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STABILIZATION OF SECOND–ORDER SYSTEMS BY NON–LINEAR FEEDBACK

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A stabilization problem of second-order systems by non-linear feedback is considered. We discuss the case when only position feedback is available. The non-linear stabilizer is constructed by placing actuators and sensors in the same location and by using a parallel compensator. The stability of the closed-loop system is proved by LaSalle's theorem. The distinctive feature of the solution is that no transformation to a first-order system is invoked. The results of analytic and numerical computations are included to verify the theoretical analysis and the mathematical formulation.

Keywords: second-order system, stability theory, non-linear feedback

1. Introduction

The modelling of many dynamic systems results in second-order differential equations. Also distributed parameter systems, very often, due to the lack of computational techniques, are discretized to second-order systems. Then the problem is solved for this reduced-order model.

Velocity feedback is not generally available. In this paper, stabilization without using velocity feedback is considered (Kobayashi, 2001; Mitkowski, 2003). We present our results here for the single-input case only. As the second-order system may have eigenvalues on the imaginary axis (Datta *et al.*, 2000), it cannot be stabilized by position feedback only. In this case, additionally, the use of a parallel compensator is necessary. The role of the compensator is to reconstruct the velocity of the output signal using feedforward compensation. This stabilization method will result in savings in sensors (tachometers) or observers. The next advantage of this approach is that the problem is solved completely in the second-order setting, i.e., no transformation to a first-order system is invoked. Retaining the model in the second-order form is also computationally efficient as the dimension of the system is lower than that of the first-order form. In the second-order form, acceleration feedback can be used in its original form, which is not possible in the first-order form because the available states are the displacement and the velocity.

The paper is organized as follows: In Section 2, we present the system under study and the reasoning behind it. Some properties of the second-order system are analyzed in Section 3. In Section 4, non-linear position feedback is given. Simulation examples are presented in Section 5 before summing up in Section 6.

2. System Description

Let us consider a finite-dimensional control system whose state equation is of the form

$$E\ddot{x}(t) + (F+G)\dot{x}(t) + Ax(t) = Bu(t), \tag{1}$$

and the observation equation is given by

$$y(t) = Cx(t), \tag{2}$$

where

$$\begin{aligned} &E \in \mathbb{R}^{n\times n}, && E = E^T > 0,\\ &F \in \mathbb{R}^{n\times n}, && F = F^T \geq 0,\\ &G \in \mathbb{R}^{n\times n}, && G = -G^T,\\ &A \in \mathbb{R}^{n\times n}, && A = A^T > 0,\\ &B \in \mathbb{R}^{n\times 1}, && C = B^T, \end{aligned}$$

and $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$, $y(t) \in \mathbb{R}$. Here $\mathbb{R}^n$ and $\mathbb{R}$ are real vector spaces of column vectors, $x(t)$, $u(t)$ and $y(t)$ are vectors of states, inputs and outputs, respectively, and $x(0)$ and $\dot{x}(0)$ are vectors of initial values.

In the mathematical formulation of mechanical systems, E is called the mass or inertia matrix, F is the damping matrix, G is the skew-symetric (gyroscopic) matrix, A represents the stiffness matrix and B is the input (control) matrix applied to the structure.

3. Some Properties of Oscillatory Systems

The properties of second-order systems are well known (Diwekar and Yedavalli, 1999; Klamka, 1990). For the sake of completeness, we re-develop here some results needed for our analysis.

The system (1) can equivalently be rewritten using the first-order differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\widetilde{x}(t) = \widetilde{A}\widetilde{x}(t) + \widetilde{B}u(t), \tag{3}$$

where we set for convenience

$$\widetilde{A} = \begin{bmatrix} 0 & I \\ -E^{-1}A & -E^{-1}(F+G) \end{bmatrix}, \tag{4}$$

$$\widetilde{B} = \begin{bmatrix} 0 \\ E^{-1}B \end{bmatrix}, \tag{5}$$

and $\widetilde{x}(t) = \operatorname{col}(x(t), \dot{x}(t))$, I being an identity matrix.

The output equation (2) can be given by

$$y(t) = \widetilde{C}\widetilde{x}(t), \tag{6}$$

where $\widetilde{C} = [C \quad 0]$ stands for the output matrix of the system (3).

Let us now consider the second-order system (1) with neglected damping, i.e., $F = 0$ (Datta *et al.*, 2000). Such systems are often called gyroscopic systems. The eigensolution of the undamped system (1) can be written as

$$(\lambda_i^2 E + \lambda_i G + A)\upsilon_i = 0, \tag{7}$$

for $i = 1, 2, \dots, n$, where λ_i is the i-th eigenvalue and υ_i is the corresponding complex eigenvector.

Theorem 1. *The eigenvalues of* (7) *are different from zero, pairwise conjugated and located on the imaginary axis.*

Proof. See, e.g., (Datta *et al.*, 2000; Diwekar and Yedavalli, 1999). ■

In the next part of this section we show some results for the second-order system (1)–(2) in the case of $E = I$, $F = G = 0$.

From (4) we have $\widetilde{A} = \left[\begin{smallmatrix} 0 & I \\ -A & 0 \end{smallmatrix}\right]$, and the following result is true:

Lemma 1. *The k-th power of the matrix $\widetilde{A}$ has the following form:*

$$\widetilde{A}^{2k} = \begin{bmatrix} (-A)^k & 0 \\ 0 & (-A)^k \end{bmatrix}, \tag{8}$$

$$\widetilde{A}^{2k-1} = \begin{bmatrix} 0 & (-A)^{k-1} \\ (-A)^k & 0 \end{bmatrix}, \tag{9}$$

where $k = 1, 2, \dots$.

Proof. The lemma can be easily proved using mathematical induction. ■

Theorem 2. *A pair $(\widetilde{C}, \widetilde{A})$ is observable if and only if the pair (C, A) is observable, where*

$$\widetilde{A} = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}, \quad \widetilde{C} = [C \quad 0].$$

Proof. It is well known, see, e.g., (Mitkowski, 1991), that the observability of the pair (C, A) specifies the criteria $\operatorname{rank} M = n$, where the matrix M is given by

$$M = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}. \tag{10}$$

In a similar way, for the pair $(\widetilde{C}, \widetilde{A})$ we have $\operatorname{rank} \widetilde{M} = 2n$ with the matrix $\widetilde{M}$ determined by

$$\widetilde{M} = \begin{bmatrix} \widetilde{C} \\ \widetilde{C}\widetilde{A} \\ \widetilde{C}\widetilde{A}^2 \\ \vdots \\ \widetilde{C}\widetilde{A}^{2n-1} \end{bmatrix}. \tag{11}$$

Using (8) and (9) in (11), after some elementary recalculations, we obtain the following condition for the observability of the pair $(\widetilde{C}, \widetilde{A})$:

$$\operatorname{rank} \begin{bmatrix} C & 0 \\ CA & 0 \\ \vdots & 0 \\ CA^{n-1} & 0 \\ 0 & C \\ 0 & CA \\ 0 & \vdots \\ 0 & CA^{n-1} \end{bmatrix} = 2n, \tag{12}$$

which can be rewritten in the form

$$\operatorname{rank} \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix} = 2n. \tag{13}$$

Therefore, if $\operatorname{rank} M = n$, then $\operatorname{rank} \widetilde{M} = 2n$, and vice versa. ■

Theorem 3. *The pair* $(\widetilde{A}, \widetilde{B})$ *is controllable if and only if the pair* (A, B) *is controllable, where*

$$\widetilde{A} = \begin{bmatrix} 0 & I \\ -A & 0 \end{bmatrix}, \quad \widetilde{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}.$$

Proof. The theorem can be proved using the same method as in Theorem 2. Instead of the observability matrix M, we have to use the controllability matrix Q (Mitkowski, 1991):

$$Q = \begin{bmatrix} B & AB & A^2B & \dots & A^{n-1}B \end{bmatrix}. \tag{14}$$

■

4. Stabilization by Non-Linear Feedback

4.1. Non-Linear Feedback I

This section is mainly devoted to the stabilization of the system (1) and (2) in the case when only position feedback is available. We design a non-linear stabilizer for the second-order system with single input and single output. This case is very often investigated in control theory. We will assume that the system (1), (2) is observable.

Let us consider the non-linear dynamical feedback given by the formula

$$u(t) = -k[w(t) + y(t)]^{2s+1}, \tag{15}$$

$$\dot{w}(t) + \alpha w(t)^{2p+1} = \beta u(t), \tag{16}$$

with $k > 0$, $\alpha > 0$, $\beta > 0$, $w(0) = 0$, $s = 0, 1, 2, \dots$, $p = 0, 1, 2, \dots$. In our approach, the condition $w(0) = 0$ is essential for the following arguments: The closed-loop system is given by

$$\dot{z}(t) = Lz(t) + \widetilde{L}, \tag{17}$$

where $z(t) = \operatorname{col}(x(t), \dot{x}(t), w(t))$,

$$L = \begin{bmatrix} 0 & I & 0 \\ -E^{-1}A & -E^{-1}(F+G) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{18}$$

and the non-linear part is

$$\widetilde{L} = \begin{bmatrix} 0 \\ -E^{-1}Bk[w(t) + y(t)]^{2s+1} \\ -\alpha w(t)^{2p+1} - \beta k[w(t) + y(t)]^{2s+1} \end{bmatrix}. \tag{19}$$

Theorem 4. *The substate* $\operatorname{col}(x(t), \dot{x}(t))$ *of the closed-loop system* (17) *tends to* 0 *as* $t \to \infty$ *for all initial conditions* $\operatorname{col}(x(0), \dot{x}(0)) \in \mathbb{R}^{2n}$ *and parameters* (s, p), $s, p = 0, 1, 2, \dots$.

Proof. The asymptotic stability of the substate vector $\operatorname{col}(x(t), \dot{x}(t))$ of the closed-loop system (17) will be proved by LaSalle's theorem (LaSalle and Lefschetz, 1966). Consider the Lyapunov function

$$\begin{aligned} V\big(x(t), \dot{x}(t), w(t)\big) &= \frac{1}{2}\dot{x}(t)^T E\dot{x}(t) + \frac{1}{2}x(t)^T Ax(t) \\ &\quad + \frac{1}{2(p+1)}\frac{\alpha}{\beta}w(t)^{2p+2} \\ &\quad + \frac{k}{2(s+1)}\big[w(t) + B^T x(t)\big]^{2s+2}. \end{aligned} \tag{20}$$

We can notice that $V(x, \dot{x}, w) = 0$ if and only if $\operatorname{col}(x, \dot{x}, w) = 0$. Otherwise, $V(x, \dot{x}, w) > 0$. Moreover, $V(x, \dot{x}, w) \to \infty$ if $\|\operatorname{col}(x, \dot{x}, w)\| \to \infty$.

Along the solution of the system (17) we have

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}V\big(x(t), \dot{x}(t), w(t)\big) &= \dot{x}(t)^T E\ddot{x}(t) + \dot{x}^T(t)Ax(t) \\ &\quad + \frac{\alpha}{\beta}w(t)^{2p+1}\dot{w}(t) \\ &\quad + k\big[w(t) + B^T x(t)\big]^{2s+1}\big[\dot{w}(t) + B^T\dot{x}(t)\big]. \end{aligned} \tag{21}$$

After some elementary calculations, we obtain

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}V\big(x(t), \dot{x}(t), w(t)\big) &= -\dot{x}(t)^T(F+G)\dot{x}(t) \\ &\quad - \beta\left\{\frac{\alpha}{\beta}w(t)^{2p+1} + k[w(t) + B^T x(t)]^{2s+1}\right\}^2. \end{aligned} \tag{22}$$

Note that for the skew-symetric (gyroscopic) matrix G the following holds:

$$\dot{x}(t)^T G\dot{x}(t) = \frac{1}{2}\dot{x}(t)^T(G + G^T)\dot{x}(t) = 0. \tag{23}$$

Using this result in (22) we have

$$\frac{\mathrm{d}}{\mathrm{d}t}V\big(x(t), \dot{x}(t), w(t)\big) \le 0, \quad t \ge 0. \tag{24}$$

According to LaSalle's principle (LaSalle and Lefschetz, 1966), all solutions of (17) asymptotically tend to the maximal invariant subset of the set

$$S = \left\{(x, \dot{x}, w) : \dot{V} = 0\right\}. \tag{25}$$

Hence, to prove that $\mathrm{col}\,(x(t), \dot{x}(t))$ tends to the origin $0 \in \mathbb{R}^{2n}$ as $t \to \infty$, it is sufficient to show that S contains only the zero solution, which is a typical procedure in the application of LaSalle's invariance principle.

From $\dot{V} = 0$ it follows that

$$\frac{\alpha}{\beta} w(t)^{2p+1} + k\big[w(t) + B^T x(t)\big]^{2s+1} = \frac{\alpha}{\beta} w(t)^{2p+1} - u(t) = 0, \quad (26)$$

and

$$\dot{x}(t)^T F \dot{x}(t) = 0. \quad (27)$$

The substitution of (26) into (16) yields

$$\dot{w}(t) = -\alpha w(t)^{2p+1} + \beta \frac{\alpha}{\beta} w(t)^{2p+1} = 0. \quad (28)$$

Thus $w(t) = 0$ because $w(0) = 0$, and $u(t) = 0$ for $t \geq 0$. Consequently, $B^T x(t) = 0$. In this case, from (1) and (2) we obtain

$$\begin{cases} E\ddot{x}(t) + (F+G)\dot{x}(t) + Ax(t) = 0, \\ y(t) = B^T x(t) = 0 \end{cases} \quad (29)$$

for all $t \geq 0$. Because the system (1), (2) is observable, it follows that $x(t) = 0$, $\dot{x}(t) = 0$, $t \geq 0$, and (27) holds, too. Thus, the largest invariant set inside S is the origin $S = \{0\}$. We have proved the theorem. ■

Remark 1. The system (1), (2) can be stabilized by the linear dynamical feedback of the following form:

$$u(t) = -k\big[w(t) + y(t)\big], \quad (30)$$

$$\dot{w}(t) + \alpha w(t) = \beta u(t), \quad (31)$$

with $k > 0$, $\alpha > 0$, $\beta > 0$, $w(0) = 0$.

4.2. Non-Linear Feedback II

Let us consider again the second-order system with a single input and a single output given by (1) and (2). Let us assume that the system is observable.

This time, in order to stabilize the system, we use the following non-linear feedback together with the one-dimensional parallel compensator:

$$u(t) = -k y_w(t)^{2s+1} - \gamma y_w(t)^{4s+3}, \quad (32)$$

$$y_w(t) = w(t) + y(t), \quad (33)$$

$$\dot{w}(t) + \alpha w(t)^{2p+1} = \beta u(t), \quad (34)$$

where $k > 0$, $\gamma > 0$, $\alpha > 0$, $\beta > 0$, $w(0) = 0$, $s = 0, 1, 2, \ldots$, $p = 0, 1, 2, \ldots$. The condition $w(0) = 0$ plays an important role for the arguments. The role of the compensator is to reconstruct the velocity of the output signal using feedforward compensation.

The closed-loop system corresponding to (1), (2) is

$$\dot{z}(t) = Lz(t) + \widetilde{L}, \quad (35)$$

where $z(t) = \mathrm{col}\,(x(t), \dot{x}(t), w(t))$, L is given by (18), $\widetilde{L} = \mathrm{col}\,(0, \widetilde{L}_2, \widetilde{L}_3)$ and

$$\widetilde{L}_2 = -E^{-1}Bky_w(t)^{2s+1} - E^{-1}B\gamma y_w(t)^{4s+3}, \quad (36)$$

$$\widetilde{L}_3 = -\alpha w(t)^{2p+1} - \beta k y_w(t)^{2s+1} - \beta\gamma y_w(t)^{4s+3}. \quad (37)$$

Theorem 5. *The substate* $\mathrm{col}\,(x(t), \dot{x}(t))$ *of the closed-loop system* (35) *tends to* 0 *as* $t \to \infty$ *for all initial conditions* $\mathrm{col}\,(x(0), \dot{x}(0)) \in \mathbb{R}^{2n}$ *and parameters* (s, p), $s, p = 0, 1, 2, \ldots$.

Proof. We define the following Lyapunov function:

$$V\big(x(t), \dot{x}(t), w(t)\big) = \frac{1}{2}\dot{x}(t)^T E \dot{x}(t) + \frac{1}{2} x(t)^T A x(t) + \frac{1}{2(p+1)}\frac{\alpha}{\beta} w(t)^{2p+2} + \frac{1}{4(s+1)\gamma}\left[k + \gamma y_w(t)^{2s+2}\right]^2 - \frac{k^2}{4(s+1)\gamma}. \quad (38)$$

We notice that $V(x, \dot{x}, w) = 0$ if and only if $\mathrm{col}\,(x, \dot{x}, w) = 0$. Otherwise, $V(x, \dot{x}, w) > 0$. Moreover, $V(x, \dot{x}, w) \to \infty$ if $\|\mathrm{col}\,(x, \dot{x}, w)\| \to \infty$.

The derivative of the Lyapunov function becomes

$$\frac{\mathrm{d}}{\mathrm{d}t} V\big(x(t), \dot{x}(t), w(t)\big) = \dot{x}(t)^T E \ddot{x}(t) + \dot{x}^T(t) A x(t) + \frac{\alpha}{\beta} w(t)^{2p+1} \dot{w}(t) + \Big[k + \gamma y_w(t)^{2s+2)}\Big] y_w(t)^{2s+1} \dot{y}_w(t). \quad (39)$$

Since $\dot{x}(t)^T G \dot{x}(t) = 0$, after some easy calculations we finally obtain

$$\dot{V}\big(z(t)\big) = -\dot{x}(t)^T F \dot{x}(t) - \beta \widetilde{w}(t)^2 \leq 0, \quad (40)$$

where

$$\widetilde{w}(t) = \frac{\alpha}{\beta} w(t)^{2p+1} + \left[k + \gamma y_w(t)^{2s+2}\right] y_w(t)^{2s+1}. \quad (41)$$

To prove that $\mathrm{col}\,(x(t), \dot{x}(t))$ converges to zero $0 \in \mathbb{R}^{2n}$ as $t \to \infty$, we use LaSalle's invariance principle (LaSalle and Lefschetz, 1966). According to this principle, the trajectory of (35) enters the largest invariant set in the region:

$$S = \left\{(x, \dot{x}, w) : \dot{V} = 0\right\}. \quad (42)$$

The condition $\dot{V} = 0$ holds if and only if

$$\frac{\alpha}{\beta} w(t)^{2p+1} + \left[k + \gamma y_w(t)^{2s+2}\right] y_w(t)^{2s+1} = \frac{\alpha}{\beta} w(t)^{2p+1} - u(t) = 0, \quad (43)$$

and

$$\dot{x}(t)^T F \dot{x}(t) = 0. \quad (44)$$

The substitution of (43) into (34) yields

$$\dot{w}(t) = -\alpha w(t)^{2p+1} + \beta \frac{\alpha}{\beta} w(t)^{2p+1} = 0. \quad (45)$$

Because $w(0) = 0$, we get $w(t) = 0$ and $u(t) = 0$. Consequently, we have $y(t) = 0$ for all $t \geq 0$. In this case, from (1) and (2) we obtain

$$\begin{cases} E\ddot{x}(t) + (F+G)\dot{x}(t) + Ax(t) = 0, \\ y(t) = B^T x(t) = 0 \end{cases} \quad (46)$$

for all $t \geq 0$. Since the system (1), (2) is observable, we have $x(t) = 0$, $\dot{x}(t) = 0$, and the equality (44) holds, too. Thus, the largest invariant set inside S contains only the zero solution. We have proved the theorem. ■

The non-linear feedback (32)–(34) for $s = 1$, $p = 1$ and for the matrices $E = I$, $F = G = 0$ was analyzed in (Mitkowski, 2003) for an LC ladder network.

5. Modelling and Examples

To illustrate our theory, we consider a simple flexible structure shown in Fig. 1. It is modelled by a system of three masses and four springs. Suppose that the mass m_1 is connected to the wall by the spring k_1; the masses m_1 and m_2 are connected by the spring k_2; the masses m_2 and m_3 are connected by the spring k_3; and the mass m_3 is linked with the wall through the spring k_4. The

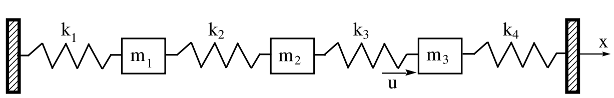

Fig. 1. Three mass system.

displacement vector $x(t) = \mathrm{col}\,(x_1(t), x_2(t), x_3(t))$ of the three-mass system satisfies

$$E\ddot{x}(t) + Ax(t) = Bu(t), \quad (47)$$

where

$$A = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 + k_4 \end{bmatrix}, \quad (48)$$

$$E = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \quad (49)$$

Let $y(t)$ be the scalar output defined as the displacement of the third mass m_3,

$$y(t) = x_3(t) = B^T x(t). \quad (50)$$

The control input $u(t)$ acts on the mass m_3.

The matrices E and A are symmetric and positive definite. The system (47) has the spectrum

$$\left\{ \pm 1.1468j, \pm 0.8681j, \pm 0.4632j \right\}. \quad (51)$$

It can be easily checked that the system (47), (50) is observable.

We will use the numerical values $m_1 = 1.0$, $m_2 = 1.5$, $m_3 = 2.0$, $k_1 = 0.4$, $k_2 = 0.5$, $k_3 = 0.6$, $k_4 = 0.7$, $x_1(0) = -1.5$, $\dot{x}_1(0) = 0$, $x_2(0) = 1.5$, $\dot{x}_2(0) = 0$, $x_3(0) = 1.5$, $\dot{x}_3(0) = 0$.

In order to stabilize the system, a one-dimensional parallel compensator is necessary,

$$\dot{w}(t) + 10w(t)^3 = 0.5u(t), \quad w(0) = 0. \quad (52)$$

For the augmented system (47), (50) and (52), we have designed the following controller:

$$u(t) = -100[w(t) + B^T x(t)]^3 - 100[w(t) + B^T x(t)]^7. \quad (53)$$

Simulation results are presented in Figs. 2 and 3. Figure 2 shows the displacements of three masses m_1, m_2 and m_3. In Fig. 3 the trajectory of the closed-loop system is shown. These simulation results show the effectiveness of the proposed controller.

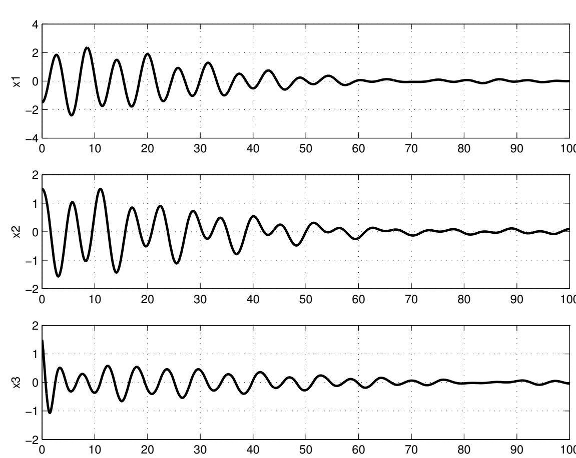

Fig. 2. Displacements of the masses m_1, m_2 and m_3.

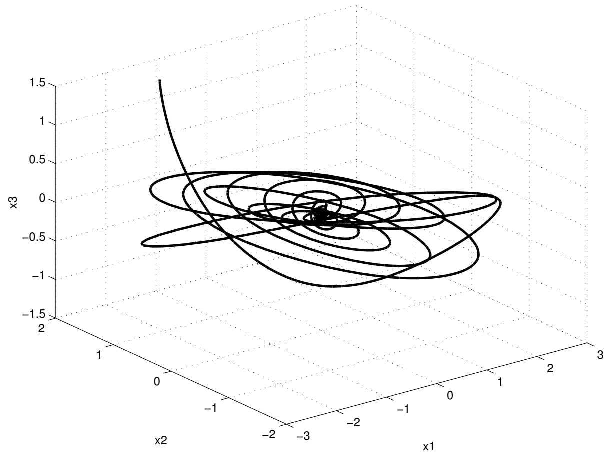

Fig. 3. Trajectory of the closed-loop system.

6. Concluding Remarks

We have investigated the stabilization of second-order systems by non-linear feedback in the case where only position feedback is available. Undamped systems have eigenvalues on the imaginary axis. We have proved that the closed-loop system is asymptotically stable when the non-linear position feedback together with the paraller compensator is applied. In the proof we have used LaSalle's invariance principle.

The theoretical results were validated by computer simulations conducted in the MATLAB/Simulink environment.

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