Int. J. Appl. Math. Comput. Sci., 2003, Vol. 13, No. 2, 179–184

QUANTITATIVE L^p STABILITY ANALYSIS OF A CLASS OF LINEAR TIME-VARYING FEEDBACK SYSTEMS

PINI GURFIL*

* Department of Mechanical and Aerospace Engineering
Princeton University, Princeton, NJ 08544, U.S.A.
e-mail: pgurfil@princeton.edu

The L^p stability of linear feedback systems with a single time-varying sector-bounded element is considered. A sufficient condition for L^p stability, with $1 \leq p \leq \infty$, is obtained by utilizing the well-known small gain theorem. Based on the stability measure provided by this theorem, quantitative results that describe output-to-input relations are obtained. It is proved that if the linear time-invariant part of the system belongs to the class of proper positive real transfer functions with a single pole at the origin, the upper bound on the output-to-input ratio is constant. Thus, an explicit closed-form calculation of this bound for some simple particular case provides a powerful generalization for the more complex cases. The importance of the results is illustrated by an example taken from missile guidance theory.

Keywords: L^p stability, time-varying Lur'e systems, functional analysis

1. Introduction

The subject of feedback systems stability has been extensively dwelt upon in the literature. On the one hand, the theory of Lyapunov functions has evolved rapidly (see, e.g., (Vidyasagar, 1993) and the references therein). On the other hand, the techniques of functional analysis, pioneered by Sandberg (1964; 1965) and Zames (1990), have developed equally rapidly and generated a large number of results concerning the input-output properties of nonlinear feedback systems. The latter approach is aimed at the determination of output bounds given the characteristics of the feedback system and its input. Both the input and the output bounds are defined in some normed spaces. Thus, the issue of input-output stability is referred to as an L^p stability analysis.

Specific attention has been given to the L^p stability analysis of the so-called Lur'e systems, which are control systems consisting of a linear-time invariant part and a single, memoryless, nonlinear time-varying element. The L^p stability analysis of such systems yielded several celebrated results (Vidyasagar, 1993), such as the circle criterion, the Popov criterion, the passivity approach and the small gain theorem (Zames, 1990).

Although L^p stability theory has been widely addressed in the literature (Mossaheb, 1982; Sandberg, 1965; Sandberg and Johnson, 1990; Zames, 1990), the discussion usually excludes the *quantitative* aspects of the L^p stability of nonlinear time-varying Lur'e systems, i.e., the upper bound on the output-to-input norm ratio (the L^p gain) is not calculated explicitly.

The main goal of this note is to present a simple technique for the explicit calculation of the L^p gain, for a certain class of linear time-varying Lur'e feedback control systems. It will be shown that when the linear time-invariant portion of the system is a positive real transfer function with a single pole at the origin, a certain stability measure that stems from the small gain theorem can be calculated exactly. This measure is then utilized for the derivation of theL^p gain. Thus, this note proves that the specialized properties of positive real functions with a pole at the origin permit the derivation of an *exact* value of the loop input/output gain using the small gain theorem. This, in turn, allows an *exact* calculation of the upper bound on the system output. This fact further implies that if the exact L^p gain is found by means of solving the simplest case possible, i.e. ideal dynamics, the same L^p gain would still be valid for an arbitrary transfer function with a pole at the origin. This important property is illustrated using a practical engineering example taken from missile guidance theory.

2. Mathematical Preliminaries

In the sequel, functional analysis is extensively implemented. Therefore, some well known definitions of frequently used signal and system norms are hereby presented in brief.

Let E be a linear space defined over the field of real numbers $\mathbb{R}$. The following signal norms are defined on appropriate subsets of E for some causal signal $x(t)$ (Desoer and Vidyasagar, 1975):

$$\|x\|_p \triangleq \left(\int_0^{t_f} |x(t)|^p \, \mathrm{d}t \right)^{1/p}, \quad 1 \le p < \infty \tag{1}$$

$$\|x\|_\infty \triangleq \operatorname*{ess\,sup}_{t\in[0,t_f]} |x(t)| , \tag{2}$$

where, as usual, $\operatorname{ess\,sup}_{t\in[0,t_f)} |x(t)| \triangleq \inf\{ k |\, |x(t)| \le k \text{ almost everywhere}\}$[1].

The corresponding normed spaces are denoted, respectively, by $L^p[0,t_f]$ and $L^\infty[0,t_f]$. It will be said that $x(t) \in L^p[0,t_f]$ if $x(t)$ is locally (Lebesgue) integrable and, in addition,

$$\|x\|_p < \infty, \quad p \in [1,\infty). \tag{3}$$

Accordingly, $x(t) \in L^\infty[0,t_f]$ if

$$\|x\|_\infty < \infty. \tag{4}$$

We consider system norms as well. The systems are assumed to be linear, time-invariant, causal and finite-dimensional. In the time domain, input-output models for such systems have the form of a convolution equation,

$$y = h * u = \int_{-\infty}^{\infty} h(t-\tau)u(\tau)\,\mathrm{d}\tau, \tag{5}$$

where, due to causality, $h(t) = 0$ for $t < 0$.

Let $H(s)$ denote the transfer function, i.e., the Laplace transform of $h(t)$. The following system norms are defined (Doyle *et al.*, 1992):

$$\|h\|_1 \triangleq \int_0^\infty |h(t)|\,\mathrm{d}t, \tag{6}$$

$$\|h\|_2 \triangleq \left(\int_{-\infty}^{\infty} |h(t)|^2 \,\mathrm{d}t \right)^{1/2}$$

$$= \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} |H(j\omega)|^2 d\omega \right)^{1/2} = \|H\|_2 , \tag{7}$$

$$\|H\|_\infty \triangleq \sup_{\omega\in\mathbb{R}} |H(j\omega)| . \tag{8}$$

The corresponding normed spaces are denoted, respectively, by L^1, L^2 and L^∞. The notation $h \in L^1$, $h \in L^2$ and $H \in L^\infty$ means that the system norm in the appropriate normed space is finite. For the implementation of the input-output stability theorem, the following additional definitions are required:

Definition 1. A function $h(t) : [0,\infty] \to \mathbb{R}$ is said to satisfy $h(t) \in A_1$ if and only if

$$(1+t)h(t) \in L^1 \cap L^2. \tag{9}$$

Definition 2. A function $h(t) : [0,\infty] \to \mathbb{R}$, with $h(t) = h_1(t) + h_2(t)$ and $H_2(s)$ the Laplace transform of $h_2(t)$, is said to satisfy $h(t) \in A_2$ if and only if

$$h_1(t) \in A_1, H_2(s) \text{ is strictly proper.} \tag{10}$$

Definition 3. It is said that some stable transfer function $H(s)$ is positive real , i.e., it satisfies $H(s) \in \{PR\}$ if and only if

$$\operatorname{Re} H(j\omega) \ge 0, \quad \forall \omega \in \mathbb{R}. \tag{11}$$

3. Problem Formulation

Consider the linear time-varying (LTV) feedback system depicted in Fig. 1. The input to the system is $u(t) \in \mathbb{R}$ and the output is $z(t) \in \mathbb{R}$. These signals are defined for $t \in [0,t_f]$. $K_1(s), K_2(s), K_3(s)$ are linear time-invariant (LTI), whereas $\psi(t)$ is a time-varying operator satisfying $\psi : \mathbb{R}_+ \to \mathbb{R}$. It is assumed that $\psi(t)$ is continuous and sector-bounded,

$$0 \le \psi(t) \le \beta, \quad \beta \in \mathbb{R}. \tag{12}$$

Also, let

$$H(s) \triangleq K_1(s)K_2(s)K_3(s). \tag{13}$$

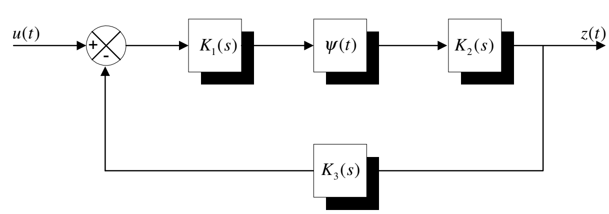

Fig. 1. Linear time-varying feedback system.

Referring to the system of Fig. 1, the following input-output stability definition is used (Desoer and Vidyasagar, 1975):

Definition 4. The system of Fig. 1 is said to be L^p-stable, $1 \le p \le \infty$, if and only if $u(t) \in L^p[0,t_f]$ implies $z(t) \in L^p[0,t_f]$ and, moreover,

$$\|z(t)\|_p \le \mu \|u(t)\|_p , \quad \forall u(t) \in L^p[0,t_f], \quad \mu \ne \mu(u). \tag{14}$$

[1] In the sequel, we use the notation ‘sup’ instead of ‘ess sup’.

L^p stability theory has been widely discussed in the literature (Mossaheb, 1982; Sandberg, 1965; Sandberg and Johnson, 1990; Zames, 1990). However, the discussion usually excludes the *quantitative* aspects of L^p stability of LTV system, i.e., the upper bound μ is not calculated. Thus, the main problem dwelt upon in the paper is the establishment of guidelines for an explicit calculation of μ in a certain class of LTV systems.

4. L^p Stability Sufficiency Theorem

Practically, it is most difficult to obtain quantitative information regarding the L^p stability of a general LTV feedback system. Nonetheless, useful results may be rendered when a specific class of LTV systems is considered. The class of systems considered herein is characterized by certain specific properties of the LTI portion, which are expressed by means of the following assumptions:

Assumption 1. $H(s)$ is a proper transfer function.

Assumption 2. $H(s)$ is stable.

Assumption 3. $H(s)$ can be written in the form $H(s) = kG(s)/s$ with $G(0) = 1$ and $G(s)$ asymptotically stable (thus $H(s)$ is stable).

Assumptions 1–3 constitute the class of systems discussed hereafter. Consequently, this class is defined as follows:

Definition 5. It is said that $H(s) \in \mathcal{H}$ if Assumptions 1–3 hold.

Note that although Assumptions 1–3 seem restrictive, in many cases the input-output linear time-invariant portion of the system dynamics can be shaped to satisfy Assumptions 1–3 by the design of a suitable controller (Doyle *et al.*, 1992).

Several theorems providing sufficient conditions for L^p stability can be found in the literature (Mossaheb, 1982; Sandberg, 1965; Zames, 1990). The theorem used herein is based upon the well-known small gain theorem (Desoer and Vidyasagar, 1975; Mossaheb, 1982). The small gain approach states a sufficient stability condition of the L^p stability of a closed-loop system, based upon the L^P induced norms of the forward and feedback paths.

Theorem 1. *Consider the system depicted in Fig. 1. Under Assumption 2, if*

$$\gamma = \frac{\beta}{2} \left\| \frac{H(j\omega)}{1 + (\beta/2)\, H(j\omega)} \right\|_\infty \le 1, \tag{15}$$

and

$$h(t) \in A_2, \tag{16}$$

then the system is L^p stable and $\|z\|_p \le \mu(\gamma)\, \|u\|_p$, where the constant $\mu(\gamma)$ is, at most, a function of γ only.

Proof. See (Mossaheb, 1982). ■

Remark 1. A celebrated L^2 stability theorem for the system of Fig. 1, known as the circle criterion, was obtained in (Sandberg, 1964) based upon (15). It can be shown that the circle criterion is an application of the small-gain theorem (Desoer and Vidyasagar, 1975). However, to extend the result to L^p stability with $1 \le p \le \infty$, the additional condition $e^{\varepsilon t} h(t) \in L^1 \cap L^2,\ \varepsilon > 0$, together with the shifted Nyquist plot of $H(j\omega)$, was used (Zames, 1990). In the more recent work (Mossaheb, 1982), it was shown that when the system is LTV, Eqn. (16) could be used as an additional condition needed for L^p stability. This condition is less conservative than the previous results. Furthermore, the shifted Nyquist plot need not be used.

5. Main Results

An important step towards achieving the goal of quantitative L^p stability is the explicit characterization of transfer functions which satisfy the first condition of Theorem 1, i.e., Eqn. (15).

Lemma 1. *Equation (15) is satisfied if*

$$\mathrm{Re}\ H(j\omega) \ge -\frac{1}{\beta}, \quad \forall \omega \in \mathbb{R}.$$

Proof. Notice that (15) can be re-written as

$$\frac{\beta}{2}\, |\mathrm{H}(j\omega)| \le \left| 1 + \frac{\beta}{2} \mathrm{H}(j\omega) \right|, \quad \forall \omega \in \mathbb{R}. \tag{17}$$

Thus,

$$(\beta/2) \sqrt{\mathrm{Re}^2 H(j\omega) + \mathrm{Im}^2 H(j\omega)} \le \sqrt{[1 + (\beta/2)\, \mathrm{Re}\, H(j\omega)]^2 + [(\beta/2)\, \mathrm{Im} H(j\omega)]^2}, \quad \forall \omega \in \mathbb{R}.$$

Simplifying both parts of the inequality yields

$$\mathrm{Re}\, H(j\omega) \ge -\frac{1}{\beta}, \quad \forall \omega \in \mathbb{R}.$$

■

Note that the corollary of Lemma 1 is that (15) is satisfied if

$$\mathrm{Re}\, H(j\omega) \ge 0, \quad \forall \omega \in \mathbb{R}, \tag{18}$$

or, equivalently, (cf. Definition 3)

$$H(s) \in \{PR\}. \tag{19}$$

We have shown thus far that if $H(s) \in \{PR\}$, the first condition of Theorem 1 is satisfied. We proceed with the second condition of this theorem, Eqn. (16).

Lemma 2. *If* $H(s) \in \mathcal{H}$, *then* $h(t) \in A_2$.

Proof. Assumption 3 assures that the residue of the pole $s = 0$ is k, so $H(s)$ can be written in the following partial fraction description:

$$H(s) = H_1(s) + \frac{k}{s}, \tag{20}$$

with $H_1(s)$ strictly proper. Since $H(s)$ is stable, $H_1(s)$ consists of a sum of asymptotically stable transfer functions. Therefore, $\|H_1\|_2 < \infty$ (see, e.g., (Doyle *et al.*, 1992)), which implies $h_1(t) \in L^2$. From the same reasons, it stems that $\|H_1\|_1 < \infty$ (Doyle *et al.*, 1992). Consequently,

$$h_1(t) \in L^1 \cap L^2. \tag{21}$$

Now, it is required to show that $th_1(t) \in L^1 \cap L^2$. This will be done by applying the following characteristic of the Laplace transform:

$$\mathcal{L}\big[th_1(t)\big] = -\frac{\mathrm{d}H_1(s)}{\mathrm{d}s}. \tag{22}$$

$H_1(s)$ is a rational function, i.e., $H_1(s) = N(s)/D(s)$. Let $\deg[N(s)] = q$ and $\deg[D(s)] = p$. Since $H_1(s)$ is strictly proper, its relative order satisfies

$$r\big[H_1(s)\big] = p - q > 0. \tag{23}$$

Note that

$$\begin{aligned} r\left[\frac{\mathrm{d}H_1(s)}{\mathrm{d}s}\right] &= r\left[\frac{(\mathrm{d}N(s)/\mathrm{d}s)D(s) - (\mathrm{d}D(s)/\mathrm{d}s)N(s)}{D^2(s)}\right] \\ &= 2p - (q - 1 + p) = (p - q) + 1 > 0. \end{aligned} \tag{24}$$

The last inequality in (24) results from (23).

Equation (24) shows that $-\mathrm{d}H_1(s)/\mathrm{d}s$ is strictly proper. Since $H_1(s)$ is asymptotically stable, $-\mathrm{d}H_1(s)/\mathrm{d}s$ is asymptotically stable as well, because the differentiation does not alter the denominator polynomial. Thus, we have

$$\frac{-\mathrm{d}H_1(s)}{\mathrm{d}s} \in L^1 \cap L^2 \Rightarrow th_1(t) \in L^1 \cap L^2. \tag{25}$$

Equations (21) and (25) yield

$$h_1(t) \in A_1. \tag{26}$$

Now, consider (20). The term k/s is strictly proper. Together with (26), we obtain $h(t) \in A_2$ (see Definition 2). ■

Consequently, we have shown that if $H(s) \in \{PR\} \cap \mathcal{H}$, the system under consideration is L^p stable. We proceed with the main result, which is formulated as follows:

Theorem 2. *If* $H(s) \in \{PR\} \cap \mathcal{H}$, *then* $\gamma = 1$.

Proof. Notice that (15) could be re-formulated as follows:

$$f(\omega) \triangleq \frac{\beta}{2}\left|\frac{H(j\omega)}{1 + (\beta/2)\,H(j\omega)}\right| \le 1, \quad \forall \omega \in \mathbb{R}, \tag{27}$$

which is satisfied if $H(s) \in \{PR\}$ and Assumption 2 holds. Next, we use Assumption 3 and substitute

$$H(j\omega) = \frac{kG(j\omega)}{j\omega}$$

into (27) to obtain

$$f(\omega) = \frac{\beta}{2}\left|\frac{kG(j\omega)}{j\omega + (\beta/2)\,kG(j\omega)}\right| \le 1. \tag{28}$$

It is simple to note that assigning $\omega = 0$ into (28) gives

$$f(0) = \frac{\beta}{2}\left|\frac{k}{k\,\beta/2}\right| = 1. \tag{29}$$

But according to (27), $f(\omega) \le 1$, so we have

$$\gamma = \frac{\beta}{2}\sup_{\omega}\left|\frac{H(j\omega)}{1 + (\beta/2)\,H(j\omega)}\right| = f(0) = 1.$$

■

The results obtained thus far may be interpreted as follows: An LTV system of the general form described in Fig. 1 is L^p stable if $H(s)$ is positive real. Moreover, if $H(s) = kG(s)/s$, then $\gamma = 1$.

Since $\mu(\gamma)$ is a function of γ only, we have the *same* $\mu(\gamma)$ for *any* system dynamics $H(s)$ which is positive real and satisfies $H(s) = kG(s)/s$. This information could be of practical engineering importance, as illustrated in the next section.

6. Illustrative Example

We shall illustrate the main result by considering an example taken from missile guidance theory. The most commonly used method for missile guidance is proportional navigation (Zarchan, 1990). In this case, the following equivalence between the discussed system and the missile guidance loop exists:

$$u(t) = a_T(t), \quad z(t) = a_M(t), \tag{30}$$

$$K_1(s) = \frac{1}{s^2}, \quad K_2(s) = N\,s\,G(s), \quad K_3(s) = 1, \tag{31}$$

$$\psi(t) = \frac{1}{t_f - t} \in [0, \infty), \quad \forall t \in [0, t_f), \tag{32}$$

$$H(s) = K_1(s)K_2(s)K_3(s) = \frac{NG(s)}{s}, \tag{33}$$

where $a_T(t)$ and $a_M(t)$ are the target and missile maneuver accelerations, respectively, $G(s)$ denotes the asymptotically stable missile autopilot dynamics, t_f is the time of flight, and N is the so-called effective proportional navigation coefficient, which is the total gain of the linear part (i.e., $G(0) = 1$).

In a conventional proportional navigation guidance (PNG) system, it is known (Gurfil *et al.*, 1998; Shinar, 1976) that an infinite missile acceleration is required near to intercept $(t \to t_f)$. This means that saturation is always reached. It will be shown hereafter that Theorem 2 characterizes a set of PNG systems, in which saturation is *avoided*.

Consequently, it is necessary to find some bound μ on the required missile-target maneuver ratio, μ_r, defined as

$$\mu_r \triangleq \frac{\sup\limits_{t\in[0,t_f]} |a_M(t)|}{\sup\limits_{t\in[0,t_f]} |a_T(t)|}. \tag{34}$$

If μ is found to be smaller than the *a-priori* known missile-target maneuver ratio, no saturation will occur. This problem can be directly formulated as a quantitative L^∞ stability problem: Find a constant μ, such that

$$\sup_{t\in[0,tf]} |a_M(t)| = \|a_M(t)\|_\infty \le \mu \|a_T(t)\|_\infty$$
$$= \mu \sup_{t\in[0,tf]} |a_T(t)|. \tag{35}$$

The desired result is obtained as follows:

First, the required maneuver acceleration of a PNG missile with ideal dynamics, i.e., $G(s) = 1$ and $H(s) = N/s$, against a constantly maneuvering target is (Zarchan, 1990)

$$\frac{a_M(t)}{a_T} = \frac{N}{N-2}\left[1 - \left(1 - \frac{t}{t_f}\right)^{N-2}\right]. \tag{36}$$

Note that in this case

$$\mu(\gamma) = \frac{\sup\limits_{t\in[0,tf]} |a_M|}{\sup\limits_{t\in[0,tf]} |a_T|} = \frac{\sup\limits_{t\in[0,tf]} |a_M|}{a_T}$$
$$= \frac{N}{N-2}, \quad \forall N > 2, \ \ a_T = \text{const.} \tag{37}$$

However, the case $H(s) = N/s$ is a particular case of $H(s) \in \{PR\} \cap \mathcal{H}$. According to Theorem 1, $\mu(\gamma)$ is a function of γ *only* $\forall a_T \in L^\infty[0, t_f]$. Theorem 2 states that for *any* $H(s) \in \{PR\} \cap \mathcal{H}$, we have $\gamma = 1$. Thus, $\mu(\gamma)$ has the value given in (37) $\forall H(s) \in \{PR\} \cap \mathcal{H}$ and $\forall a_T \in L^\infty[0, t_f]$, i.e.,

$$\|a_M\|_\infty \le \frac{N}{N-2} \|a_T\|_\infty, \quad \forall a_T \in L^\infty[0, t_f],$$
$$\forall H(s) \in \{PR\} \cap \mathcal{H}. \tag{38}$$

The consequence of (38) should be interpreted as follows. If the PNG system is designed such that $H(s) \in \{PR\} \cap \mathcal{H}$, and $N/(N-2)$ is chosen to be higher than the *a-priori* known missile-target maneuver ratio, acceleration saturation will be avoided. Equation (38) expands the results thus known in the literature, since it shows that the required missile-target maneuver ratio should be $N/(N-2)$ not only for an ideal missile and a constant target maneuver, but also for any missile dynamics satisfying $\operatorname{Re} H(j\omega) \ge 0, \ \forall \omega \in \mathbb{R}$ and any target maneuver with bounded maximal value.

7. Conclusions

In this paper, the L^p stability of linear feedback systems with a single time-varying sector-bounded element was considered. A sufficient condition for L^p stability, with $1 \le p \le \infty$, was obtained by utilizing the well-known small gain theorem. The main highlights of the results are:

(a) If the LTI part of the LTV feedback system is a proper, positive real transfer function, the system is L^p stable for $1 \le p \le \infty$.

(b) If in addition the transfer function has a single pole at the origin, the stability measure provided by the small gain theorem is the same for any system dynamics.

(c) Property (b) plays an important roll in the explicit calculation of the bound on the output-to-input ratio, due to the fact that this bound can be calculated for some particular simple case, and then generalized to the entire class.

Consequently, this paper showed that the specialized properties of positive real functions with a pole at the origin permit the derivation of an *exact* value of the loop input/output gain using the small gain theorem. This, in turn, allows an *exact* calculation of the upper bound on the system output. This fact further implies that if the exact L^p gain is found by means of solving the simplest case possible, i.e., $G(s) = 1$ and, accordingly, $H(s) = 1/s$, the same L^p gain would still be valid for an arbitrary $H(s)$ with a pole at the origin.

This important property was illustrated using a practical engineering example taken from missile guidance theory, which proved that the main result of this paper allowed not only to explicitly find the L^p gain of Lur'e-type time-varying systems, but moreover, to *synthesize* a family of non-saturating systems.

References

Desoer C.A. and Vidyasagar M. (1975): *Feedback Systems: Input-Output Properties*. — New York: Academic Press.

Doyle J.C., Francis B.A. and Tannenbaum A.R. (1992): *Feedback Control Theory*. — New York: Macmillan Publishing.

Gurfil P., Jodorkovsky M. and Guelman M. (1998): *Finite time stability approach to proportional navigation systems analysis*. — J. Guid. Contr. Dynam., Vol. 21, No. 6, pp. 853–861.

Mossaheb S. (1982): *The circle criterion and the L^P stability of feedback systems*. — SIAM J. Contr. Optim., Vol. 20, No. 1, pp. 144–151.

Sandberg I.W. (1964): *A frequency domain condition for the stability of feedback systems containing a single time-varying nonlinear element*. — Bell Syst. Tech. J., Vol. 43, pp. 1601–1608.

Sandberg I.W. (1965): *Some results on the theory of physical systems governed by nonlinear functional equations*. — Bell Syst. Tech. J., Vol. 44, No. 5, pp. 871–898.

Sandberg I.W. and Johnson K.K. (1990): *Steady state errors and the circle criterion*. — IEEE Trans. Automat. Contr., Vol. 35, No. 1, pp. 530–534.

Shinar J. (1976): *Divergence range of homing missiles*. — Israel J. Technol., Vol. 14, pp. 47–55.

Vidyasagar M. (19): *Nonlinear Systems Analysis, 2nd Ed.*. — New Jersey: Upper Saddle River.

Zames G. (1990): *On input-output stability of time-varying nonlinear feedback systems-Part II: Conditions involving circles in the frequency plane and sector nonlinearities*. — IEEE Trans. Automat. Contr., Vol. AC–11, No. 2, pp. 465–476.

Zarchan P. (1990): *Tactical and Strategic Missile Guidance*. — Washington: AIAA.

Received: 10 August 2002
Revised: 16 November 2002