delta_k = \min_i\{m_i(\tau_{k-1})/\sum_r Post(p_i, t_r)v_r(\tau_{k-1})\}$.

4. If $ET_0 = \emptyset$, then apply update $\tau_k := \tau_k + \Delta_k$, $m_i(\tau_k) := m_i(\tau_k) + \sum_r Pre(p_i, t_r)v_r(\tau_k) - \sum_r Post(p_i, t_r)v_r(\tau_k)$, and go to Step 2.

5. Find transition $t_j \in ET_0$ satisfying $1/V_{j\max} = \min_r\{1/V_{r\max}|t_r \in ET_0\}$.

6. If $1/V_{j\max} \geq \Delta_k$, then update $\tau_k := \tau_k + \Delta_k, m_i(\tau_k) := m_i(\tau_k) + \sum_r Pre(p_i, t_r)v_r(\tau_k) - \sum_r Post(p_i, t_r)v_r(\tau_k)$, and go to Step 2.

7. Update $\tau_k := \tau_k + 1/V_{j\max}$ and $m_i(\tau_k) := m_i(\tau_k) + \sum_r Pre(p_i, t_r)v_r(\tau_k) - \sum_r Post(p_i, t_r)v_r(\tau_k)$, $V(\tau_k) := V(\tau_k),\ v_j(\tau_k) := V_{j\max}$ and go to Step 2.

Consider the ICPN in Fig. 4(a). The speed ranges of transitions and initial markings of places are specified. The behavioural evolution can be analyzed as follows: At the initial time $\tau = 0$, $m_1(0) = 10$, $m_2(0) = 20$ and $m_3(0) = m_4(0) = 0$. Obviously, transition t_1 is strongly enabled or 2-level enabled, and $v_1(0) = 5$. Since place p_4 is supplied by a flow of $(5+v_2(0))$, transitions t_3 and t_4 are weakly enabled. By the proportional policy, their IFSs could be $v_3(0) = v_4(0) = (5 + v_2(0))/2 \geq 2$, and thus transitions t_3 and t_4 are 1-level enabled. The fired transition t_4 can supply place p_3 at a speed of over 2.5. Then, transition t_2 is 1-level enabled, and fired at speed $v_2(0) = 2$. Now, the total supply of place p_4 can be determined as $(5 + 2) = 7$. Therefore, transitions t_3 and t_4 can be fired at speed $v_3(0) = v_4(0) = 3$. From time $\tau = 0$, the ICPN's behaviour is governed by the following equations:

$$\begin{cases} m_1(\tau) = 10 - 2\tau, \\ m_2(\tau) = 20 - 2\tau, \\ m_3(\tau) = m_4(\tau) = \tau. \end{cases}$$

At time $\tau = 5$, $m_1(0) = 0$, $m_2(0) = 10$ and $m_3(0) = m_4(0) = 5$. Transitions t_2, t_3 and t_4 are strongly enabled, and their IFSs are reset as $v_2(5) = 2$ and $v_3(5) = v_4(5) = 3$, respectively. Since place p_1 is supplied by a flow of 3, transition t_1 is 1-level enabled, and can be fired at $v_1(5) = 3$. From time $\tau = 0$, the ICPN's behaviour is governed by the following equations:

$$\begin{cases} m_1(\tau) = 0, \\ m_2(\tau) = 20 - 2\tau, \\ m_3(\tau) = \tau, \\ m_4(\tau) = \tau. \end{cases}$$

At time $\tau = 10$, we have $m_1(0) = 0$, $m_2(0) = 0$ and $m_3(0) = m_4(0) = 10$. Transition t_2 is disabled, and $v_2(10) = 0$. By the 0^+ enabled rule (David and Alla, 2001), transitions t_1 and t_1 are 1-level enabled, and their IFS are reset as $v_1(10) = v_3(10) = 3$. From time $\tau = 10$, the ICPN reaches a steady state which is governed by the following equations:

$$\begin{cases} m_1(\tau) = m_2(\tau) = 0, \\ m_3(\tau) = m_4(\tau) = 10. \end{cases}$$

The dynamic evolution of the ICPN's behaviour is graphically shown in Fig. 4(b), and the markings of the continuous places are presented in Fig. 4(c).

4. Chemical Process Example

A chemical process with 4 units and 4 operations is shown in Fig. 5(a). Two kinds of material are processed in Unit 1 (Operation 1) and Unit 2 (Operation 2), respectively, and then fed to Unit 3, where Operation 3 is undertaken. The feed flow from Unit 1 to Unit 3 is limited within the interval $[2, 3]$, and the feed flow from Unit 2 to Unit 3 is within the interval $[3, 5]$. The intermediate product is fed from Unit 3 to Unit 4 at a flow of $[4, 6]$. There are two output flows of Unit 4: one is the final product flow at a speed of $[3, 4]$, and the other is the recycled flow to Unit 3 at a speed of $[1, 2]$. The capacity of Unit 3 is limited by 30, and its initial volume is 10.

This process can be modelled as the ICPN shown in Fig. 5(b). From the ICPN model, we can analyze the dynamic behaviour as follows:

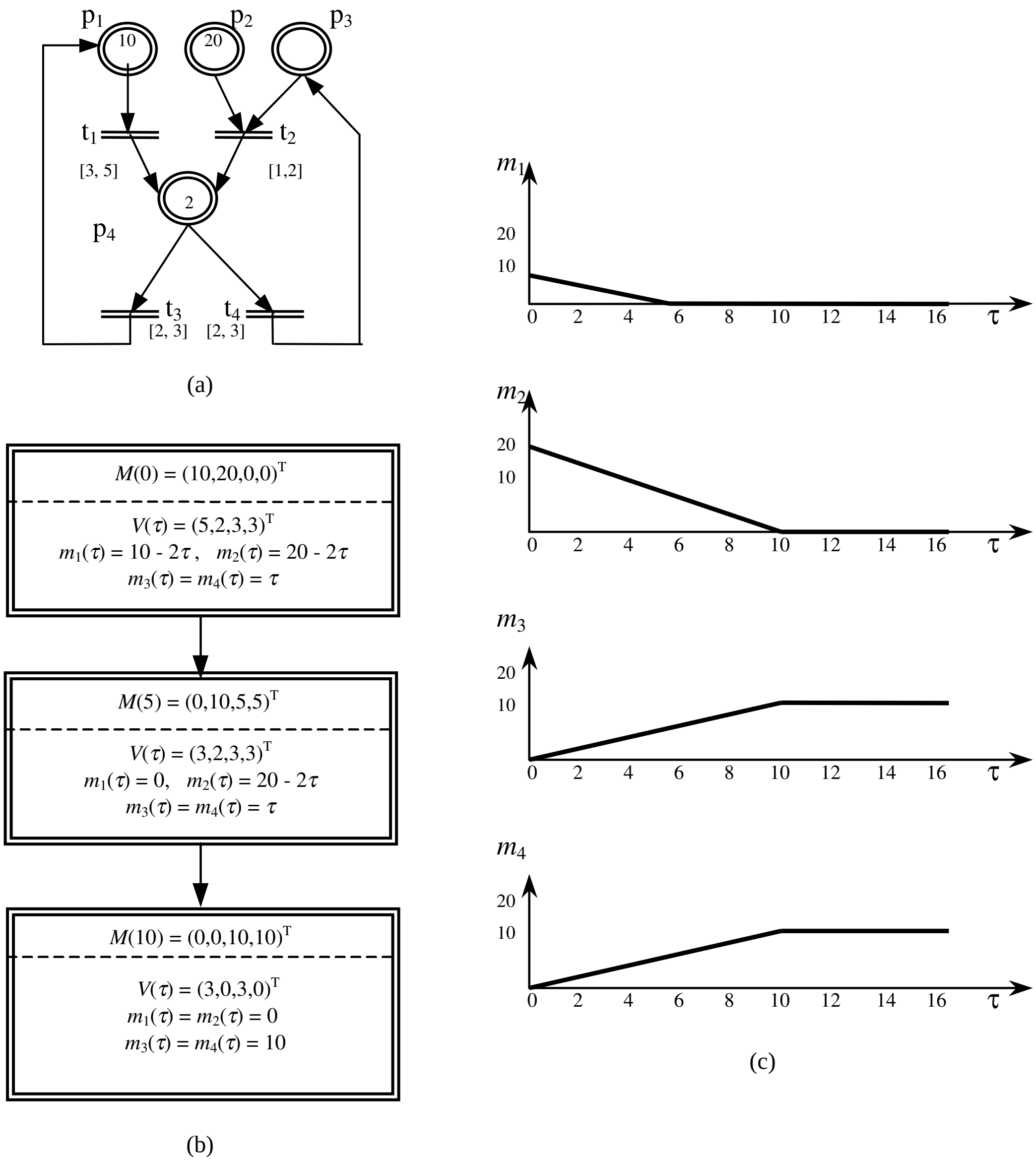


Fig. 4. Illustrative example.

At $\tau = 0$ we have $m_1(0) = 10$, $m_2(0) = 20$ and $m_3(0) = 0$. Then $ET_2 = t_1, t_2, t_3$, $ET_1 = t_4, t_5$, and $v_1(0) = 3$, $v_2(0) = 5$, $v_3(0) = 6$, $v_4(0) = 4$, $v_5(0) = 2$. From time $\tau = 0$, the ICPN's behaviour is governed by the following equations:

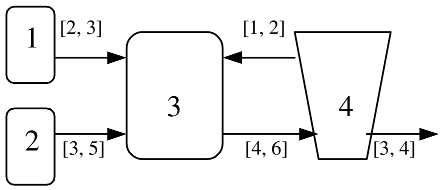
$$\begin{cases} m_1(\tau) = 10 + 4\tau, \\ m_2(\tau) = 20 - 4\tau, \\ m_3(\tau) = 0. \end{cases}$$

At time $\tau = 5$ we have $m_1(5) = 30$, $m_2(5) = 0$ and $m_3(5) = 0$, and transition t_3 is still strongly enabled, and $ET_2 = t_3$, $ET_1 = t_2, t_4, t_5$, $ET_0 = t_1$, $v_1(5) = 0$, $v_2(5) = 3$, $v_3(5) = 6$, $v_4(5) = 4$, $v_5(5) = 1.2$. From time $\tau = 5$, the ICPN's behaviour is governed by the following equations:

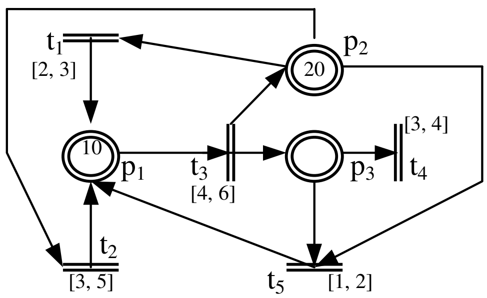
$$\begin{cases} m_1(\tau) = 39 - 1.8\tau, \\ m_2(\tau) = 1.8\tau - 9, \\ m_3(\tau) = 0.8\tau - 4. \end{cases}$$

At time $\tau = 5.5$, the 0-level enabled transition t_1 is fired at $v_1(5.5) = 2$. The IFSs of other transitions remain unchanged, and $m_1(5.5) = 29.1$, $m_2(5.5) = 0.9$, $m_3(5.5) = 0.4$. Thus, from time $\tau = 5.5$, the equations governing the ICPN's behaviour are changed to

$$\begin{cases} m_1(\tau) = 28 + 0.2\tau, \\ m_2(\tau) = 2 - 0.2\tau, \\ m_3(\tau) = 0.8\tau - 4. \end{cases}$$



(a)



(b)

Fig. 5. Chemical process.

At time $\tau = 10$ we have $m_1(10) = 30$, $m_2(10) = 0$ and $m_3(10) = 4$. Then $ET_2 = \{t_3, t_4\}$, $ET_1 = \{t_2, t_5\}$, $ET_0 = \{t\}$, and $v_1(10) = 0$, $v_2(10) = 3$, $v_3(10) = 6$, $v_4(10) = 4$, $v_5(10) = 1.2$. Thus, from time $\tau = 10$, the marking equations of the ICPN are

$$\begin{cases} m_1(\tau) = 48 - 1.8\tau, \\ m_2(\tau) = 1.8\tau - 18, \\ m_3(\tau) = 0.8\tau - 4. \end{cases}$$

At time $\tau = 10.5$, the 0-level enabled transition t_1 is fired at $v_1(10.5) = 2$. The IFSs of other transitions remain unchanged, and $m_1(10.5) = 29.1$, $m_2(10.5) = 0.9$, $m_3(10.5) = 4.4$. Thus, from time $\tau = 5.5$, the equations governing the ICPN's behaviour are changed to

$$\begin{cases} m_1(\tau) = 28 + 0.2\tau, \\ m_2(\tau) = 2 - 0.2\tau, \\ m_3(\tau) = 0.8\tau - 4. \end{cases}$$

Obviously, after time $\tau = 5.5$, the ICPN reaches the following periodical behaviour:

$$\begin{cases} v_1(\tau) = 0, v_2(\tau) = 3, v_3(\tau) = 6, v_4(\tau) = 4, \\ v_5(\tau) = 1.2 \ \text{when} \quad \tau \in [5k, 5k + 0.5], \\ \qquad\qquad\qquad\qquad k = 1, 2, \dots \\ v_1(\tau) = 2, v_2(\tau) = 3, v_3(\tau) = 6, v_4(\tau) = 4, \\ v_5(\tau) = 1.2 \ \text{when} \quad \tau \in [5k + 0.5, 5(k + 1)], \\ \qquad\qquad\qquad\qquad k = 1, 2, \dots \end{cases}$$

5. Conclusions

Continuous flows with maximal and minimal bounds are important characteristic quantities in either approximating discrete event systems or describing continuous processes. Aiming at approximating time Petri nets, the concept of an Interval Speed Continuous Petri Net is developed in this paper. The ICPN can be considered as a general formalism of continuous processes. When the minimal speed limit is relaxed, the ICPN reduces to a CPN.

Associating maximal and minimal firing speeds with continuous transitions implies that the dynamics and properties in ICPNs are much more complicated than in traditional CPNs. Further efforts are required to establish a more theoretical foundation regarding net dynamics and structural properties of ICPNs. In addition to that, a formal proof to compare the ICPN with the original net model is under way, as the optimization and control of continuous and hybrid processes via ICPNs are interesting topics.

Acknowledgments

The authors are very grateful to the anonymous reviewers for their helpful comments. This work was supported by the National Natural Science Foundation of China (grant no. 60243002), the Scientific Research Foundation for the Returned Overseas Scholars of the Chinese Education Ministry, and the Natural Science Foundation of Guangxi Province (grant No. 0448072).

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Received: 31 December 2003
Revised: 17 June 2004