# SYSTEM IDENTIFICATION FROM MULTIPLE-TRIAL DATA CORRUPTED BY NON-REPEATING PERIODIC DISTURBANCES

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Iterative learning and repetitive control aim to eliminate the effect of unwanted disturbances over repeated trials or cycles. The disturbance-free system model, if known, can be used in a model-based iterative learning or repetitive control system to eliminate the unwanted disturbances. In the case of periodic disturbances, although the unknown disturbance frequencies may be the same from trial to trial, the disturbance amplitudes, phases, and biases do not necessarily repeat. Furthermore, the system may not return to the same initial state at the end of each trial before starting the next trial. In spite of these constraints, this paper shows how to identify the system disturbance-free dynamics from disturbance-corrupted input-output data collected over multiple trials without having to measure the disturbances directly. The system disturbance-free model can then be used to identify the disturbances as well, for use in learning or repetitive control. This paper represents the first extension of the interaction matrix approach to the multiple-trial environment of iterative learning control.

**Keywords:** system identification, disturbance identification, iterative learning control, repetitive control, interaction matrix

## 1. Introduction

Iterative learning control (ILC) and repetitive control (RC) improve the tracking error of a repetitive process by compensating for unwanted disturbances that are present in a repetitive process (Bien and Xu, 1998; Moore, 1993). Since there are two independent variables (repetition and time), learning control can be viewed from the perspective of 2-D systems theory (Amann *et al.*, 1994), and, under appropriate conditions, repetitive control can be viewed this way as well. At one end of the spectrum, there are learning and repetitive controllers that guarantee convergence to zero tracking error without requiring knowledge of the plant and the disturbances. Such a general approach, although attractive in theory, has limited applications in practice because these model-independent controllers may exhibit unacceptable learning behavior while converging to zero tracking error. At the other end of the spectrum there are performance-oriented model-based controllers. These controllers require the knowledge of the plant and possibly of the disturbance environment in their design. One does not expect that such information can be derived accurately from analytical modeling alone, especially when the disturbances may not be predicted accurately beforehand. Consequently, system identification in one form or another is used to provide the information needed. System identification has a unique advantage that the identified model reflects the true dynamics of the system under consideration. For example, an experimentally identified model naturally incorporates actuator or sensor dynamics that may be left out in an analytical model. Thus, in the context of designing model-based learning or repetitive controllers, one asks the following question: To what extent can the system disturbance-free dynamics be identified from input-output data that are corrupted by unknown periodic disturbances? Knowledge of the disturbance-free dynamics allows real time estimation of the unwanted disturbance contribution to the system output because the disturbance contribution is simply the difference between the measured disturbance-corrupted output and the computed disturbance-free output. A disturbance-rejection control system can make use of this information to cancel the estimated disturbance contribution. Alternately, one can use the disturbance-free model to design a disturbance observer to estimate the equivalent disturbance input at the control actuators (Tesfaye *et al.*, 1997) so that the necessary disturbance cancellation control input can be generated. It is clear that a comprehensive answer to the identification question will significantly contribute towards making learning and repetitive control attractive in the real world. The primary emphasis of this paper is the identification problem. The control aspect will be addressed in a later work.

In the following we provide practical motivation for the identification problem being examined in this paper. In the standard ILC problem, it is assumed that the system always returns to the same initial condition before the start of the next trial. There are situations where the system goes to what one considers the same starting point, but the disturbance sources are not at the same phase. In other words, the disturbances have the same frequencies each repetition, but the phase does not necessarily repeat. We can divide the situation into two classes.

The first class can be illustrated by considering the timing belt drive system used in repetitive control experiments in (Hsin *et al.*, 1997). This consists of an input shaft driving a timing belt (a belt with teeth). This belt drives a gear on one end of an idler shaft, and a larger gear on the other end drives a second belt. The other end of the second belt drives the output shaft. The disturbances to the output motion are small errors due to imperfect manufacture of the gears, imperfect mounting for position and angle, disturbance dynamics as the belt teeth mesh with the gear teeth, etc. This produces a rich set of disturbance frequencies at the fundamental and harmonics for motion once around of each shaft and each belt. In this situation, when the output shaft is put in the desired starting angle, it does not mean that all other gears and belts are back to their same starting position. Depending on the gear ratios, one may have to rotate the output shaft many times before one gets to a common period for the entire system; with closely spaced periods for different parts of the system, the common period can be very long. In this case one has several options. One is to use the very long common period as the learning period. This can require recording and using long data sets, the long period means that learning will be correspondingly slow, and one may need to go through a large adjustment in the system to get the system to a proper starting point for the next run. Another option makes use of the fact that there are a finite number of initial conditions for all the system disturbances, associated with having the output shaft at the desired angle. If one knows how many of these there are, one can do a separate learning control process for each. Again, this makes for slow learning, and the starting points cycle through the possibilities. And the third alternative is to use a method such as the one developed here that identifies the disturbance-free dynamics in order to infer the phases of the disturbances each time. One can have substantially faster learning as a result.

The second class of ILC problems where the phase does not repeat can be illustrated by a couple of situations. One of these is the belt steering problem treated in (Hsin *et al.*, 1998). This hardware uses a simple belt, not a timing belt. And in practice the belt will have some drift with respect to the angle of the roller driving it, and hence the phase of the disturbances will drift. This would also be the case in the timing belt drive system discussed above, if the timing belts were replaced by simple belts, both drifting somewhat over time. Another example comes from the timing belt system when one considers periodic errors due to imperfections in the ball or roller bearings. The disturbance period is nominally a fraction of the shaft velocity. But again the relationship is not perfect due to something analogous to the belt slipping. Because the slip is not something one can predict accurately, the method of this paper becomes more important as a means of addressing this class of problems.

The method developed here not only handles the cases above, but also handles the still more general case of non-repeating initial conditions and non-repeating disturbance amplitudes, provided the disturbances always have the same set of frequencies. System identification is performed off line, and it is indicated how one must do this to get the disturbance-free system model from multiple-trial data that is corrupted by the non-repeating disturbances.

Central to the proposed identification method is the derivation of a relationship between the excitation or control input and the disturbance-corrupted output. Through the so-called "interaction matrix", terms related to the unknown disturbance inputs and explicit state dependence are eliminated from this input-output model. This matrix describes how the coefficients of the identified model become corrupted by the unknown disturbances, yet the system disturbance-free dynamics can still be correctly recovered without having to determine the disturbance inputs first. The interaction matrix had its origin in the open-loop state-space system identification problem (Juang and Phan, 2001; Phan *et al.*, 1997). It has substantially developed into a general framework spanning the problems of closed-loop state-space identification (Juang and Phan, 1994; Phan *et al.*, 1994), observer and Kalman filter identification (Juang *et al.*, 1993; Phan *et al.*, 1995), disturbance identification and rejection by feedforward and feedback control (Goodzeit and Phan, 2000b; Phan *et al.*, 1997; Goodzeit and Phan, 2000a), predictive control (Lim and Phan, 1997; Phan *et al.*, 1998; Phan *et al.*, 1999). For each problem there is a corresponding interaction matrix. This paper is the first extension of the interaction matrix formulation to the multiple-trial environment of iterative learning control.

## 2. Problem Statement

Consider an $n$-th order, $r$-input, $q$-output discrete-time system

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + B_d d(k), \\ y(k) &= Cx(k) + Du(k), \end{aligned} \tag{1}$$

where $d(k)$ is a disturbance input made up of a finite number of unknown frequencies, amplitudes, and phases. Available for system identification there are $s$ sets of data, each $\ell$ samples long, consisting typically of excitation input $u^{(j)}(k)$ and disturbance-corrupted output $y^{(j)}(k)$, where $j$ denotes the trial number, $j = 1, 2, \dots, s$, and $k$ the time index, $k = 0, 1, 2, \dots, \ell - 1$. After each trial the system may not return exactly to the same initial state before the next trial starts. Likewise, the amplitudes, phases, and biases of the harmonic components of the disturbance $d^{(j)}(k)$ may vary from trial to trial,

$$d^{(j)}(k) = \sum_{i=1}^{f} A^{(j)} \cos\left(\omega_i k \Delta t + \phi_i^{(j)}\right) + B^{(j)}. \tag{2}$$

Note that these assumptions are more general than those normally assumed in the standard ILC theory, where the system is assumed to return to the same initial state before the next trial is carried out, and any disturbance, if present, is assumed to be repeating from trial to trial. The objective of our identification problem is to recover the dynamics of the disturbance-free system. In discrete-time, the system dynamics is completely characterized by the sequence of Markov parameters $Y(k) = CA^k B$, $k = 0, 1, 2, \dots$, which are the sampled unit pulse response of the disturbance-free system. Once the Markov parameters are known, a state-space model of the system can be obtained by various realization techniques (Juang, 1994).

Other than the given sets of disturbance-corrupted data, nothing else is known about the system. Although we do not know the exact order of the system or the exact number of disturbance frequencies, upper bounds are assumed known for the purpose of identification. Furthermore, the disturbance frequencies may coincide with any number of system natural frequencies.

## 3. Input-Output Representation via the Interaction Matrix

In the following we derive an equation that relates the excitation input data to disturbance-corrupted output data via the so-called interaction matrix. This matrix is later used to show how the disturbance information can be embedded in the model where the disturbance input is explicitly absent. From (1), by repeated substitution,

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{C} u_p(k) + \mathcal{C}_d d_p(k), \\ y_p(k) &= \mathcal{O} x(k) + \mathcal{T} u_p(k) + \mathcal{T}_d d_p(k), \end{aligned} \tag{3}$$

where

$$u_p(k) = \begin{bmatrix} u(k) \\ u(k+1) \\ \vdots \\ u(k+p-1) \end{bmatrix},$$

$$y_p(k) = \begin{bmatrix} y(k) \\ y(k+1) \\ \vdots \\ y(k+p-1) \end{bmatrix}, \tag{4}$$

$$d_p(k) = \begin{bmatrix} d(k) \\ d(k+1) \\ \vdots \\ d(k+p-1) \end{bmatrix},$$

$$\begin{aligned} \mathcal{C} &= \left[A^{p-1}B, \cdots, AB, B\right], \\ \mathcal{C}_d &= \left[A^{p-1}B_d, \cdots, AB_d, B_d\right], \end{aligned} \tag{5}$$

$$\mathcal{T} = \begin{bmatrix} D & & & \\ CB & D & & \\ \vdots & \ddots & \ddots & \\ CA^{p-2}B & \cdots & CB & D \end{bmatrix},$$

$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{p-1} \end{bmatrix}, \tag{6}$$

$$\mathcal{T}_d = \begin{bmatrix} 0 & & & \\ CB_d & 0 & & \\ \vdots & \ddots & \ddots & \\ CA^{p-2}B_d & \cdots & CB_d & 0 \end{bmatrix}.$$

We will discuss the requirement placed on $p$ later. The interaction matrix $M$ is introduced by adding and subtracting the product $My_p(k)$ from the right-hand side of (3) to produce

$$\begin{aligned} x(k+p) &= A^p x(k) + \mathcal{C}u_p(k) + \mathcal{C}_d d_p(k) \\ &\quad + M\big[\mathcal{O}x(k) + \mathcal{T}u_p(k) + \mathcal{T}_d d_p(k)\big] \\ &\quad - My_p(k) \\ &= (A^p + M\mathcal{O})x(k) + (\mathcal{C} + M\mathcal{T})u_p(k) \\ &\quad + (\mathcal{C}_d + M\mathcal{T}_d)d_p(k) - My_p(k). \end{aligned} \tag{7}$$

We do not need to determine the interaction matrix $M$, we are only concerned with its existence in this analysis. With $M$ introduced, the output equation becomes

$$\begin{aligned} y(k+p) &= (CA^p + CM\mathcal{O})x(k) + (C\mathcal{C} + CM\mathcal{T})u_p(k) \\ &\quad + (C\mathcal{C}_d + CM\mathcal{T}_d)d_p(k) \\ &\quad - CMy_p(k) + Du(k+p). \end{aligned} \tag{8}$$

Equation (8) describes the disturbance-corrupted output at time step $k+p$ in terms of the system state at time step $k$, excitation input from time step $k$ to $k+p$, the system output from time step $k$ to $k+p-1$, and the disturbance input from time step $k$ to $k+p-1$ as expected.

## 4. State and Disturbance-Input Independent Model

From the identification point of view, Eqn. (8) cannot be used because it involves the system state and the disturbance input, both of which are unknown. We now use the freedom introduced by the interaction matrix to eliminate terms involved with these unknown quantities in the equation.

Specifically, we are looking to impose the conditions for $M$, or the product $CM$, such that the explicit dependence of the disturbance-corrupted output $y(k)$ on the state $x(k)$ and unknown disturbance input $d(k)$ is eliminated not only for all $k \geq p$ during each trial, but also for all available trials, $j = 1, 2, \dots, s$.

$$CA^p + CM\mathcal{O} = 0, \tag{9}$$

$$(C\mathcal{C}_d + CM\mathcal{T}_d)d_p^{(j)}(k) = 0. \tag{10}$$

Equation (9) is disturbance-input independent, but (10) is not. Thus, it is not clear whether such an interaction matrix exists. Let us examine (10) in greater detail. It can be re-written as

$$[C\mathcal{C}_d + CM\mathcal{T}_d]\mathcal{D} = 0, \tag{11}$$

$$\mathcal{D} = [\mathcal{D}^{(1)}, \mathcal{D}^{(2)}, \mathcal{D}^{(3)}, \dots], \tag{12}$$

where each $\mathcal{D}^{(j)}$ is a matrix of time-shifted disturbance input time histories associated with trial $j$,

$$\mathcal{D}^{(j)} = \begin{bmatrix} d_p^{(j)}(0) & d_p^{(j)}(1) & d_p^{(j)}(2) & \cdots \end{bmatrix}. \tag{13}$$

Equations (11) can be viewed as a set of constraint equations that $M$ or the product $CM$ must satisfy for us to relate the excitation input to the disturbance-corrupted output, without explicit dependence on the system state or input disturbance. These constraints must be met not only for each trial, but also for all available trials. The existence of such an interaction matrix is now examined.

In the following we will show that the rank of $\mathcal{D}$ is limited by the total number of disturbance frequencies present in the data as long as the disturbance inputs are periodic. This observation effectively limits the number of constraint equations that $CM$ (or $M$) must satisfy. Specifically, if the total number of distinct disturbance frequencies over all trials is $f$, then the rank of $\mathcal{D}$ is at most $2f+1$, where the 1 accounts for any possible constant bias in the disturbances. This is a two-part argument. First, the rank of each matrix $\mathcal{D}^{(j)}$ is bounded by the number of disturbance frequencies that are present for that trial. To see this, consider the case where the disturbance is a single frequency harmonic input. Since the rows of $\mathcal{D}^{(j)}$ are time-shifted versions of this harmonic input, its row rank can be at most 2 (or 3 if a constant bias is present). The same argument can be applied to multiple harmonic disturbance inputs. Second, we extend the argument to show that the rank of $\mathcal{D}$ itself is limited by the total number of disturbance frequencies present over all available trials. This second part of the argument is more difficult to see. Although each $\mathcal{D}^{(j)}$ is rank limited, the rank of $\mathcal{D}$ will not increase beyond $2f+1$ to become full (row) rank eventually as long as the disturbances are periodic. To see why, consider the typical case where all disturbance frequencies are present in each trial data, and the amplitudes of the harmonic components of the disturbance input do not change from one trial to the next. Then the matrix $\mathcal{D}$ formed by the disturbance input from multiple"short" trials is the same as the extended $\mathcal{D}^{(1)}$ (or $\mathcal{D}^{(2)}$, or $\mathcal{D}^{(3)}$) with certain columns removed if the experiment for that trial were allowed to continue beyond the actual trial duration. The removed columns correspond to the fact that the disturbance input at the beginning of each trial is not simply a continuation of the disturbance input at the end of the previous trial. Because the rank of the extended $\mathcal{D}^{(1)}$ (or $\mathcal{D}^{(2)}$, or $\mathcal{D}^{(3)}$) is bounded by $2f+1$, having certain columns removed cannot possibly increase its rank beyond $2f+1$. The same argument can be extended to the case where the amplitudes of the harmonic components of the disturbance may change from trial to trial. We thus arrive at the desired conclusion that the number of constraint equations that the interaction matrix must satisfy is indeed bounded from above.

We now look for a specific condition to guarantee the existence of such an interaction matrix. Let $\mathcal{D}_f$ denote a matrix formed by $2f+1$ linearly independent columns of $\mathcal{D}$. The equation that $CM$ must satisfy is therefore

$$CM[\mathcal{O}, \mathcal{T}_d\mathcal{D}_f] = -[CA^p, C\mathcal{C}_d\mathcal{D}_f]. \tag{14}$$

Since (14) is a set of linear equations, the existence of $CM$ is guaranteed as long as the matrix $[\mathcal{O}, \mathcal{T}_d\mathcal{D}_f]$ is full rank, and the number of unknowns in the product $CM$ is at least equal to the number of constraint equations. Specifically, the number of scalar unknowns in the product $CM$ is $pq^2$. The number of scalar equations is $q(n+2f+1)$. Thus the condition on $p$ for the existence of $CM$ (or $M$) is

$$pq \geq n + 2f + 1. \tag{15}$$

Thus from an assumed upper bound on the order of the system and an assumed upper bound on the number of distinct disturbance frequencies, $p$ can be chosen such that the above condition is met. Then as long as $[\mathcal{O}, \mathcal{T}_d\mathcal{D}_f]$ is full rank, $CM$ exists to satisfy (14). As long as such a $CM$ exists, (8) becomes

$$\begin{aligned} y(k+p) = {} & (C\mathcal{C} + CM\mathcal{T})y_p(k) \\ & - CMy_p + Du(k+p). \end{aligned} \tag{16}$$

We have just argued the existence of (16), which is a linear input-output equation that relates excitation input data to disturbance-corrupted data in terms of $D$, $-CM$, and $(C\mathcal{C} + CM\mathcal{T})$. Notice the absence of the system state and the disturbance input in this equation. Because we have shown that there exists a $CM$ that is common for all available trials, input-output data from all available trials can now be used collectively to identify these unknown parameter combinations of the model. Essentially, the disturbance information is partly embedded in the product $CM$; we refer to (16) as a disturbance-corrupted model. In the following sections, we will show how the identification of $D$, $-CM$, and $(C\mathcal{C} + CM\mathcal{T})$ can be carried out from multiple-trial data, and then how the disturbance-free dynamics can in fact be recovered from these identified coefficients.

## 5. Identification of Disturbance-Corrupted Model Coefficients

Recall that $s$ sets of disturbance-corrupted input-output data are available, one set from each trial, $j = 1, 2, \ldots, s$, and $\ell$ denotes the duration of each trial, $k = 0, 1, \ldots, \ell - 1$. From (16), we can write

$$Y = PV, \tag{17}$$

where

$$P = \big[D, -CM, (C\mathcal{C} + CM\mathcal{T})\big], \tag{18}$$

$$\begin{aligned} Y &= \big[Y^{(1)}, Y^{(2)}, \ldots, Y^{(s)}\big], \\ V &= \big[V^{(1)}, V^{(2)}, \ldots, V^{(s)}\big]. \end{aligned} \tag{19}$$

The superscript $j$ denotes the trial number, $j = 1, 2, \ldots, s$,

$$Y^{(j)} = \big[y^{(j)}(p), y^{(j)}(p+1), \ldots, y^{(j)}(\ell-1)\big], \tag{20}$$

$$V^{(j)} = \begin{bmatrix} u^{(j)}(p) & u^{(j)}(p+1) & \ldots & u^{(j)}(\ell-1) \\ y_p^{(j)}(0) & y_p^{(j)}(1) & \ldots & y_p^{(j)}(\ell-1-p) \\ \vdots & \vdots & \vdots & \vdots \\ u_p^{(j)}(0) & u_p^{(j)}(1) & \ldots & u_p^{(j)}(\ell-1-p) \end{bmatrix}. \tag{21}$$

With sufficient data, $P$ can be solved from

$$P = YV^+, \tag{22}$$

where $V^+$ denotes the pseudo-inverse of $V$ computed via its singular value decomposition.

## 6. Recovery of Disturbance-Free Dynamics

We are interested in recovering the disturbance-free model from the disturbance-corrupted coefficients of the previous section. It is possible to do so without actually knowing the disturbances themselves. Let the identified parameter combinations $-CM$, $C\mathcal{C} + CM\mathcal{T}$ be partitioned as

$$[\alpha_p, \alpha_{p-1}, \ldots, \alpha_1] = -CM, \tag{23}$$

$$[\beta_p, \beta_{p-1}, \ldots, \beta_1] = C(\mathcal{C} + M\tau), \quad \beta_0 = D. \tag{24}$$

By algebraic manipulation, the first $p$ Markov parameters can be recovered as follows:

$$\begin{aligned} D &= \beta_0, \\ CB &= \beta_1 + \alpha_1 D, \\ CAB &= \beta_2 + \alpha_2 D + \alpha_1 CB, \\ &\;\vdots \\ CA^{p-1}B &= \beta_p + \alpha_p D + \alpha_{p-1}CB + \ldots \\ &\quad + \alpha_1 CA^{p-2}B. \end{aligned} \tag{25}$$

To recover the additional Markov parameters, we make use of the condition $CA^p + CM\mathcal{O} = 0$ by post-multiplying it by $B$, then $AB$, etc., so that

$$\begin{aligned} CA^pB &= \alpha_1 CA^{p-1}B + \cdots + \alpha_p CB, \\ CA^{p+1}B &= \alpha_1 CA^pB + \cdots + \alpha_p CAB, \\ &\;\vdots \end{aligned} \tag{26}$$

Any additional Markov parameters can be recovered in the same manner. As mentioned, the Markov parameters completely describe the system disturbance-free dynamics. Once a sufficient number of Markov parameters is obtained, the step of producing a state-space model from the Markov parameters is straightforward. A realization of the system state-space model $A$, $B$, $C$ can be found by any standard realization procedure as described in (Juang, 1994).

The above procedure shows that although the disturbance information is present in the model coefficients, it does not in principle hinder the ability to recover the system disturbance-free dynamics. In the presence of noise, an over-parameterized model contains true identifiable modes of the system, uncontrollable disturbance "modes", and uncontrollable modes due to the over-parameterization. The discrimination of identifiable system modes, over-parameterization modes, and disturbance modes can be based on the fact that the identifiable system modes are both observable and controllable, disturbance modes and over-parameterization modes are observable but uncontrollable, and unlike over-parameterization modes, disturbance modes are undamped. In the presence of noise, these modes can be distinguished by examining models with increasing over-parameterization (by the parameter $p$). A detailed description of this discrimination process can be found in (Goodzeit and Phan, 2000b; Phan *et al.*, 1997), which include extensive experimental confirmation on a flexible truss and a structural acoustic testbed.

## 7. Relationship between Interaction Matrix Methods and Subspace Methods

Recently there has been considerable interest in the literature regarding a new class of system identification methods known as subspace methods (Van Overschee and De Moor, 1996). The interaction matrix method recovers the system Markov parameters first before finding a state-space representation. The subspace methods, in contrast, find the system state-space models directly from input-output data through an oblique projection technique. In ILC, the relationship from an input history to an output time history during each trial is governed by the system Markov parameters (Phan and Frueh, 1998; Elci *et al.*, 2002). Thus finding the Markov parameters directly (instead of computing them from the system state space matrices) represents a more direct approach to ILC problems. In doing so, one avoids issues associated with finding a state-space model representation. These include system order determination, the choice of coordinates for the state variables, and the nonlinear step of determining the individual state-space matrices from input-output data. Moreover, no subspace method specifically designed to address the specific problem of this paper has been developed, namely system identification from multiple-trial data that are corrupted by unknown non-repeating periodic disturbances for use in iterative learning control.

In spite of the above observation, it is perhaps still useful to see how the indirect interaction matrix approach differs from the direct subspace approach at a general level. In the interaction matrix approach, the first step of finding the Markov parameters is a linear problem, which takes advantages of the benefits associated with solving a linear problem. The second step of finding a state-space model from the identified Markov parameters is nonlinear but this problem can be addressed by the well-developed realization theory. At the Markov parameters step, one need not be concerned with issues such as the dimensions of the state vector or which coordinate system it is in because the Markov parameters are invariant with respect to a coordinate transformation of the state vector, and the dimensions of the Markov parameters are fixed by the number of inputs and outputs. The situation is actually more complicated because the interaction matrix technique does not solve for the system Markov parameters directly, but rather a different set of Markov parameters from which the system state-space model and an optimal observer or Kalman filter gain or controller gains can be recovered in the second step by realization (Phan *et al.*, 1997). Another difference between the two approaches is that in the interaction matrix approach, the linear first step makes it easier to analyze which error is being minimized, and to study the associated conditions for optimality. Depending on the specific technique used this could be the equation error, output error, or Kalman filter residual error. Errors associated with the second nonlinear realization step are associated with truncating non-zero singular values for model order determination. The characterization of errors introduced during this step is considerably more difficult. The same difficulty is encountered in the direct subspace approach.

In terms of mathematical derivation, there could be dramatic differences in the two approaches. There have been numerous variations within both approaches, but to illustrate the point, we compare a typical interaction matrix method to a subspace method described in (Franklin *et al.*, 1998), which is a generalization of the Ho-Kalman realization to general input-output data. Both methods set up input-output equations for subsequent manipulation. The subspace method then proceeds to eliminate the matrix of system Markov parameters to isolate the unknown initial states, whereas the interaction matrix method eliminates the unknown initial states to isolate the (observer) Markov parameters (then goes back to get the initial states if needed). So fundamentally the two approaches take different paths from the very beginning. Efforts are now being made to compare and contrast the two approaches more rigorously.

Finally, we note that the interaction matrix approach is not limited to the problem of open-loop state-space system identification. As mentioned, the general technique has found its applications in the problems of closed-loop state-space identification (Juang and Phan, 1994; Phan *et al.*, 1994), observer and Kalman filter identification (Juang *et al.*, 1993; Phan *et al.*, 1995), disturbance identification and rejection by feedforward and feedback control (Goodzeit and Phan, 2000a; 2000b; Phan *et al.*, 1997), and predictive control (Lim and Phan, 1997; Phan *et al.*, 1998; Phan *et al.*, 1999).

## 8. Numerical Illustration

We now illustrate the developed identification algorithm by a numerical example. Consider the system

$$x(k+1) = Ax(k) + B\big[u(k) + d(k)\big],$$
$$y(k) = Cx(k) + Du(k),$$

where the discrete-time model $A = e^{A_c \Delta t}$, $B = (A-I)^{-1} A_c B_c$ describes the dynamics of an undamped spring-mass system with the force input and the position output using a sampling interval $\Delta t = 0.02\,\text{sec}$,

$$A_c = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 1/m \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0.$$

The disturbance-free dynamics of the above system is completely characterized by its Markov parameters $D$, $CB$, $CAB$, $CA^2B$, etc. which are its unit pulse response samples. These Markov parameters are to be recovered by the developed identification technique.

In this numerical illustration, we use $m = 1\,\text{kg}$, $k = 100\,\text{N/m}$. The system natural frequency is thus $\omega = 10\,\text{Rad/sec}$. At each trial $j$, suppose the unknown disturbance input $d(k)$ consists of two harmonic components and a bias,

$$d_j(k) = A^{(j)} \cos\big(\omega_1 k \Delta t + \phi_1^{(j)}\big) + B^{(j)} \cos\big(\omega_2 k \Delta t + \phi_2^{(j)}\big) + C^{(j)}, \tag{27}$$

where, for illustration, the first unknown disturbance frequency $\omega_1$ is chosen to coincide with the system natural frequency, $\omega_1 = 10\,\text{Rad/sec}$, and the second unknown disturbance frequency is taken to be $\omega_2 = 5\,\text{Rad/sec}$. The amplitudes $A^{(j)}$, $B^{(j)}$, the bias $C^{(j)}$, and the phase angles $\phi_i^{(j)}$ are also unknown, and they are allowed to vary from trial to trial. Furthermore, the system does not return to the same initial state after one trial before another data collection trial is performed. Available for system identification are $s = 10$ sets of input-output data collected from 10 trials. At each trial, only the excitation input $u(k)$ and the disturbance-corrupted output $y(k)$ are known. For this system the safe minimum value of $p$ developed above is $n + 2f + 1 = 7$. Any $p$ larger than or equal to this value can be used in the identification. In practice, one only has a rough guess of the order of the system and the number of disturbance frequencies. It is considered safe practice to use a value of $p$ that is well above the minimum value. In this illustration, $p = 10$ is used although any value of $p \geq 7$ can be used. As shown in the problem formulation, there is an interaction matrix $M$ that is common to all trials, so that input-output data from 10 trials can be used collectively to determine the coefficients of the disturbance-corrupted model. In this example, they are found to be

$$D = 0,$$
$$-CM = \big[0.7023 \;\; -2.9794 \;\; 3.5917 \;\; 1.0090 - 3.5916 \;\; 1.7923 \;\; 4.0586 \;\; 3.0724 \;\; -7.8751 \;\; 4.8043\big],$$
$$C\mathcal{C} + CM\mathcal{T} = \big[-1.4000 \;\; 1.7947 \;\; 3.6973 \;\; -3.7182 \;\; -5.8375 \;\; 3.0081 \;\; 7.2163 \;\; -3.0779 \;\; -3.6761 \;\; 1.9933\big] \times 10^{-4}.$$

Any Markov parameters of the disturbance-free system can be recovered correctly using (25) and (26) as follows:

$$D = 0,$$
$$CB = \beta_1 + \alpha_1 D = 1.9933 \times 10^{-4},$$
$$CAB = \beta_2 + \alpha_2 D + \alpha_1 CB = 5.9006 \times 10^{-4},$$
$$CA^2B = \beta_3 + \alpha_3 D + \alpha_2 CB + \alpha_1 CAB = 9.5725 \times 10^{-4},$$
$$\vdots$$

As mentioned, from the identified Markov parameters, a state-space model of the disturbance-free system can be easily obtained using any standard realization techniques.

## 9. Conclusions

This paper presents the first extension of the interaction matrix formulation to identify the system disturbance-free dynamics from disturbance-corrupted data in the multiple-trial setup of iterative learning control. We have shown that with assumed upper bounds on the order of the system and the number of disturbance frequencies, the disturbance-free dynamics can be identified from such data. As long as the unknown disturbances have a finite number of frequencies, such identification is possible even if the disturbance frequencies coincide with the system natural frequencies, and the disturbance amplitudes, phases, biases, and the system initial conditions do not necessarily repeat from trial to trial. The key theoretical development here is to show the existence of an interaction matrix that is common not only for all time steps during each trial, but also for all trials, so that data from all trials can be used collectively in the identification. Such an interaction matrix allows us to identify certain parameter combinations from all available input-output data sets. From the identified parameter combinations, the disturbance-free dynamics can be recovered without having to determine the disturbance input first. System identification results obtained here can later be used in an iterative learning or repetitive control system to cancel unwanted periodic disturbances. The scope of this paper is limited to the identification problem. The control aspect will be addressed in future work.

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