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COMPUTING WITH WORDS AND LIFE DATA

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The problem of statistical inference on the mean lifetime in the presence of vague data is considered. Situations with fuzzy lifetimes and an imprecise number of failures are discussed.

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1. Introduction

One of the most important problems of reliability analysis is to estimate the *mean lifetime* of the item under study. In technical applications this parameter is also called the *mean time to failure* (*MTTF*) and is often included in the specification of a product. For example, producers are interested whether this time is sufficiently large, as a large *MTTF* allows them to extend a warranty time. Classical estimators require precise data obtained from strictly controlled reliability tests (for example, those performed by a producer at his or her laboratory). In such a case a failure should be precisely defined, and all tested items should be continuously monitored. However, in a real situation these requirements might not be fulfilled. In an extreme case, the reliability data come from users whose reports are expressed in a vague way. The vagueness of the data has many different sources: it might be caused by subjective and imprecise perception of failures by a user, by imprecise records of reliability data, by imprecise records of the rate of usage, etc. Therefore we need different tools appropriate for modelling vague data, and suitable statistical methodology to handle these data as well.

Grzegorzewski and Hryniewicz (1999) considered the generalization of the exponential model which admits vagueness in lifetimes or censoring times but requires precise information about the number of observed failures (i.e. one knows whether a given item failed or whether it survived). In their paper fuzzy sets were used for modelling the vagueness of the lifetimes. However, sometimes we face situations when the number of observed failures is also vague. For example, it may be due to an imprecise definition of the failure. We can also consider partial failures or information about the scale of the failure expressed by colloquial words. Hence in the present paper we suggest another generalization of the classical exponential model. We consider not only fuzzy lifetimes but situations in which the number of failures is fuzzy as well.

2. Classical Approach

The mean lifetime may be efficiently estimated by the sample average from the sample of the times to failure $W_1, \ldots, W_n$ of n tested items, i.e.

$$MTTF = \frac{W_1 + \cdots + W_n}{n}. \tag{1}$$

However, in the majority of practical cases the lifetimes of all tested items are not known, as the test is usually terminated before the failure of all items. It means that exact lifetimes are known only for a portion of the items under study, while the remaining lifetimes are known only to exceed certain values. This feature of lifetime data is called *censoring*. More formally, a fixed censoring time $Z_i > 0$, $i = 1, \ldots, n$ is associated with each item. We observe W_i only if $W_i \le Z_i$. Therefore our lifetime data consist of pairs $(T_1, Y_1), \ldots, (T_n, Y_n)$, where

$$T_i = \min\{W_i, Z_i\}, \tag{2}$$

$$Y_i = \begin{cases} 1 & \text{if} \quad T_i = W_i, \\ 0 & \text{if} \quad T_i = Z_i. \end{cases} \tag{3}$$

Numerous parametric models are used in the lifetime data analysis. Among them the most widely used are the exponential, Weibull, gamma and lognormal distribution models. Historically, the exponential model was the first lifetime model extensively developed and widely used in many areas of lifetime analysis: from studies on the lifetimes of various types of manufactured items to research involving survival or remission times in chronic diseases.

In this model the lifetime T is described by the probability density function

$$f(t) = \begin{cases} \frac{1}{\theta} e^{-t/\theta} & \text{if} \quad t > 0, \\ 0 & \text{if} \quad t \leq 0, \end{cases} \tag{4}$$

where $\theta > 0$ is the mean lifetime. It is worth noticing that the hazard function in the model considered is constant. Although this assumption is very restrictive, the exponential model is still frequently used in practice because of two important features: its parameter θ is easily estimated, and for lifetimes described by a probability distribution with increasing hazard it gives a conservative approximation for the mean lifetime. Thus, in this paper we assume that the exponential distribution model is the mathematical model which describes lifetimes of tested items. Note that

$$T = \sum_{i=1}^{n} T_i = \sum_{i \in O} W_i + \sum_{i \in C} Z_i \tag{5}$$

is the total survival time (sometimes called a total time on test), where O and C denote the sets of items for which exact lifetimes are observed and censored, respectively. Moreover, let

$$r = \sum_{i=1}^{n} Y_i \tag{6}$$

denote the number of observed failures. In the exponential model considered the statistic (r, T) is a minimally sufficient statistic for θ, and the maximum likelihood estimator of the mean lifetime θ is (assuming $r > 0$)

$$\widehat{\theta} = \frac{T}{r}. \tag{7}$$

It can be shown (Cox, 1953) that the statistic $2r\widehat{\theta}/\theta$ is approximately chi-square distributed with $2r+1$ degrees of freedom. This approximation is used for constructing satisfactory confidence intervals even for quite small sample sizes (e.g., see Lawless, 1982). For example, the one-sided confidence interval with the upper limit for θ on the confidence level $1-\delta$ is given by

$$\left(0, \frac{2T}{\chi^2_{2r+1,\delta}}\right], \tag{8}$$

where $\chi^2_{m,\delta}$ is the quantile of order δ of the chi-square distribution with m degrees of freedom.

Practitioners are usually interested in testing the hypothesis $H\colon \theta \geq \theta_0$ that the mean lifetime is no less than a given value θ_0 (e.g., a given requirement on the *MTTF*) against $K\colon \theta < \theta_0$. The desired test can be constructed easily using (8). Namely, the hypothesis H should be rejected on the significance level δ if

$$\frac{2T}{\theta_0} \leq \chi^2_{2r+1,\delta}. \tag{9}$$

3. Vague Data

3.1. Fuzzy Survival Times

Now suppose that the lifetimes (times to failures) and censoring times are not necessarily crisp but may be vague as well. A generalization of the exponential model which admits vagueness in lifetimes was considered by Grzegorzewski and Hryniewicz (1999). They model imprecise lifetimes by fuzzy numbers. In this paper we adopt a slightly more general assumption that censoring times may also be vague. We assume, however, that the values of the indicators $Y_1, Y_2, \ldots, Y_n$ are equal either to 0 or to 1, i.e., in every case we know if the test has been terminated by censoring or as a result of a failure. In order to describe the vagueness of life data we use the notion of a fuzzy number.

Let us recall some basic concepts and notation connected with the fuzzy numbers and fuzzy random variables.

Definition 1. The fuzzy subset A of the real line $\mathbb{R}$, with the membership function $\mu_A\colon \mathbb{R} \to [0,1]$, is a *fuzzy number* iff

(a) A is normal, i.e., there exists an element x_0 such that $\mu_A(x_0) = 1$;

(b) A is fuzzy convex, i.e. $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$, $\forall x_1, x_2 \in \mathbb{R}$, $\forall \lambda \in [0,1]$;

(c) μ_A is upper semicontinuous;

(d) $\operatorname{supp} A$ is bounded.

It is known that for any fuzzy number A there exist four numbers $a_1, a_2, a_3, a_4 \in \mathbb{R}$ and two functions $\eta_A, \zeta_A\colon \mathbb{R} \to [0,1]$, where η_A is nondecreasing and ζ_A is nonincreasing, such that we can describe a membership function μ_A in the following manner:

$$\mu_A(x) = \begin{cases} 0 & \text{if} \quad x < a_1, \\ \eta_A(x) & \text{if} \quad a_1 \leq x < a_2, \\ 1 & \text{if} \quad a_2 \leq x \leq a_3, \\ \zeta_A(x) & \text{if} \quad a_3 < x \leq a_4, \\ 0 & \text{if} \quad a_4 < x. \end{cases} \tag{10}$$

Functions η_A and ζ_A are called the left side and the right side of a fuzzy number A, respectively.

The notion of the fuzzy number was introduced by Dubois and Prade (1978). Some authors using this concept in their papers do not quote requirement (d) given above. They just adopt a more general assumption (e.g., see Chanas, 2001)

$$\text{(d}'\text{)} \qquad \int_{-\infty}^{+\infty} \mu_A(x)\,\mathrm{d}x < +\infty.$$

However, others (especially practitioners) argue that (d) is more natural than (d′) since it means that real numbers less than a_1 or greater than a_4 surely do not belong to A. Hence in our paper we adopt (d), although from a mathematical point of view the requirement (d′) is sufficient.

A useful notion for dealing with a fuzzy number is a set of its α-cuts. The α-cut of a fuzzy number A is a nonfuzzy set defined as

$$A_\alpha = \{x \in \mathbb{R} : \mu_A(x) \geq \alpha\}. \tag{11}$$

A family $\{A_\alpha : \alpha \in (0,1]\}$ is a set representation of the fuzzy number A. Based on the resolution identity, we get

$$\mu_A(x) = \sup\{\alpha \mathbb{I}_{A_\alpha}(x) : \alpha \in (0,1]\}, \tag{12}$$

where $\mathbb{I}_{A_\alpha}(x)$ denotes the characteristic function of A_α. According to the definition of the fuzzy number it is easily seen that every α-cut of a fuzzy number is a closed interval. Hence we have $A_\alpha = [A_L(\alpha), A_U(\alpha)]$, where

$$\begin{aligned} A_\alpha^L &= \inf\{x \in \mathbb{R} : \mu_A(x) \geq \alpha\}, \\ A_\alpha^U &= \sup\{x \in \mathbb{R} : \mu_A(x) \geq \alpha\}. \end{aligned} \tag{13}$$

If the sides of the fuzzy number A are strictly monotone then by (10) one can easily see that A_α^L and A_α^U are inverse functions of η_A and ζ_A, respectively. In general, we may adopt the convention that $\eta_A(x)^{-1} = \inf\{x \in \mathbb{R} : \mu_A(x) \geq \alpha\} = A_\alpha^L$ and $\zeta_A(x)^{-1} = \sup\{x \in \mathbb{R} : \mu_A(x) \geq \alpha\} = A_\alpha^U$.

As in the classical arithmetic, we can add, subtract, multiply and divide fuzzy numbers. Since all these operations become rather complicated if the sides of fuzzy numbers are not very regular, simple fuzzy numbers, e.g. with linear or piecewise linear sides, are preferred in practice. Such fuzzy numbers with simple membership functions also have more natural interpretation. Therefore the most often used fuzzy numbers are the so-called *trapezoidal fuzzy numbers*, i.e. fuzzy numbers whose both sides are linear. Trapezoidal fuzzy numbers can be used for the representation of expressions such as, e.g., “more or less between 5 and 7”, “approximately between 10 and 15”, etc. Trapezoidal fuzzy numbers with $a_2 = a_3$ are called *triangular fuzzy numbers* and are often used for modelling expressions such as, e.g., “about 6”, “more or less 8”, etc. Triangular fuzzy numbers with only one side may be useful when describing situations like “just before 50” ($a_2 = a_3 = a_4$) or “just after 30” ($a_1 = a_2 = a_3$). If $a_1 = a_2$ and $a_3 = a_4$, then we get the so-called *rectangular fuzzy numbers*, which may represent expressions such as, e.g., “between 20 and 25.” In the case of $a_1 = a_2 = a_3 = a_4 = a$ we get a crisp number, i.e., a fuzzy number which is no longer vague but represents a precise value that can be identified with the proper real number a.

A space of all fuzzy numbers will be denoted by $\mathbb{FN}$. Of course, $\mathbb{FN} \subset \mathbb{F}(\mathbb{R})$, where $\mathbb{F}(\mathbb{R})$ denotes the space of all fuzzy sets on the real line.

Definition 2. A fuzzy number $A \in \mathbb{FN}$ is *non-negative* if $\mu_A(x) = 0$ for all $x < 0$, and *positive* if $\mu_A(x) = 0$ for all $x \leq 0$.

Equivalently, we may say that $A \in \mathbb{FN}$ is non-negative if $A_{\alpha=0}^L \geq 0$ and is positive if $A_{\alpha=0}^L > 0$. The space of all non-negative fuzzy numbers will be denoted by $\mathbb{NFN}$, while the space of all positive fuzzy numbers will be denoted by $\mathbb{PFN}$.

The notion of a fuzzy random variable was introduced by Kwakernaak (1978; 1979). Other definitions of fuzzy random variables are due to Kruse (1982) or to Puri and Ralescu (1986). Our definition is similar to those of Kwakernaak and Kruse. Suppose that a random experiment is described, as usual, by a probability space $(\Omega, \mathbb{A}, P)$, where Ω is the set of all possible outcomes of the experiment, $\mathbb{A}$ is a σ-algebra of subsets of Ω (the set of all possible events) and P is a probability measure.

Definition 3. A mapping $X\colon \Omega \to \mathbb{FN}$ is called a *fuzzy random variable* if it satisfies the following properties:

(a) $\{X_\alpha(\omega) : \alpha \in [0,1]\}$ is a set representation of $X(\omega)$ for all $\omega \in \Omega$,

(b) for each $\alpha \in [0,1]$ both $X_\alpha^L = X_\alpha^L(\omega) = \inf X_\alpha(\omega)$ and $X_\alpha^U = X_\alpha^U(\omega) = \sup X_\alpha(\omega)$ are usual real-valued random variables on $(\Omega, \mathbb{A}, P)$.

Thus a fuzzy random variable X is considered as a perception of an unknown usual random variable $V\colon \Omega \to \mathbb{R}$, called the *original* of X. Let $\mathcal{V}$ denote the set of all possible originals of X. If only vague data are available, it is of course impossible to show which of the possible originals is the true one. Therefore, we can define a fuzzy set on $\mathcal{V}$, with a membership function $\iota\colon \mathcal{V} \to [0,1]$ given as follows:

$$\iota(V) = \inf\{\mu_{X(\omega)}(V(\omega)) : \omega \in \Omega\}, \tag{14}$$

which corresponds to the grade of acceptability that a fixed random variable V is the original of the fuzzy random variable in question (see Kruse and Meyer, 1987).

Similarly, an n-dimensional fuzzy random sample $X_1, \ldots, X_n$ may be treated as a fuzzy perception of the usual random sample $V_1, \ldots, V_n$ (where $V_1, \ldots, V_n$ are independent and identically distributed crisp random variables). The set $\mathcal{V}^n$ of all possible originals of that fuzzy random sample is, in fact, a fuzzy set with the membership function

$$\iota(V_1, \ldots, V_n) = \min_{i=1,\ldots,n} \inf \big\{ \mu_{X_i(\omega)}(V_i(\omega)) : \omega \in \Omega \big\}. \tag{15}$$

Although a random variable is completely characterized by its probability distribution, very often we are interested only in some parameters of this distribution. Let us consider a parameter $\theta = \theta(V)$ of a random variable V. This parameter may be viewed as an image of a mapping $\Gamma\colon \mathbb{P} \to \mathbb{R}$, which assigns each random variable V having distribution $P_\theta \in \mathbb{P}$ the analysed parameter θ, where $\mathbb{P} = \{P_\theta : \theta \in \Theta\}$ is a family of distributions. However, if we deal with a fuzzy random variable, we cannot observe parameter θ but only its vague image. Using this reasoning together with Zadeh's extension principle, Kruse and Meyer (1987) introduced the notion of the *fuzzy parameter of the fuzzy random variable*, which may be considered as a *fuzzy perception* of the unknown parameter θ. It is defined as a fuzzy set with the membership function

$$\mu_{\Lambda(\theta)}(t) = \sup \big\{ \iota(V) : V \in \mathcal{V}, \theta(V) = t \big\}, \quad t \in \mathbb{R}, \tag{16}$$

where $\iota(V)$ is given by (14). This notion is well defined because if our data are crisp, i.e., $X = V$, we get $\Lambda(\theta) = \theta$. Similarly, for a random sample of size n we get

$$\mu_{\Lambda(\theta)}(t) = \sup \big\{ \iota(V_1, \ldots, V_n) : (V_1, \ldots, V_n) \in \mathcal{V}^n, \theta(V_1) = t \big\}, \quad t \in \mathbb{R}. \tag{17}$$

One can easily obtain α-cuts of $\Lambda(\theta)$:

$$\Lambda_\alpha(\theta) = \big\{ t \in \mathbb{R} : \exists (V_1, \ldots, V_n) \in \mathcal{V}^n, \theta(V_1) = t \big\}, \tag{18}$$

such that $V_i(\omega) \in (X_i(\omega))_\alpha$ for $\omega \in \Omega$ and for $i = 1, \ldots, n$. For more information, we refer the reader to (Kruse and Meyer, 1987).

3.2. Failures and Partial Failures

It happens very often in practice that we deal not only with critical failures but also with non-critical failures that are usually described using a common language. For example, one is anxious because of a strange noise in the car. However, he or she can still drive this car. Such a situation corresponds to a failure which is not critical (at least at this moment).

In order to take into account such non-critical failures let us describe the state of each observed item at the time Z_i. Let G denote the set of all items which are capable at their censoring times Z_i. Therefore we can assign to each item $i = 1, \ldots, n$ its degree of belongingness $g_i = \mu_G(i)$ to G, where $g_i \in [0, 1]$. When the item has not failed before the censoring time Z_i, i.e., it works perfectly at Z_i, we set $g_i = 1$. On the other hand, if a critical failure has occurred before or exactly at time moment L_i, we set $g_i = 0$. If $g_i \in (0, 1)$, then the item under study neither works perfectly nor is completely failed. We may consider this situation as a *partial failure* of the considered item. Let us notice that G can be considered now as a fuzzy set with a finite support.

The way in which we define the values of g_i in practice is beyond to the scope of this paper. For example, it is possible to describe formally some performance measures, and to evaluate the value of a certain aggregated quality index. For this evaluation we can use the notions of *possibility theory* such as the *necessity of dominance* or the *possibility of dominance* indices, which are useful when measuring the degree to which some imprecisely defined requirements are fulfilled. However, in the majority of practical situations, we describe partial failures linguistically using notions such as, e.g., “slightly possible”, “highly possible”, “nearly sure”, etc. In such a case we may assign arbitrary weights $g_i \in (0, 1)$ to such imprecise expressions.

Alternatively, one can consider a set D of faulty items and, in the simplest case, the degree of belongingness to D equals $d_i = \mu_D(i) = 1 - g_i$. Further on, we will call g_i and d_i the degrees of the up and down states, respectively.

Now we have to find a fuzzy counterpart of the number of observed failures r. Grzegorzewski (2001) proposed several methods for failure counting. Depending on the output, we can divide them into two groups: crisp or fuzzy methods.

3.3. Crisp Failure Counting

Let $g_1, \ldots, g_n$ $(d_1, \ldots, d_n)$ denote the degrees of up states (down states) of all items tested. The most natural way for counting failures is to either consider only critical failures or to treat all kinds of failures similarly. These two approaches correspond to optimistic (or liberal) and pessimistic (conservative) viewpoints, respectively. Thus the number of failures observed in accordance with the optimistic viewpoint is

$$\widetilde{r}_{\text{opt}} = \sum_{i=1}^{n} I(d_i = 1) = n - \sum_{i=1}^{n} I(g_i > 0), \tag{19}$$

where I is the indicator function, while the number of failures obtained in accordance with the pessimistic viewpoint is

$$\widetilde{r}_{\text{pes}} = n - \sum_{i=1}^{n} I(g_i = 1) = \sum_{i=1}^{n} I(d_i > 0). \tag{20}$$

More generally, one can take into account only failures with some degree of the down state (up state). Then we get

$$\widetilde{r}_{\xi} = \sum_{i=1}^{n} I(d_i > \xi) = n - \sum_{i=1}^{n} I(g_i \geq 1 - \xi), \tag{21}$$

where $\xi \in (0,1)$. Measures (19)–(21) are crisp, since $\widetilde{r}_{\text{opt}}, \widetilde{r}_{\text{pes}}, \widetilde{r}_{\xi} \in N \cup \{0\}$. It is clear that $\widetilde{r}_{\text{opt}} \leq \widetilde{r}_{\xi} \leq \widetilde{r}_{\text{pes}}$ for each $\xi \in (0,1)$. The methods of failure counting described above are in some sense reductive. Actually, they abandon the whole information on particular degrees of up or down states and utilize only part of that information—whether these degrees exceed a given level. However, sometimes it would be useful to take into account all accessible information. Then the following method for failure counting might be used:

$$\widetilde{r}^{c} = \sum_{i=1}^{n} d_i = |D| = n - \sum_{i=1}^{n} g_i = n - |G|, \tag{22}$$

where $|D|$ and $|G|$ denote the cardinalities of fuzzy sets D and G, respectively.

3.4. Fuzzy Failure Counting

A basic advantage of the methods for counting failures given above is that they are easy to handle, since their outputs are crisp. Unfortunately, such an approach does not reflect the reality very well, especially that the test results are often non-precise but vague. Moreover, the requirements are sometimes vague, too. It seems that the best way to summarize fuzzy descriptions of the test results is to use fuzzy failure counting measures. We consider the observed degrees of down states and count the number of failures we would get if the rejection limit were fixed on each degree of the down state (naturally, the lower the rejection limit, the more failures we observe). Thus we get the following (fuzzy) number of failures:

$$\widetilde{r}^{f}_{\text{opt}} = |D|_f\,, \tag{23}$$

where $|D|_f$ denotes the fuzzy cardinality of a fuzzy set D. We may also start from up states. Therefore

$$\widetilde{r}^{f}_{\text{pes}} = n - |G|_f\,, \tag{24}$$

where $|G|_f$ denotes the fuzzy cardinality of a fuzzy set G. However, contrary to the crisp counting, $|D|_f \neq n - |G|_f$. It is obvious that such a fuzzy number of observed failures is a finite fuzzy set. It is also a normal fuzzy set (since we assume, as in the classical approach, that there exists at least one critical failure).

Example. In order to explain the concept of fuzzy failure counting, let us consider a simple example. Suppose that 10 items were put under a test, and at the test termination moments (caused either by a failure or by censoring) their degrees of belongingness to G were 1, 0.5, 0.9, 1, 1, 0.2, 0, 0.9, 1, and 0, respectively. This means that only four items survived the test without any sign of failure, in four cases there was an evidence of partial failures, and two items surely failed. Obviously, the degrees of their belongingness to D were 0, 0.5, 0.1, 0, 0, 0.8, 1, 0.1, 0, 1, respectively. Using crisp failure counting methods we may get the following results:

$$\widetilde{r}_{\text{opt}} = 2, \quad \widetilde{r}_{\text{pes}} = 6,$$
$$\widetilde{r}_{0.5} = 3, \qquad \widetilde{r}^{c} = 3.5.$$

However, in the case of fuzzy failure counting we have

$$\widetilde{r}^{f}_{\text{opt}} = 1|2 + 0.8|3 + 0.5|4 + 0.1|6$$

and

$$\widetilde{r}^{f}_{\text{pes}} = 0.2|2 + 0.5|3 + 0.9|4 + 1|6.$$

As can be seen, in the optimistic case we assign the highest plausibility measure to the failures that were revealed with certainty. On the other hand, in the pessimistic case, we assign the highest plausibility measure to all cases with even slight symptoms of failure. ♦

4. Statistical Inference

4.1. *MTTF* Point Estimation

Now we consider fuzzy lifetimes $\widetilde{T}_1, \ldots, \widetilde{T}_n$ described by their membership functions $\mu_1(t), \ldots, \mu_n(t) \in \mathbb{NFN}$. Thus applying the extension principle to (5) we get the fuzzy total survival lifetime $\widetilde{T}$ (which is also a fuzzy number)

$$\widetilde{T} = \sum_{i=1}^{n} \widetilde{T}_i, \tag{25}$$

with the membership function

$$\mu_{\widetilde{T}}(t) = \sup_{t_1,\ldots,t_n \in \mathbb{R}^+ : t_1 + \cdots + t_n = t} \big\{\mu_1(t_1) \wedge \cdots \wedge \mu_n(t_n)\big\}. \tag{26}$$

Using the Minkowski operation on α-cuts, we may find the set representation of $\widetilde{T}$ given as follows:

$$\begin{aligned}\widetilde{T}_\alpha &= (T_1)_\alpha + \cdots + (T_n)_\alpha \\ &= \big\{t \in \mathbb{R}^+ : t = t_1 + \cdots + t_n, \\ &\qquad \text{where } t_i \in (T_i)_\alpha,\ i = 1, \ldots, n\big\},\end{aligned} \tag{27}$$

where $\alpha \in (0,1]$. Now, using the extension principle once more, we may define a fuzzy estimator of the mean lifetime $\widehat{\Theta}$ in the presence of vague lifetimes as

$$\widetilde{\theta} = \frac{\widetilde{T}}{r}. \tag{28}$$

Since $r \in \mathbb{N}$, we can easily find a set representation of $\widehat{\Theta}$:

$$\widetilde{\theta}_\alpha = \left\{ t \in \mathbb{R}^+ : t = \frac{x}{r},\ \text{where } x \in \widetilde{T}_\alpha \right\}. \tag{29}$$

For more details and the discussion on fuzzy confidence intervals, we refer the reader to (Grzegorzewski and Hryniewicz, 1999).

By the extension principle, we may also define a fuzzy estimator of the mean lifetime $\widetilde{\widetilde{\theta}}$ in the presence of fuzzy lifetimes and a vague number of failures. Namely, for crisp failure counting methods we get the following formula:

$$\widetilde{\widetilde{\theta}} = \frac{\widetilde{T}}{\widetilde{r}}, \tag{30}$$

where $\widetilde{T}$ is the fuzzy total survival time and $\widetilde{r}$ denotes the number (crisp) of vaguely defined failures. Actually, (30) provides a family of estimators that depend on the choice of $\widetilde{r}$. Namely, one can choose his or her preferred measure of failure counting $\widetilde{r} \in \{\widetilde{r}_{\text{opt}}, \widetilde{r}_{\text{pes}}, \widetilde{r}_\xi, \widetilde{r}^c\}$ and get, as a result, an estimator $\widetilde{\widetilde{\theta}}_{\text{opt}}$, $\widetilde{\widetilde{\theta}}_{\text{pes}}$, $\widetilde{\widetilde{\theta}}_\xi$ $(0 < \xi < 1)$, $\widetilde{\widetilde{\theta}}^c$, respectively. It is not difficult to prove that $\widetilde{\widetilde{\theta}}$ is a fuzzy number.

However, in the case of fuzzy failure counting methods, i.e. for $\widetilde{r} \in \{\widetilde{r}^f_{\text{opt}}, \widetilde{r}^f_{\text{pes}}\}$, we have

$$\widetilde{\widetilde{\theta}} = \frac{\widetilde{T}}{\operatorname{conv}(\widetilde{r})}, \tag{31}$$

where $\operatorname{conv}(\widetilde{r})$ is the convex hull of the fuzzy set $\widetilde{r}$ defined as follows:

$$\operatorname{conv}(\widetilde{r}) = \inf \big\{ A \in \mathbb{NFN} : \widetilde{r} \subseteq A \big\}. \tag{32}$$

Since now the denominator of (31) is a fuzzy number, our estimators $\widetilde{\widetilde{\theta}}^f_{\text{opt}}$ and $\widetilde{\widetilde{\theta}}^f_{\text{pes}}$ of the mean lifetime are fuzzy numbers, too.

4.2. Confidence Intervals for *MTTF*

Besides finding the fuzzy estimator of the mean lifetime, we can also construct fuzzy confidence intervals for *MTTF*. First of all, we should realize what information is yielded with the one-sided confidence interval with an upper limit for the mean lifetime. Roughly speaking, it tells us that with high probability (confidence) the true mean lifetime of the individual under study does not exceed the given value (upper confidence limit). For the crisp case this confidence limit can be easily found from (8), i.e. $\pi = 2T/\chi^2_{2r+1,\delta}$. For example, $\pi = 100$ hours means that the true mean lifetime "almost surely" does not exceed 100 hours—it is possible that the mean lifetime is equal to 99.5 hours, or 57.3 or 13.2 or even 0.5 hour—it remains unknown, although it is "almost sure" that it is not equal to, e.g., 150 hours (expression "almost surely" might be a linguistic interpretation of the confidence level equal to, e.g., 0.95).

Unfortunately, in the presence of fuzzy data, T is no longer crisp but fuzzy, and therefore π would be also fuzzy. Moreover, if our data are fuzzy, we lose that natural and simple interpretation of the one-sided confidence interval with the upper limit for the mean lifetime given above. Since if we get, e.g., $\pi =$ "about 100 hours" (described by the triangular fuzzy number with $a_1 = 95$, $a_2 = a_3 = 100$ and $a_4 = 105$), it is possible that the true mean lifetime is equal, e.g., to 5 hours or 50 hours, it is "almost sure" that it is not equal to, e.g., 150 hours, but it is not clear whether or not it is possible that it is equal to 97 or 103 hours. An optimist would say: "yes, it is possible that the true mean lifetime is equal to 97 hours" and "it might be possible that the true mean lifetime is equal to 103 hours." However, a pessimist would be more cautious and would answer: "no, it might be possible that the true mean lifetime is equal to 97 hours" but "it is not possible that the true mean lifetime is equal to 103 hours." This example shows that we have neither clear nor unique interpretation of the upper confidence limit for vague life data. Below we will suggest how to handle situations like those described above. Our proposal is based on the above-mentioned difference in the attitude, i.e., optimism and pessimism.

To begin with, we have to consider vague data again. It is seen at once that there are doubts, analogous to that mentioned above, how to qualify our conviction about the survival time of any individual whose vague lifetime is, e.g., $T =$ "between 1000 and 1050 hours" (described by the rectangular fuzzy number $a_1 = a_2 = 1000$, $a_3 = a_4 = 1050$). And again, according to an optimistic or a pessimistic attitude to that lifetime one may or may not be convinced that it is possible that the individual under discussion survived, for example, 1020 hours. Hence, together with given vague life data $\widetilde{T}_1, \ldots, \widetilde{T}_n$, we will also consider the so-called survival data of two types: optimistic and pessimistic.

Now let $\widetilde{T}_1, \ldots, \widetilde{T}_n$, $\widetilde{T}_i \in \mathbb{NFN}$ denote fuzzy lifetimes and $(\widetilde{T}_i)_\alpha = [(\widetilde{T}_i)^L_\alpha, (\widetilde{T}_i)^U_\alpha]$ be an α-cut of $\widetilde{T}_i$, $\alpha \in (0,1]$, $i = 1, \ldots, n$. Consider two operators: $\text{Opt}, \text{Pes}\colon \mathbb{NFN} \to \mathbb{NFN}$ defined as follows:

$$(\text{Opt}\, \widetilde{T}_i)_\alpha = \big(0, (\widetilde{T}_i)^U_\alpha\big], \quad \alpha \in (0,1] \tag{33}$$

and

$$(\operatorname{Pes}\widetilde{T}_i)_\alpha = \big(0, (\widetilde{T}_i)^L_{1-\alpha}\big], \quad \alpha \in (0,1]. \tag{34}$$

If the quantities $\widetilde{T}_1, \dots, \widetilde{T}_n$ denote fuzzy lifetimes, then $\operatorname{Opt}\widetilde{T}_1, \dots, \operatorname{Opt}\widetilde{T}_n$, defined by (33), and $\operatorname{Pes}\widetilde{T}_1, \dots, \operatorname{Pes}\widetilde{T}_n$, defined by (34), are optimistic and pessimistic survival times, respectively. Of course, if all the data are crisp, then both optimistic and pessimistic survival times are identical and might be identified with life times.

Kruse and Meyer (1987; 1988) proposed a general method for deriving fuzzy confidence intervals for fuzzy data $X_1, \dots, X_n$ if one knows how to construct a usual (i.e., crisp) confidence interval for the parameter under discussion. Particularly, if $(-\infty, \pi]$ is the crisp one-sided confidence interval with the upper limit for θ on a confidence level of $1-\delta$, where $\pi = \pi(V_1, \dots, V_n)$, then a fuzzy set $\Pi = \Pi(X_1, \dots, X_n)$ with the membership function

$$\mu_\Pi(t) = \sup\big\{\alpha \mathbb{I}_{(-\infty, \Pi^U_\alpha]}(t) : \alpha \in (0,1]\big\}, \tag{35}$$

where

$$\begin{aligned}\Pi^U_\alpha &= \Pi^U_\alpha(X_1, \dots, X_n)\\ &= \sup\big\{u \in \mathbb{R} : \forall i \in \{1, \dots, n\} \exists x_i \in (X_i)_\alpha\\ &\qquad \text{such that } \pi(x_1, \dots, x_n) \geq u\big\}\end{aligned} \tag{36}$$

is the one-sided fuzzy confidence interval with the upper limit for θ on a confidence level of $1-\delta$. For details we refer the reader to (Kruse and Meyer, 1987; 1988).

In the case of our vague life data we will construct two one-sided fuzzy confidence intervals with upper limits for the mean lifetime based on either optimistic or pessimistic survival times. To begin with, let us assume that the number of failures r is crisp. According to Kruse and Meyer's method, the *optimistic one-sided fuzzy confidence interval with the upper limit* $\Pi^{\text{opt}} = \Pi^{\text{opt}}(\operatorname{Opt}\widetilde{T}_1, \dots, \operatorname{Opt}\widetilde{T}_n)$ for the mean lifetime on the confidence level $1-\delta$ has the following membership function:

$$\mu_{\Pi^{\text{opt}}}(t) = \sup\big\{\alpha \mathbb{I}_{(0, (\Pi^{\text{opt}})^U_\alpha]}(t) : \alpha \in (0,1]\big\}, \tag{37}$$

where

$$(\Pi^{\text{opt}})^U_\alpha = \frac{2}{\chi^2_{2r+1,\delta}} \Big(\sum_{i=1}^n \operatorname{Opt}\widetilde{T}_i\Big)^U_\alpha, \tag{38}$$

while the *pessimistic one-sided fuzzy confidence interval with the upper limit* $\Pi^{\text{pes}} = \Pi^{\text{pes}}(\operatorname{Pes}\widetilde{T}_1, \dots, \operatorname{Pes}\widetilde{T}_n)$ for the mean lifetime on the confidence level $1-\delta$ has the following membership function:

$$\mu_{\Pi^{\text{pes}}}(t) = \sup\big\{\alpha I_{(0, (\Pi^{\text{pes}})^U_\alpha]}(t) : \alpha \in (0,1]\big\}, \tag{39}$$

where

$$(\Pi^{\text{pes}})^U_\alpha = \frac{2}{\chi^2_{2r+1,\delta}} \Big(\sum_{i=1}^n \operatorname{Pes}\widetilde{T}_i\Big)^U_{1-\alpha}. \tag{40}$$

By (25) one can easily check that $(\sum_{i=1}^n \operatorname{Opt}\widetilde{T}_i)^U_\alpha = (\widetilde{T})^U_\alpha$ and $(\sum_{i=1}^n \operatorname{Pes}\widetilde{T}_i)^U_\alpha = (\widetilde{T})^L_{1-\alpha}$, so α-cuts (38) and (40) of the optimistic and pessimistic one-sided confidence intervals with upper limits for the mean lifetime might be calculated from the equations

$$(\Pi^{\text{opt}})^U_\alpha = \frac{2}{\chi^2_{2r+1,\delta}} (\widetilde{T})^U_\alpha, \tag{41}$$

$$(\Pi^{\text{pes}})^U_\alpha = \frac{2}{\chi^2_{2r+1,\delta}} (\widetilde{T})^L_{1-\alpha}, \tag{42}$$

respectively. Hence it is seen that the optimistic one-sided fuzzy confidence interval with the upper limit Π^{opt} is entirely based on the right side of the total lifetime $\widetilde{T}$ and disregards completely the left side of $\widetilde{T}$. So it corresponds only to that optimistic attitude to life data described in the example given above. In contrast to Π^{opt}, Π^{pes} is based solely on the left side of the total lifetime $\widetilde{T}$ and disregards the right side of $\widetilde{T}$, and thus it represents the pessimistic attitude to the life data.

It is also not surprising that $\Pi^{\text{pes}} \subseteq \Pi^{\text{opt}}$. Moreover, if all life data are crisp, both optimistic and pessimistic fuzzy confidence intervals coincide and reduce to the traditional crisp one-sided confidence interval with upper limit.

Equations (41) and (42) were obtained taking into account only one aspect of vague data, i.e., imprecise lifetimes. However, we may easily generalize these formulae to situations with vaguely defined failures.

Let us now consider a fuzzy case, and let $\widetilde{r}$ denote the number (crisp or fuzzy) of observed failures given by formulae (19)–(24). Now, in accordance with the optimistic or pessimistic attitudes to the life data (i.e., using $\operatorname{Opt}\widetilde{T}_1, \dots, \operatorname{Opt}\widetilde{T}_n$ or $\operatorname{Pes}\widetilde{T}_1, \dots, \operatorname{Pes}\widetilde{T}_n$) and choosing a failure counting method $\widetilde{r}$ (where $\widetilde{r} \in \{\widetilde{r}_{\text{opt}}, \widetilde{r}_{\text{pes}}, \widetilde{r}_\xi, \widetilde{r}^c, \widetilde{r}^f_{\text{opt}}, \widetilde{r}^f_{\text{pes}}\}$) we could get formulae for confidence intervals corresponding to different combinations of lifetimes/numbers of failure descriptions. We have to bear in mind, however, that in order to arrive at optimistic (pessimistic) bounds of the mean lifetime, we have to use consistently $\{\widetilde{r}_{\text{opt}}, \widetilde{r}^f_{\text{opt}}\}$ for optimistic bounds and $\{\widetilde{r}_{\text{pes}}, \widetilde{r}^f_{\text{pes}}\}$ for pessimistic bounds. In the intermediate case we can use either $\widetilde{r}_\xi$ or $\widetilde{r}^c$. Depending on whether we consider optimistic or pessimistic survival times, the α-cuts of the upper bound of the confidence interval on the confidence level $1-\delta$ are as follows:

$$(\widetilde{\Pi}^{\text{opt}}_{\widetilde{r}})^U_\alpha = \frac{2}{\chi^2_{\text{df},\delta}} (\widetilde{T})^U_\alpha \tag{43}$$

or

$$(\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}})^U_\alpha = \frac{2}{\chi^2_{\mathrm{df},\delta}}(\widetilde{T})^L_{1-\alpha}, \tag{44}$$

where the number of degrees of freedom df depends on the failure counting method $\widetilde{r}$. Namely, if $\widetilde{r} \in \{\widetilde{r}_{\mathrm{opt}}, \widetilde{r}_{\mathrm{pes}}, \widetilde{r}_\xi\}$, $\xi \in (0,1)$, then

$$\mathrm{df} = 2\widetilde{r} + 1. \tag{45}$$

If $\widetilde{r} = \widetilde{r}^c$, then

$$\mathrm{df} = 2\lceil\widetilde{r}\rceil + 1, \tag{46}$$

where $\lceil\widetilde{r}\rceil$ stands for the least integer greater than $\widetilde{r}$. Moreover, if $\widetilde{r} = \widetilde{r}^f_{\mathrm{pes}}$, then we get

$$\mathrm{df} = 2\widetilde{r}^U_\alpha + 1, \tag{47}$$

where $\widetilde{r}^U_\alpha = \max\{x \in \mathbb{N} : \mu_{\widetilde{r}^f_{\mathrm{pes}}}(r) \geq \alpha\}$, and $\mu_{\widetilde{r}^f_{\mathrm{pes}}}$ is the membership function of $\widetilde{r}^f_{\mathrm{pes}}$. Finally, if $\widetilde{r} = \widetilde{r}^f_{\mathrm{opt}}$, then we get

$$\mathrm{df} = 2\widetilde{r}^L_\alpha + 1, \tag{48}$$

where $\widetilde{r}^L_\alpha = \min\{x \in \mathbb{N} : \mu_{\widetilde{r}^f_{\mathrm{opt}}}(r) \geq \alpha\}$ and $\mu_{\widetilde{r}^f_{\mathrm{opt}}}$ is the membership function of $\widetilde{r}^f_{\mathrm{opt}}$.

4.3. Testing Hypotheses on *MTTF*

In the present section we will consider the problem of how to design a statistical test to verify a hypothesis that the mean lifetime of the individual under study is no less than a certain fixed value. Thus we are interested in testing a hypothesis H: $\theta \geq \theta_0$, where θ_0 is a given requirement for the mean lifetime, against K: $\theta < \theta_0$.

Grzegorzewski (2000) proposed a general method for deriving fuzzy tests for testing hypotheses with vague data. In the case of testing the one-sided null hypothesis H: $\theta \geq \theta_0$ against K: $\theta < \theta_0$ with vague data $X_1, \ldots, X_n$, $X_i \in \mathbb{FN}$, $i = 1, \ldots, n$, we get a fuzzy test φ: $(\mathbb{FN})^n \to \mathbb{F}(\{0,1\})$ on the significance level δ with the membership function

$$\begin{aligned}\mu_\varphi(t) &= \mu_\Pi(\theta_0)\mathbb{I}_{\{0\}}(t) + \mu_{\neg\Pi}(\theta_0)\mathbb{I}_{\{1\}}(t)\\ &= \mu_\Pi(\theta_0)\mathbb{I}_{\{0\}}(t) + \big(1 - \mu_\Pi(\theta_0)\big)\mathbb{I}_{\{1\}}(t),\\ &\quad t \in \{0,1\},\end{aligned} \tag{49}$$

where Π denotes the one-sided fuzzy confidence interval with the upper limit for the parameter θ on the confidence level $1-\delta$ given by (35) and (36), and $\mathbb{I}$ is an indicator function. Since in our case of vague life data we have to consider different attitudes to the information on the lifetimes and different failure counting methods, we may also construct a desired test in several ways.

First, if we consider exact information on the number of failures r, then we have two tests $\phi^{\mathrm{opt}}, \phi^{\mathrm{pes}}$: $(\mathbb{NFN})^n \to \mathbb{F}(\{0,1\})$ on the significance level δ with the membership functions

$$\begin{aligned}\mu_{\phi^{\mathrm{opt}}}(t) &= \mu_{\Pi^{\mathrm{opt}}}(\theta_0)\mathbb{I}_{\{0\}}(t) + \mu_{\neg\Pi^{\mathrm{opt}}}(\theta_0)\mathbb{I}_{\{1\}}(t)\\ &= \mu_{\Pi^{\mathrm{opt}}}(\theta_0)\mathbb{I}_{\{0\}}(t) + \big(1 - \mu_{\Pi^{\mathrm{opt}}}(\theta_0)\big)\mathbb{I}_{\{1\}}(t),\\ &\quad t \in \{0,1\}\end{aligned} \tag{50}$$

and

$$\begin{aligned}\mu_{\phi^{\mathrm{pes}}}(t) &= \mu_{\Pi^{\mathrm{pes}}}(\theta_0)\mathbb{I}_{\{0\}}(t) + \mu_{\neg\Pi^{\mathrm{pes}}}(\theta_0)\mathbb{I}_{\{1\}}(t)\\ &= \mu_{\Pi^{\mathrm{pes}}}(\theta_0)\mathbb{I}_{\{0\}}(t)\\ &\quad + \big(1 - \mu_{\Pi^{\mathrm{pes}}}(\theta_0)\big)\mathbb{I}_{\{1\}}(t), \quad t \in \{0,1\},\end{aligned} \tag{51}$$

where fuzzy confidence intervals Π^{opt} and Π^{pes}, which correspond to the optimistic and pessimistic attitude to the life data, are given by (41) and (42), respectively.

Second, if we consider imprecise information on the number of failures $\widetilde{r}$ (where $\widetilde{r} \in \{\widetilde{r}_{\mathrm{opt}}, \widetilde{r}_{\mathrm{pes}}, \widetilde{r}_\xi, \widetilde{r}^c, \widetilde{r}^f_{\mathrm{opt}}, \widetilde{r}^f_{\mathrm{pes}}\}$), then we get a family of tests $\phi^{\mathrm{opt}}_{\widetilde{r}}, \phi^{\mathrm{pes}}_{\widetilde{r}} : (\mathbb{NFN})^n \to \mathbb{F}(\{0,1\})$ on the significance level δ with the membership functions

$$\begin{aligned}\mu_{\phi^{\mathrm{opt}}_{\widetilde{r}}}(t) &= \mu_{\widetilde{\Pi}^{\mathrm{opt}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{0\}}(t) + \mu_{\neg\widetilde{\Pi}^{\mathrm{opt}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{1\}}(t)\\ &= \mu_{\widetilde{\Pi}^{\mathrm{opt}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{0\}}(t)\\ &\quad + \big(1 - \mu_{\widetilde{\Pi}^{\mathrm{opt}}_{\widetilde{r}}}(\theta_0)\big)\mathbb{I}_{\{1\}}(t), \quad t \in \{0,1\}\end{aligned} \tag{52}$$

or

$$\begin{aligned}\mu_{\phi^{\mathrm{pes}}_{\widetilde{r}}}(t) &= \mu_{\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{0\}}(t) + \mu_{\neg\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{1\}}(t)\\ &= \mu_{\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}}}(\theta_0)\mathbb{I}_{\{0\}}(t)\\ &\quad + \big(1 - \mu_{\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}}}(\theta_0)\big)\mathbb{I}_{\{1\}}(t), \quad t \in \{0,1\},\end{aligned} \tag{53}$$

where fuzzy confidence intervals $\widetilde{\Pi}^{\mathrm{opt}}_{\widetilde{r}}$ and $\widetilde{\Pi}^{\mathrm{pes}}_{\widetilde{r}}$ are given by (43) and (44), respectively.

As one may expect, $\mu_{\phi^{\mathrm{opt}}}(1) \leq \mu_{\phi^{\mathrm{pes}}}(1)$ and $\mu_{\phi^{\mathrm{opt}}}(0) \geq \mu_{\phi^{\mathrm{pes}}}(0)$ for any life data. Similarly, $\mu_{\phi^{\mathrm{opt}}_{\widetilde{r}}}(1) \leq \mu_{\phi^{\mathrm{pes}}_{\widetilde{r}}}(1)$ and $\mu_{\phi^{\mathrm{opt}}_{\widetilde{r}}}(0) \leq \mu_{\phi^{\mathrm{pes}}_{\widetilde{r}}}(0)$. Moreover, if all life data are crisp, both optimistic and pessimistic fuzzy tests coincide and reduce to the traditional test for one-sided hypotheses.

It is easily seen that in contrast to the classical crisp test, our fuzzy tests do not lead to the binary decision—to accept or to reject the null hypothesis—but to a fuzzy decision. We may get $\varphi = 1/0+0/1$, which indicates that we shall accept H, or $\varphi = 0/0 + 1/1$, which means that H must be rejected. But we may also get $\varphi = \mu_0/0 + (1-\mu_0)/1$, where $\mu_0 \in (0,1)$, which can be interpreted as a degree of conviction that we should accept (μ_0) or reject ($\mu_1 = 1 - \mu_0$) the hypothesis H. We suggest to categorize all the possible outcomes of our fuzzy tests in the following way:

if $0 \le \mu_0 < 0.1$ (i.e., $0.9 < \mu_1 \le 1$)	then	H must be rejected
if $0.1 \le \mu_0 < 0.2$ (i.e., $0.8 < \mu_1 \le 0.9$)	then	H should be rejected
if $0.2 \le \mu_0 < 0.3$ (i.e., $0.7 < \mu_1 \le 0.8$)	then	H may be rejected
if $0.3 \le \mu_0 < 0.4$ (i.e., $0.6 < \mu_1 \le 0.7$)	then	H might be rejected
if $0.4 \le \mu_0 \le 0.6$ (i.e., $0.4 \le \mu_1 \le 0.6$)	then	we do not know what to do
if $0.6 < \mu_0 \le 0.7$ (i.e., $0.3 \le \mu_1 < 0.4$)	then	H might be accepted
if $0.7 < \mu_0 \le 0.8$ (i.e., $0.2 \le \mu_1 < 0.3$)	then	H may be accepted
if $0.8 < \mu_0 \le 0.9$ (i.e., $0.1 \le \mu_1 < 0.2$)	then	H should be accepted
if $0.9 < \mu_0 \le 1$ (i.e., $0 \le \mu_1 < 0.1$)	then	H shall be accepted.

The situation when μ_0 is close to μ_1 was classified as "we do not know what to do." This means that using our data we can neither reject nor accept H. These data are simply too vague.

5. Conclusions

Zadeh's idea of "computing with words" is not a well-defined concept. In one of its interpretations it could be understood as data processing when both input and output data are given in linguistic terms. If we accept this definition, the problem considered in this paper is a practical realization of this idea. Using the example from the area of life-testing, we propose a method for processing imprecise statistical data. First, we proposed a method for the description of statistical data of different type by fuzzy sets. Then we developed algorithms for building estimators, confidence intervals, and statistical decision functions for such data. Finally, we proposed a method to communicate the results of very complicated computations in a user-friendly manner, just by giving advice using a common language.

The method proposed in this paper is not an example of merely number-crunching. It reflects problems which are important while dealing with imprecise data like, e.g., the decision-maker's attitude. The proposed method could be generalized so as to be applied to solving other statistical decision problems.

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