Int. J. Appl. Math. Comput. Sci., 2002, Vol.12, No.3, 449–462

SYNTHESIS AND EVALUATION ANALYSIS OF THE INDICATOR INFORMATION IN NUCLEAR SAFEGUARDS APPLICATIONS BY COMPUTING WITH WORDS

JUN LIU*, DA RUAN**, ROLAND CARCHON***

* Manchester School of Management, UMIST, Manchester, UK
e-mail: j.liu@umist.ac.uk

** Belgian Nuclear Research Centre (SCK●CEN), Boeretang 200, B–2400 Mol, Belgium
e-mail: druan@sckcen.be

*** IAEA, Vienna, e-mail: r.carchon@iaea.org

This paper aims at the handling and treatment of nuclear safeguard relevant information by using a linguistic assessment approach. This is based on a hierarchical analysis of a State's nuclear activities in a multi-layer structure of the evaluation model. After a hierarchical analysis of the State's nuclear activities on the basis of the IAEA Physical Model, the addressed objective is divided into several less complex levels. The overall evaluation can be obtained step by step from those lower levels. Special emphasis is put on the synthesis and evaluation analysis of the Physical Model indicator information. Accordingly, the aggregation process with the consideration of the different kinds of qualitative criteria is in focus. Especially, the symbolic approach is considered by the direct computation on linguistic values instead of the approximation approach using the associated membership function. In this framework, several kinds of ordinal linguistic aggregation operators are presented and analyzed. The application of these linguistic aggregation operators to the combination of the Physical Model indicator information is provided. An example is given to support and clarify the mathematical formalism.

Keywords: safeguards, computing with words, decision-making, physical model

1. Introduction

As a part of its efforts to strengthen international safeguards, including enhancing its ability to detect any undeclared nuclear activities, the International Atomic Energy Agency (IAEA) is using an increased amount of information on some State's nuclear and nuclear-related activities: information provided by the State, information collected by the IAEA, and information from open sources (e.g., media, etc.). The information can be of very different nature, it can be incomplete, imprecise, not fully reliable, conflicting, etc. In order to allow an adequate interpretation of the information and to reach a conclusion on undeclared activities and facilities in the State, there is a need to establish an evaluation method that enables the IAEA to check that there has been no diversion of nuclear material and that there are no undeclared nuclear activities.

Hence, it was considered advantageous to have a mathematical framework available that provides a basis for synthesis across multidimensional information of varying quality, especially to deal with information that may be not quantifiable due to its nature, and that may be imprecise, too complex, ill-defined, etc., for which the traditional quantitative approach (e.g., the statistical approach) does not give an adequate answer.

Our focus is on how to combine the indicator information to get the assurance of the presence of a nuclear process. A flexible and realistic approach is used in this work, i.e., the use of a linguistic assessment based on fuzzy set theory. Fuzzy logic (Bellman and Zadeh, 1970; Zadeh, 1975) provides a systematic way to handle fuzziness and to represent qualitative aspects as linguistic labels by means of linguistic variables, which can be viewed as complementary to traditional methods.

A summary of the work is described in the following steps:

- *Establishment of a hierarchical structure of the evaluation model.* After a hierarchical analysis of the State's nuclear activities on the basis of the IAEA Physical Model, the objective to be evaluated is divided into several less complex levels, and its hierarchical structure is established.
- *Linguistic assessments of vague or imprecise information instead of numerical values.* The symbolic approach acting by direct computation on linguistic terms is applied, where an extended symbolic approach, i.e., the 2-tuple linguistic representation approach, is used to deal with linguistic information without loss of information during the fusion and combination process.

- *Aggregation operators for combining linguistic information.* The operators of combination of the linguistic values are presented and analyzed. These operators are based on the direct computation, and direct application of these aggregation operators to the fusion of safeguards indicator information is provided.

Based on these technical steps, a linguistic evaluation model for strengthened safeguard information based on the symbolic approach is established, where the overall evaluation can be obtained step by step from several lower levels. It would be a multi-level, multi-criteria, multi-expert linguistic evaluation model for strengthened safeguards information.

The paper is organised as follows: In Section 2, an evaluation structure for the State's nuclear activities is outlined based on the IAEA Physical Model. Specific emphasis on and the detailed analysis of the evaluation of the Physical Model indicator information is given in Section 3. The paper is concluded in Section 4.

2. Evaluation Structure for the State's Nuclear Activities Based on the IAEA Physical Model

To provide an effective evaluation, it is necessary to establish a systematic and comprehensive indicator system. A hierarchy structure of the evaluation model of the State's nuclear activities should be established.

The IAEA Physical Model (IAEA, 1999) of the nuclear fuel cycle will be taken as the basis for this task. It includes all the main activities that may be involved in the nuclear fuel cycle, from source material acquisition to the production of weapons-usable materials. The structure of the Physical Model of the nuclear fuel cycle is well developed, i.e., its elements and the interconnections between them are clearly defined. The Physical Model contains detailed narratives describing every known process for accomplishing each given nuclear activity represented in the fuel cycle and the links between them, i.e., it can take into account all the possible technological chains of production of *Pu* and High Enriched Uranium (*HEU*). It also identifies and describes indicators of the existence or the development of a particular process. The indicators include especially designed and dual-use equipment, nuclear and non-nuclear materials, technology/training R&D, and by-products.

The IAEA Physical Model of the nuclear fuel cycle provides a convenient structure for organizing the safeguards relevant information which will be used by IAEA experts to evaluate in a better way the safeguards-related significance of information on some State's activities. The Physical Model may also be used by safeguards inspectors to help them establish what to look for, i.e., indicators of undeclared nuclear activities or misuse of declared facilities.

The hierarchy structure of the evaluation is based on the Physical Model, and it is a multi-layer comprehensive structure. The resultant evaluation structure generally follows the steps that would be involved in the nuclear fuel cycle from source material acquisition to the production of weapons-usable material. The general evaluation structure is illustrated in Fig. 1.

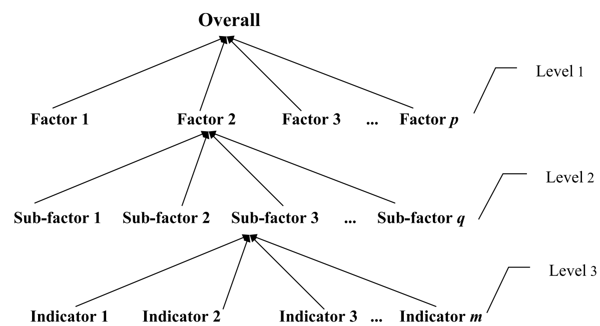

Fig. 1. Structure of the overall evaluation.

The structure of the model has several levels ranging from technologies to specific facilities. Each succeeding level, depending on the order taken, is a detailed version or a generalization of the previous level, which can be described in detail as follows:

Level 1: This level contains all the main activities that may be involved in proliferation. This level is intended to represent the general performance of the nuclear activity of a State: the level of general directions of possible production of *Pu* and *HEU*. It is in fact a technology level of processing nuclear materials, like Enrichment, Fuel Fabrication, Mining and Milling, etc. The elements of this level are linked. They reflect the possible presence of a specific technology in a country. The value of any element of this level is described by a fuzzy linguistic variable. The value of this level will be obtained from Level 2 by using the fuzzy aggregation.

Level 2: Separate processes, like gas centrifuge or Gaseous Diffusion within the enrichment technology. At this level the links between the different technologies for processing nuclear material are clearly seen. The value of any element of this level reflects the State's capability to conduct a specific process at the qualitative level and is described by a fuzzy linguistic variable. The value of this level will be obtained from Level 3 by using the fuzzy aggregation.

Level 3: This is a detailed description of Level 2 and reflects the existence of a specific capacity for processing nuclear materials, i.e., the indicator level. The value of

Table 1. Stratification of the multi-layer evaluation.

Level 0	Level 1	Level 2	Level 3
State	Mining/milling	U from ores, U from sea water, U from monazite, ... *Th* from monazite, *Th* from U ore, ...	$I_{001} - I_{083}$
	Conversion	to *UF*6 to *UF*4 to *UCl*$_4$, ...	$I_{084} - I_{199}$
	Enrichment	Gas centrifuge, Gaseous diffusion, Aerodynamic, Molecular laser, EMLIS, Electromagnetic, chemical exchange, ion exchange atomic vapor laser, plasma separation	$I_{200} - I_{415}$
	Fuel Fabrication	*Umet*, *UO*2, *MOX*	$I_{496} - I_{593}$
	Reactors	*GCR*, *AGR*, *HTGR*, *LWR*, *PWR*, *BWR*, *FBR*	$I_{594} - I_{790}$
	Reprocessing		$I_{841} - I_{914}$

this level qualitatively reflects the potential of the specific facilities used by a country to conduct a specific process for treating nuclear material. The value of any element of this level reflects the possible presence of a specific indicator and is also defined by a fuzzy linguistic variable, which is described and provided by an expert or an analyst. This will be further discussed in Section 3.

As an example, enrichment is a technology of processing nuclear materials which can be divided into independent sub-technologies determined by the nature of the raw materials: *UF*6, *UCl*$_4$, and *Umet*, i.e., Enrichment of *UF*6 (F_1), Enrichment of *UCl*$_4$ (F_2), and Enrichment of *Umet* (F_3). Moreover, each factor is determined by many sub-factors: F_1 is determined by one of the sub-factors, i.e., Gas Centrifuge (F_{11}), Gaseous Diffusion (F_{12}), Aerodynamic (F_{13}), or Molecular Laser (F_{14}); F_2 is determined by one of the sub-factors, i.e., Electromangnetic (F_{21}), Chemical Exchange (F_{22}), or Ion Exchange (F_{23}); F_3 is determined by one of the sub-factors, i.e., Atomic Vapour Laser (F_{31}), or Plasma (F_{32}). Finally, every sub-factor is determined by many indicators including especially designed and dual-use equipment, nuclear and non-nuclear materials, technology/training/R&D, and so on. The practice of this overall evaluation model is given in Table 1, where the different levels are made more explicit and directly applicable to this evaluation problem.

3. Synthesis and Evaluation Analysis of the Physical Model Indicator Information

3.1. Characteristics of the Physical Model Indicator Information

Up to 914 indicators were identified within the IAEA study throughout the whole fuel cycle, from mining to reprocessing, and they can have a different strength, but they are, in one way or another, signs of on-going activities. Indeed, the specificity of each indicator has been designated to a given nuclear activity and is used to determine the *strength* of an indicator. An indicator that is present only if the nuclear process exists or is under development or whose presence is almost always accompanied by a nuclear activity is a *strong* indicator of that activity. Conversely, an indicator that is present for many other reasons, or is associated with many other activities, is a *weak* indicator. In between there are *medium* indicators. As an example, some of the indicators related to Gaseous Diffusion Enrichment are illustrated in Table 2.

The indicators associated with each process are placed in a quasi-logical structure:

– a *strong* indicator: process P implies an indicator x and is implied by the indicator x;

– a *medium* indicator: process P implies an indicator y and the indicator y may imply process P;

– a *weak* indicator: process P may imply an indicator z and the indicator z may imply process P.

For example, consider the special process of Gaseous Diffusion Enrichment. It implies the presence of the indicator of *gaseous diffusion barriers* and is implied by the indicator of *gaseous diffusion barriers*, i.e., the presence of the indicator of *gaseous diffusion barriers* is always accompanied by the process of *Gaseous Diffusion Enrichment*, so the indicator of *gaseous diffusion barriers* is a *strong* one for the process of Gaseous Diffusion Enrichment.

As an example of a medium indicator, consider also the specific process of Gaseous Diffusion Enrichment. It implies the indicator of *gas blowers for* UF6 but is not implied by the indicator of *gas blowers for* UF6, so the indicator of *gas blowers for* UF6 is a *medium* one for the process of Gaseous Diffusion Enrichment .

As an example of a weak indicator, consider the specific process Gaseous Diffusion Enrichment. It may imply the indicator of *Feed system/product and tails withdrawal* and is not implied by the indicator of *Feed system/product and tails withdrawal*, so the indicator of *Feed*

Table 2. Specific indicators of gaseous diffusion enrichment.

Ii	Denomination	Type	Strength
266	Gaseous diffusion barriers	Especially designed equipment	strong
261	Gas blowers for *UF*6	Especially designed equipment	medium
258	Expansion bellows	Dual-use equipment	weak
259	Gasket, large	Dual-use equipment	weak
262	Rotary shaft seal	Especially designed equipment	medium
265	Compressor for pure *UF*6	Especially designed equipment	strong
267	Heat exchanger for cooling pure *UF*6	Especially designed equipment	strong
268	Feed system/product and tails withdrawal	Especially designed equipment	weak
269	Header piping system	Especially designed equipment	weak
271	Chlorine trifluoride	Non-nuclear material	medium
273	Aluminum oxide powder	Non-nuclear material	weak
272	Nickel powder, high purity	Non-nuclear material	medium
276	Large electrical switching yard	Non-nuclear material	weak
277	Large heat increases in air or water	Other	weak
279	Large specific power consumption	Other	weak

system/product and tails withdrawal is a *weak* one for the process *Gaseous Diffusion Enrichment*.

It was considered necessary to have a mathematical framework that provides a basis for synthesis across multidimensional indicator information of varying quality when considering the different strength of an indicator (this means considering indicators in combination). In the following section, we will put special emphasis on the evaluation of the Physical Model indicator information. Here we make use of a linguistic assessment based on fuzzy logic. For example, the assurance value that reflects the capacity of "conducting a specific process at a given nuclear facility" will be determined by the assessment of the "presence of related indicators", which is observed or determined by experts. Usually the assessment values are not limited to Yes or No, since an expert cannot always detect the indicators arising from the process, and instead he/she may only get certain assurance or a possibility of the existence of the indicator, which can be characterized by a fuzzy linguistic variable, and expressed, e.g., as *very low*, *low*, *high*, etc.

3.2. Fuzzy Linguistic Approaches

Here we briefly review some knowledge about fuzzy linguistic approaches:

- **Characterization of the ordinal linguistic term set**

Definition 1. (Zadeh, 1975) A linguistic variable is characterized by a quintuple $(H, T(H), U, G, M)$ in which H is the name of the variable; $T(H)$ (or simply T) denotes the term set of H, i.e., the set of names of linguistic values of H, with each value being a fuzzy variable denoted generically by X and ranging across a universe of discourse U, which is associated with the base variable u; G is a syntactic rule (which usually takes the form of a grammar) for generating the names of values of H; and M is a semantic rule for associating its meaning with each H, $M(X)$, which is a fuzzy subset of U.

The first priority ought to establish what kind of term set to use. Let $S = \{s_i\}$, $i \in \{0, \ldots, m\}$ be a finite and totally ordered term set. Any label, s_i, represents a possible value for a linguistic variable.

The semantics of the finite term set S is given by fuzzy numbers defined in the $[0, 1]$ interval, which are described by their membership functions. Moreover, it must have the following characteristics:

1. The set is ordered: $s_i \leq s_j$ if $i \leq j$.
2. There is a negation operator: $\mathrm{Neg}(s_i) = s_j$ such that $j = m - i$.
3. There is a maximization operator: $\mathrm{Max}(s_i, s_j) = s_i$ if $s_j \leq s_i$.
4. There is a minimization operator: $\mathrm{Min}(s_i, s_j) = s_i$ if $s_i \leq s_j$.

- **Classical fuzzy linguistic approach and the ordinal fuzzy linguistic approach**

The linguistic variables used in the process of Computing with Words imply their fusion, aggregation, comparison, etc. Assuming the proposed linguistic approach, two main different approaches can be found in order to aggregate linguistic values:

I. The linguistic computational approach based on the Extension Principle (Bellman and Zadeh, 1970; Zadeh, 1975), i.e., the approximation approach, uses associated membership functions. The use of an extended arithmetic based on the Extension Principle increases the vagueness of the results. Therefore the fuzzy sets obtained by the linguistic aggregation operators based on the Extension Principle are counts of information that usually do not match any linguistic term (fuzzy set) in the initial term set, so a linguistic approximation process is needed to express the result in the original expression domain.

II. The linguistic computational symbolic approach (or the ordinal fuzzy linguistic approach), acts by direct computation on labels (Delgado *et al.*, 1993; Herrera and Herrera-Viedma, 1997; Yager, 1981; 1993). This kind of methods works assuming that the linguistic term set is an ordered structure uniformly distributed on a scale. Hence the use of membership functions is unnecessary, and these methods are computationally simple and fast. Usually they use the ordered structure of the linguistic term sets, $S = s_i, \quad i \in \{0, \ldots, g\}$, where $s_i < s_j$ if and only if $i < j$, to make direct computation on labels (Delgado *et al.*, 1993). The intermediate results are numerical values, $\alpha \in [0, g]$, which must be approximated in each step of the process by means of an approximation function $\mathrm{app}(\cdot)$ to obtain a value $\mathrm{app}(\alpha) \in \{0, \ldots, g\}$ such that it indicates the index of the associated linguistic term, $s_{\mathrm{app}(\alpha)} \in S$.

Graphically, the scheme of this approach is shown in Fig. 2, where $\mathrm{app}(\cdot)$ is an approximation function used to obtain an index associated with a term in S as a value, e.g., a "round" operator. For a more detailed description of these linguistic computational models, see (Delgado *et al.*, 1993; Herrera and Herrera-Viedma, 1997; 2000; Herrera *et al.*, 2000; Yager, 1981; 1993).

- **Extended symbolic approach based on the 2-tuple representation**

We can see that both of the above computational models have a common important drawback, i.e., the loss of information, caused by the need to express the results in the initial expression domain that is discrete. In the following, a continuous linguistic representation model introduced in (Herrera and Martinez, 1999; 2000) is used. It can express any counting of information although it does not exactly match any linguistic term, i.e., the linguistic information will be represented by means of the 2-tuple, (s, α), where $s(\in S)$ is a linguistic term and $\alpha(\in [-0.5, 0.5))$ is a numerical value, which represents the translation from the original result to the closest index label in the linguistic term set S (called a symbolic translation). In the following, we will recall some basic concepts. For details about the 2-tuple linguistic representation model we refer the reader to (Herrera and Martinez, 1999; 2000).

The reason why we use linguistic 2-tuples is due to the following two aspects:

1. A need for the representation of the expert's judgement in applications (similarly to the questionaire response).

In a real application, the evaluation set is often given on a continuous scale as shown in Fig. 3. The expert needs to draw a cross on this continuous scale to indicate his/her assessment, for example, on the possibility of the presence of a certain indicator. We have to define how to represent the cross indication of assessment. For example, the inspector draws a cross at the point $\beta = 3.8$ on the continuous scale from 0 to 6. How to represent this kind of assessment by using the linguistic information is the first problem that needs to be solved in the evaluation process.

2. The continuous value is often obtained when fusion and combination processes are performed on linguistic variables.

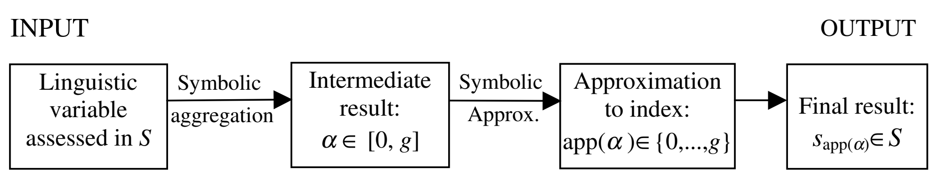

Fig. 2. Symbolic aggregation.

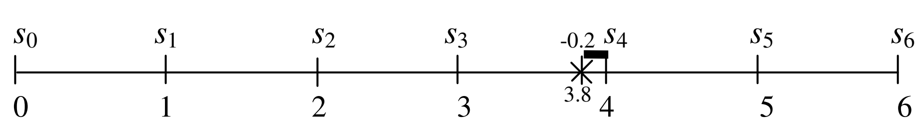

Fig. 3. Example of assessment indication of the inspector.

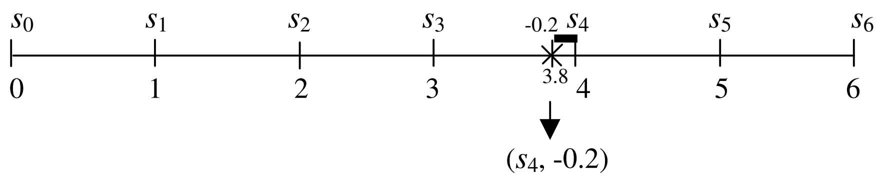

Fig. 4. Example of a symbolic translation (of indication of the inspector).

The symbolic approach acts by direct computation on the labels taking into account only the order and the properties of such linguistic assessments. This method uses a process of approximation together with its computation to obtain the results in the initial term set. In this case, the result usually does not exactly match any of the initial linguistic terms. Then an approximation process must be developed to express the result in the source expression domain (Delgado *et al.*, 1993; Herrera *et al.*, 2000; Herrera and Herrera-Viedma, 2000; Yager, 1981; 1993). This produces the consequent loss of information and hence the lack of precision. For example, let us suppose a symbolic aggregation operation over labels in $S = \{s_1, s_2, \dots, s_7\}$ that obtains as its result $\beta_1 = 4.1$ and $\beta_2 = 4.3$. Note that 4.1 is not equal to 4.3 so, using the round operators, both are equal to s_4. As we can see, the use of "round" leads us to loss of information in the aggregation process.

We shall use a linguistic representation model, which represents the linguistic information by means of 2-tuples $(s_i; \alpha_i)$, $s_i \in S$, $\alpha_i \in [-0.5, 0.5)$. Here s_i represents the linguistic label center of the information and α_i is a numerical value that represents the translation from the original result β to the closest index label in the linguistic term set S (a symbolic translation), i.e., the point $\beta = 3.8$ corresponds to the 2-tuple linguistic term $(s_4, -0.2)$. The assessment would appear as illustrated in Fig. 4.

In fact, the value of the parameter 'α' has the meaning of translation:

A positive value means a translation towards the right label; (s_2, α) has the meaning of s_2 towards s_3.

A negative value means a translation towards the left label; (s_2, α) has the meaning of s_2 towards s_1.

Here α belongs to $[-0.5, 0.5)$ and is associated with a real value from 1.5 to 2.5 (obtained via the aggregation). In this way, we consider the aggregation process in a continuous space, without loss of information in it (the use of "round" leads us to loss of information in the aggregation process).

In fact, the 2-tuple computational model is an extension of the ordinal one which uses as a representation a pair of values to avoid the loss of information, an ordinal value and a numerical translation, and therefore it always obtains at least the same or better results than the ordinal model as its refinement. In addition, due to the limitation of the ordinal symbolic approach, we can only use a few numbers of operators for aggregation. Using the 2-tuple we can use more operators because we can manage them in a continuous domain. There are several advantages of this formalism for representing the linguistic information over classical models, such as the following:

- The linguistic domain can be treated as continuous, while in the classical models it is treated as discrete.
- The linguistic computational model based on linguistic 2-tuples carries out processes of computing with words easily and without loss of information.
- The results of the processes of computing with words are always expressed in the initial linguistic domain.

Taking into account these advantages, we shall use this linguistic representation approach to accomplish our objective.

The following text defines how to convert a classical linguistic term into an equivalent 2-tuple and how to build a 2-tuple from counting of information that does not exactly express the information about a linguistic term.

Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set. If an inspector draws a cross at the point $\beta \in [0, g]$ and $\beta \notin \{0, \dots, g\}$, then an approximation function ϕ is used to express the index of the result in S.

Definition 2. (Herrera and Martinez, 1999; 2000) Let $s_i \in S$ be a linguistic term. Its equivalent 2-tuple representation is obtained by means of the function θ as

$$\theta : S \to S \times [-0.5, 0.5), \quad \theta(s_i) = (s_i, 0)/s_i \in S.$$

The function θ is defined in this way because it is evident that the symbolic translation of any linguistic term in S is 0.

Definition 3. (Herrera and Martinez, 1999; 2000) Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and $\beta \in [0, g]$ a value supporting the result of a symbolic aggregation operation. Then the 2-tuple that expresses the information equivalent to β is obtained with the following function:

$$\phi : [0; g] \to S \times [-0.5, 0.5),$$

$$\phi(\beta) = (s_{\mathrm{round}(\beta),}\alpha = \beta - \mathrm{round}(\beta)), \quad \alpha \in [-0.5, 0.5),$$

where 'round' is the usual round operation, $s_{\text{round}(\beta)}$ has the closest index label to 'β', and 'α' is the value of the symbolic translation.

Definition 4. (Herrera and Martinez, 1999; 2000) Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set and $(s_i; \alpha_i)$ be a 2-tuple. There is always a ϕ^{-1} function, such that from a 2-tuple it returns its equivalent numerical value $\beta \in [0, g]$,

$$\phi^{-1} : S \times [-0.5, 0.5) \rightarrow [0; g], \quad \phi^{-1}(s_i; \alpha_i) = \alpha + i = \beta.$$

The following are the additional necessary concepts of the 2-tuple approach:

- **Comparison of 2-tuples** (Herrera and Martinez, 1999; 2000)
  Let (s_k, α_1) and (s_l, α_2) be two 2-tuples, each representing counting of information. Then
  - If $k < l$ then (s_k, α_1) is less than (s_l, α_2),
  - If $k = l$ then
    1. If $\alpha_1 = \alpha_2$ then (s_k, α_1), (s_l, α_2) represent the same information,
    2. If $\alpha_1 < \alpha_2$ then (s_k, α_1) is less than (s_l, α_2),
    3. If $\alpha_1 > \alpha_2$ then (s_k, α_1) is greater than (s_l, α_2).
- **Negation operator of a 2-tuple** (Herrera and Martinez, 1999; 2000)
  The negation operator over 2-tuples is defined as

  $$\operatorname{Neg}\left((s_i, \alpha)\right) = \phi\left(g - \left(\phi^{-1}(s_i, \alpha)\right)\right),$$

  where g is the cardinality of S, $S = \{s_0, \dots, s_g\}$.

The following evaluation approach is mainly based on this 2-tuple symbolic approach. Now we consider the evaluation principle.

3.3. Evaluation Principles

An evaluation principle can be summarized by the multi-criteria evaluation method to get the overall linguistic assessment value for a given process with the consideration of all the indicators related to this process, as shown in Table 3. Here $E = \{E_1, \dots, E_p\}$ represents the expert activities (detection or assessment is derived from different information sources); $EW = \{EW_1, \dots, EW_p\}$ represents the importance of each expert activity; $I = \{I_{s1}, \dots I_{st}, I_{m1}, \dots I_{mr}, I_{w1}, \dots I_{wk}\}$ represents the indicators related to the process P; $A_{i,j}$ denotes the assessment value of the indicator I_i by an expert activity E_j; F_s represents the set of all *strong* indicators related to P, F_m represents the set of all *medium* indicators related to P, and F_m represents the set of all *weak* indicators related to P; $W = \{w_s, w_m, w_w\}$ represents the

Table 3. Multi-expert, multi-indicator (classified) evaluation matrix for a process P.

		EW_1	EW_2	EW_3	$\cdots$	EW_p
		E_1	E_2	E_3	$\cdots$	E_p
F_s (w_s)	I_{s1}	$A_{s1,1}$	$A_{s1,2}$	$A_{s1,3}$	$\cdots$	$A_{s1,p}$
	I_{s2}	$A_{s2,1}$	$A_{s2,2}$	$A_{s2,3}$	$\cdots$	$A_{s2,p}$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	I_{st}	$A_{st,1}$	$A_{st,2}$	$A_{st,3}$	$\cdots$	$A_{st,p}$
		$D_1(F_s)$	$D_2(F_s)$	$D_3(F_s)$		$D_p(F_s)$
F_m (w_m)	I_{m1}	$A_{m1,1}$	$A_{m1,2}$	$A_{m1,3}$	$\cdots$	$A_{m1,p}$
	I_{m2}	$A_{m2,1}$	$A_{m2,2}$	$A_{m2,3}$	$\cdots$	$A_{m2,p}$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	I_{mr}	$A_{mr,1}$	$A_{mr,2}$	$A_{mr,3}$	$\cdots$	$A_{mr,p}$
		$D_1(F_m)$	$D_2(F_m)$	$D_3(F_m)$		$D_p(F_m)$
F_w (w_w)	I_{w1}	$A_{w1,1}$	$A_{w1,2}$	$A_{w1,3}$	$\cdots$	$A_{w1,p}$
	I_{w2}	$A_{w2,1}$	$A_{w2,2}$	$A_{w2,3}$	$\cdots$	$A_{w2,p}$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
	I_{wk}	$A_{wk,1}$	$A_{wk,2}$	$A_{wk,3}$	$\cdots$	$A_{wk,p}$
		$D_1(F_w)$	$D_2(F_w)$	$D_3(F_w)$		$D_p(F_w)$
		$D_1(A)$	$D_2(A)$	$D_3(A)$	$\cdots$	$D_p(A)$
		$D(A)$				

strength of indicators. $D_i(A)$ means the overall assessment of F_s, F_m, and F_w by E_i when considering the strength of indicators. $D(A)$ means the overall assessment of $D_i(A)$ when considering the importance of each expert activity.

Moreover, notice that the linguistic labels are considered as being in ascending order, from the left to the right, e.g., $S_7 = \{s_0 = \textit{none},\ s_1 = \textit{very low},\ s_2 = \textit{low},\ s_3 = \textit{medium},\ s_4 = \textit{high},\ s_5 = \textit{very high},\ s_6 = \textit{perfect}\}$. Thus we can also meaningfully assign ascending integer values (according to their subscript indices), i.e., $\{0, 1, 2, 3, 4, 5, 6\}$. Hence, for convenience, we will use the numerical expression (which actually corresponds to the index of the linguistic value) instead of the 2-tuple representation in the following discussion. This means that each numerical value in the following actually corresponds to an equivalent 2-tuple linguistic term, e.g., the value $\beta = 3.8$ corresponds to the 2-tuple linguistic term $(s_4, -0.2)$. The corresponding 2-tuple representation can be obtained from Definitions 2–4.

For a case study, we assume that the assessment value and the importance of each expert activity are all taken from the above linguistic term set $S_7 = \{s_0 = \textit{none},\ s_1 = \textit{very low},\ s_2 = \textit{low},\ s_3 = \textit{medium},\ s_4 = \textit{high},\ s_5 = \textit{very high},\ s_6 = \textit{perfect}\}$.

We suppose that the values of $A_{i,j}$ and the importance of the expert activity are initially given by an expert, as these values should be determined according to the results of safeguards expert activities, e.g., A_i, is assessed by experts and expressed with the linguistic values. Furthermore, how to assess and express the strength of indicators is a rather complex problem, which is relevant to the one of how to combine indicators across the whole fuel cycle process. The simple arithmetic "rule based system" which was indicated in the Safeguards field suggested that, as a "rule of thumb,"

$$\begin{aligned} &3 \text{ Medium Indicators} = 1 \text{ Strong Indicator},\\ &9 \text{ Weak Indicators} = 3 \text{ Medium Indicators}\\ &\qquad\qquad\qquad\quad = 1 \text{ Strong Indicator},\\ &3 \text{ Weak plus } 2 \text{ Medium Indicators} = 1 \text{ Strong Indicator}. \end{aligned}$$

Here we consider this kind of rules as a case study and suppose that the strength of indicators is expressed in a numerical value, i.e., Strong =: 9, Medium =: 3, and Weak =: 1. We need not to unify the weights and the assessment values. In the numerical context we can compute a weighted average using the weights belonging to a term set different from the assessment values. The 2-tuple linguistic weighted average acts in the same way, although the weights and the assessment values belong to different term sets. The final results always belong to S because the weights can be normalized such that $\sum_i w_i = 1$.

3.4. Selection of Aggregation Operators for Combining Indicator Information

To manipulate the linguistic information in this context, we shall work with operators for combining the linguistic unweighted and weighted values by direct computation on labels. Specifically, we shall present and analyse the weighted operators of combination of the linguistic values based on direct computation.

In the application here, a basic problem is how to deal with the aggregation of the indicator information. Due to the diversified nature of the strength of indicators, it is necessary to aggregate the indicators with different strengths by using different aggregation operators, some of which are given below. Note that because we use the 2-tuple representation, some aggregation operators in the continuous domain can also be used. The corresponding aggregation operators within the 2-tuple framework are also introduced:

(A) *Minimum aggregation function*: Min,

(B) *Maximum aggregation function*: Max.

It should be noted that neither Min nor Max aggregation operators allow a compensation, i.e., a higher degree of satisfaction of one of the criteria cannot compensate for a lower degree of satisfaction of another criterion. Hence the following mean-type aggregation operators can be adopted:

(C) *The normative approach* (Yager, 1992; 1993). In this approach, the decision-maker adds all the values relating to every alternative, by taking the average of all the values. For the ordinal case, we have the following normative operator:

$$\mathrm{Norm}(A_1, \ldots, A_n) = \mathrm{Max}_j \left[\mathrm{Min}(w_j, b_j)\right],$$

where $A_i, i = 1, \ldots, n$ is the value to be assessed, b_j is the j-th largest value of the A_i, w_j are given such that for $j = 1, \ldots, n$ we have $w_j = s_{T(j)}$ with

$$T(j) = \mathrm{Int}\left(\frac{(m-1)j + (n-m)}{n-1}\right),$$

$\mathrm{Int}(u)$ being the integer part of u, and m the cardinality of the linguistic term set S. Note that Norm is an average-like operator used in the ordinal case.

(D) *The Hurwicz approach* (Dubois and Prade, 1985; Yager, 1992), i.e.,

$$H(A_1, \ldots, A_n) = a\,\mathrm{Max}_i[A_i] + (1-a)\,\mathrm{Min}_j[A_j],$$

where $a \in [0, 1]$. This approach attempts to strike a balance between the Max and Min strategies.

(E) *Non-weighted median aggregation* (Yager, 1993): The process of taking the median requires ordering the arguments and the elements in the middle are significant. Let $C = \{A_1, \ldots, A_n\}$ be a collection of elements drawn from S. If we order the elements in C and denote the result by $\{b_1, \ldots, b_n\}$ such that b_j is the j-th largest value of the A_i in C, then

$$\mathrm{Med}(C) = \begin{cases} b_{\frac{n+1}{2}} & \text{if } n \text{ is odd},\\ b_{\frac{n}{2}} & \text{if } n \text{ is even}. \end{cases}$$

Note that the median operation is simply based on the ordering of the elements, and it is also like the average in that it is a mean-type aggregation.

(F) *Arithmetic Mean (AM)* (Dubois and Prade, 1985; Ruan *et al.*, 1999): Let $C = \{A_1, \ldots, A_n\}$ be a set of numerical values. The arithmetic mean is obtained by dividing the sum of all values by their cardinality, i.e.,

$$AM(C) = \frac{1}{n}\sum_{i=1}^{n} A_i.$$

Due to the continuous nature of the 2-tuple representation, one way to aggregate linguistic 2-tuples may be to use the philosophy of numerical aggregation operators and to extend them to deal with linguistic 2-tuples. To extend both numerical and symbolic aggregation operators to dealing with the 2-tuple representation model, it will be neccesary to employ the functions ϕ and ϕ^{-1}, which are easily used to deal with 2-tuples. On the other hand, the philosophy of symbolic linguistic aggregation operators can also be easily used to deal with 2-tuples.

In the following, several numerical aggregation operators for combining 2-tuples are given.

(D*) *The 2-tuple Hurwicz operator* (H^*). The Hurwicz operator for linguistic information modelled by means of 2-tuples will be the following:

Definition 5. Let $A = \{(r_1, \alpha_1), \dots, (r_n, \alpha_n)\}$ be a set of 2-tuples, $a \in [0, 1]$. The extended Hurwicz operator H for linguistic 2-tuples is computed as

$$H^*\big((r_1, \alpha_1), \dots, (r_n, \alpha_n)\big) = \phi\Big(a \operatorname{Max}_j \left[\phi^{-1}(r_i, \alpha_i)\right] + (1-a) \operatorname{Min}_i \left[\phi^{-1}(r_i, \alpha_i)\right]\Big), \quad i, j = 1, \dots, n$$

(F*) *The 2-tuple Arithmetic Mean* (AM^*). The Arithmetic Mean operator for the linguistic information modelled by means of 2-tuples will be the following:

Definition 6. Let $x = \{(r_1, \alpha_1), \dots, (r_n, \alpha_n)\}$ be a set of 2-tuples. Then their extended Arithmetic Mean AM^* is computed as

$$AM^*\big((r_1, \alpha_1), \dots, (r_n, \alpha_n)\big) = \phi\left(\sum_{i=1}^{n} \frac{1}{n}\phi^{-1}(r_i, \alpha_i)\right) = \phi\left(\frac{1}{n}\sum_{i=1}^{n} \beta_i\right).$$

3.5. Proposed Procedures for Synthesis and Evaluation of Indicator Information

Now we turn to the problem of synthesis and evaluation of indicator information. The evaluation procedure can be summarized in the following different steps:

Step 1: Classification of indicators related to a given process P according to their different strengths, strong (F_s), medium (F_m), and weak (F_w).

Step 2: Aggregation of the indicators within each category.

Class 1 (*aggregation of* F_s). We will get the assessment of "conducting a specific process at a given facility". Assuming that a strong indicator is a sufficient condition (even a necessary condition) for the corresponding process, from the safe point of view, we will propose to use the Max aggregation operator. It aggregates the values on the premise of "maximum assurance or possibility of presence of those indicators." Hence we have

$$D_i(F_s) = \operatorname{Max}(A_{s1,i}, A_{s2,i} \dots, A_{st,i}), \quad i = 1, \dots, p.$$

Class 2 (*aggregation of* F_m). Assuming that a *medium* indicator is a necessary condition (not a sufficient condition) for the corresponding process, it follows that both the indicators with the maximum assurance and those with the minimum assurance are equally important, so we need to consider the Max and Min assurance simultaneously. Accordingly, there are two approaches available for this purpose: the Hurwicz approach (H^*), which attempts to strike a balance between the Max and Min strategies, and the Arithmetic Mean (AM^*), which tries to strike the balance point or center from the set of all values. Note that the Hurwicz approach puts special emphasis on the extreme assurance. In fact, it is considered reasonable to assume that the extreme values play a more important role in the aggregation process than the middle ones for the medium indicator. Hence we propose to use the Hurwicz approach when its parameter $a = 0.5$, which reflects an average of the Max and Min ones, i.e.,

$$D_i(F_m) = H^*(A_{m1,i}, \dots, A_{mr,i}), \quad a = 0.5, \quad i = 1, \dots, p.$$

But the Arithmetic Mean (AM) can still be considered available on the premise of "mean assurance or possibility of presence of those indicators," i.e.,

$$D_i(F_m) = AM^*(A_{m1,i}, \dots, A_{mr,i}), \quad i = 1, \dots, p.$$

Class 3 (*aggregation of* F_w). From the definition of the weak indicator, a single weak indicator has little sense for the overall assessment so that each assurance value of a weak indicator is in the same status as those of other weak indicators. It follows that Max, Min and Med, which take on special values (the extreme value and the middle one, respectively), are not considered reasonable for the aggregation of weak indicators. Also only the Max and Min values are considered in the Hurwicz approach, so the Hurwicz approach is not considered feasible, either. Hence we propose to use the Normative Operator (Norm) and the Arithmetical Mean, which all take the average of all the values. It aggregates the values on the premise of "normative (average) assurance," i.e.,

$$D_i(F_w) = \operatorname{Norm}(A_{w1,i}, A_{w2,i}, \dots, A_{wk,i})$$

or

$$D_i(F_w) = AM^*(A_{w1,i}, A_{w2,i}, \dots, A_{wk,i}), \ i = 1, \dots, p.$$

We use Table 4 to illustrate the aggregation result of indicators within each class by using different aggregation operators and indicate the feasibility of different aggregation operators. Without loss of generality, we use the same example for analysing strong, medium and weak indicators, respectively.

Table 4. Illustration of the aggregation of indicators within each class.

Indicators \ Experts	E_1	E_2	E_3	E_4	E_5	E_6	feasibility or acceptability		
							strong	medium	weak
I_1	1	1	6	3	2	6			
I_2	6	6	1	3	2	6			
I_3	2	1	5	3	2	6			
I_4	3	1	5	3	6	6			
I_5	5	6	5	6 (1)	6	1			
Min	1	1	1	3 (1)	2	1	N	N	N
Max	6	6	6	6 (3)	6	6	Y	N	N
Med	3	1	5	3 (3)	2	6	N	N	N
Norm	3	3	5	3 (3)	3	6	N	N	Y
Hurwicz(H^*) ($a = 0.5$)	3.5	3.5	4.5	4.5 (2)	4	3.5	N	Y	N
Arithmetic Mean (AM^*)	3.4	3	4.4	3.6 (2.6)	3.6	5	N	Y	Y

Suppose that $I_i \quad (i = 1, \dots, 5)$ in Table 4 are all medium indicators. Then the following remarks can be made:

1. For the Med operator, it can be seen from E_2 that $\mathrm{Med}(I_1, \dots, I_5) = 1$, which does not seem reasonable.

2. It was observed that the same results were obtained with the Hurwicz approach in the cases E_1, E_2, and E_3 because they have the same extreme value (Max and Min values). This means that we only strike the balance of Max and Min values and ignore the middle values. For case E_6, this value of I_5 is equal to 1, which would play a more important role than other values (all equal to 6) because I_5 is a necessary condition for a given process. But we can see that $\mathrm{Norm}(E_6) = 6$, it actually does not put more emphasis on I_5, and we have $H^*(E_6) = 3.5$ and $\mathrm{Mean}(E_6) = 5$, which are considered more reasonable. Moreover, considering the case E_4, when $I_5 = 6$, we have $\mathrm{Norm}(E_4) = 3$, $H^*(E_4) = 4.5$, $AM^*(E_4) = 3.6$; when I_5 changes considerably to 1, $H^*(E_4)$ is changed to 2, and $\mathrm{Mean}(E_4)$ is changed to 2.6, which means the H^* and AM^* reflect every change when the input is different without loss of any information. But $\mathrm{Norm}(E_4)$ is still equal to 3, which shows that the Norm operator is not sensitive to the extreme value variation due to its formulation (with several approximate processes, like Max, Min and Round operations). This is also the reason why we skip using the Norm for the aggregation of the medium indicator.

3. Compared with the Hurwicz approach, the Mean takes the same attitude on the value of each medium indicator and the final result is an average one. The Hurwicz approach puts more attention to the extreme Max and Min values.

Please note that here the numerical value equivalently corresponds to the 2-tuple term, e.g., "3.6" corresponds to the 2-tuple term $(s_4, -0.4)$, and "4.5" corresponds to the 2-tuple term $(s_5, -0.5)$.

Step 3: Aggregation of F_s, F_m and F_w by considering the corresponding strength of indicators. We need to use the weighted aggregation operator, i.e.,

$$D_i(A) = \mathrm{Agg}_W \Big((w_s, D_i(F_s)), (w_m, D_i(F_m)), (w_w, D_i(F_w)) \Big), \quad i = 1, \dots, p.$$

Here Agg_W can be taken as a weighted aggregation operator to get a final assessment $D_i(A)$. According to the following analysis, we propose to use the weighted mean operator which aggregates the value on the premise of "mean assurance under consideration of the strength."

The following are some available weighted aggregation operators:

(G) Min-*type weighted aggregation* (W-min) (Yager, 1981; 1993):

$$W\text{-min}\big((w_1, a_1), (w_2, a_2), \dots, (w_n, a_n)\big) = \mathrm{Min}\big(g(w_1, a_1), g(w_2, a_2), \dots, g(w_n, a_n)\big).$$

Here $g(w_i, a_i) = \mathrm{Max}(\mathrm{Neg}(w_i), a_i)$, $\mathrm{Neg}(w_i)$ is the negation of w_I, i.e., $\mathrm{Neg}(w_i) = w_j$ such that $j = m - i$, m is the cardinality of the linguistic term set of the weights.

(H) Max-*type weighted aggregation* (W-max) (Yager, 1981; 1993):

$$W\text{-max}\big((w_1, a_1), (w_2, a_2), \dots, (w_n, a_n)\big) = \mathrm{Max}\big(g(w_1, a_1), g(w_2, a_2), \dots, g(w_n, a_n)\big),$$

where $g(w, a) = \mathrm{Min}(w_i, a_i)$.

Table 5. Illustration of the weighted aggregation of indicators.

Indicators \ Experts	E_1	E_2	E_3	E_4	E_5	E_6	E_7
$D(F_s)$	0	2	3	4	5	6	6
$D(F_m)$	0	0	0	0	0	0	0
$D(F_w)$	0 (6)	0 (6)	0 (6)	0 (6)	0 (6)	0 (6)	0 (6)
W-min	0 (0)	1 (1)	2 (2)	3 (3)	3 (3)	3 (3)	3 (3)
W-max	0 (0)	1 (1)	2 (2)	3 (3)	4 (6)	6 (6)	6 (6)
W-med	0 (0)	1 (1)	2 (2)	3 (3)	3 (3)	3 (3)	3 (3)
W-mean*	0 (2)	0.67 (2.67)	1 (3)	1.33 (3.33)	1.67 (3.67)	2 (4)	2 (4)

(I) Med-*type weighted aggregation* (W-med) (Yager, 1993):

$$W\text{-med}\big((w_1,a_1),(w_2,a_2),\dots,(w_n,a_n)\big) = \mathrm{Med}(a_1^+,a_1^-,a_2^+,a_2^-,\dots,a_p^+,a_p^-),$$

where $a_i^+ = \mathrm{Max}(\mathrm{Neg}(w_i),a_i)$, $a_i^- = \mathrm{Min}(w_i,a_i)$.

(J) *Weighted mean aggregation operator* (W-mean) (Dubois and Prade, 1985; Ruan *et al.*, 1999):

Let $X = \{a_1,\dots,a_n\}$ be a set of numerical values and $W_X = \{w_1,\dots,w_n\}$ be their associated weights such that w_1 corresponds to a_1 and so on. The weighted mean will be

$$W\text{-mean}\big((w_1,a_1),(w_2,a_2),\dots,(w_n,a_n)\big)\frac{\sum_{i=1}^{n} a_i w_i}{\sum_{i=1}^{n} w_i}.$$

The corresponding operator using the linguistic 2-tuples is defined as follows:

Definition 7. Let $x = \{(r_1,\alpha_1),\dots,(r_n,\alpha_n)\}$ be a set of 2-tuples and $W = \{w_1,\dots,w_n\}$ be their associated weights. The extended weighted mean W-mean* is

$$W\text{-mean}^*\big(w_1,(r_1,\alpha_1),\dots,\big(w_n,(r_n,\alpha_n)\big)\big) = \phi\left(\frac{\sum_{i=1}^{n}\phi^{-1}(r_i,\alpha_i)w_i}{\sum_{i=1}^{n} w_i}\right) = \phi\left(\frac{\sum_{i=1}^{n}\beta_i w_i}{\sum_{i=1}^{n} w_i}\right),$$

where ϕ and ϕ^{-1} are given in Definitions 2–4.

We use Table 5 to illustrate the weighted aggregation result of indicators for Step 3 by using different weighted aggregation operators, and to explain the feasibility of different aggregation operators.

Remark 1. From columns E_1 to E_6 in this table, we can see that when $D_m = 0$, the D_s's are all fixed and D_w increases from 0 to 6; there is no difference in the aggregation results by using the different operator W-min, W-max and W-med. This shows that these three weighted aggregation operators are not reasonable. However, the weighted mean results seem reasonable.

Step 4: Aggregation of several detecting activities. Steps 1–3 are a procedure to get the overall assessment by each indicator-detecting activity. In Step 4, we consider the evaluation about the assessment of the process P when considering different importance of each expert activity. Note that the Min-type, Max-type or Med-type weighted aggregation operators will overstate the fused value due to the loss of too much information (as shown in Step 3). There should be a consensus degree of all expert activities. Hence we also propose to use the weighted mean operator to get a final assessment $D(A)$. It aggregates the value on the premise of “mean assurance under consideration of the importance of each expert activity,” i.e.,

$$D(A) = W\text{-mean}^*\Big(\big(EW_1, D_1(F_s)\big), \big(EW_2, D_2(F_m)\big),\dots,\big(EW_p, D_p(F_w)\big)\Big).$$

3.6. Example

As an example, we consider a specific evaluation to illustrate the method proposed here. Let it be required to evaluate the possibility of “conducting a specific process Gaseous diffusion enrichment” within the evaluation of the production of Highly Enriched Uranium (*HEU*) as shown in Table 6.

Although we have described different term sets for strength, importance and the assessment value, we usually have to unify them in order to operate on them. As has already been discussed, we take the weight vector of the indicator as $W_I = (9,3,1)$, and suppose that the importance of the expert activity is also taken from S_7. In Table 6, the

Table 6. Evaluation of the process A – Gaseous diffusion enrichment.

		E_1 (3)	E_2 (5)	E_3 (4)	E_4 (2)
F_s (9)	Compressor for pure *UF*6	4	2	4	6
	Gaseous diffusion barrier	6	5	4	6
	Heat exchanger for cooling pure *UF*6	5	3	6	6
$D(F_s)$ (Max)		6	5	6	6
F_m (3)	Diffuser housing/vessel	3	3	5	4
	Gas blower for *UF*6	3	2	3	6
	Rotary shaft seal	4	3	5	3
	Special control value (large aperture)	3	2	5	5
	Special shut-off value (large apertue)	6	3	4	5
	Chlorine trifluoride	3	2	5	4
	Nickel powder, high purity	2	2	3	4
$D(F_m)$ (Mean)		3.43	2.43	4.29	4.43
$D(F_m)$ (Hurwicz)		4	2.5	4	4.5
F_w (1)	Gasket, large	2	3	5	3
	Feed system/product and tails withdrawal	1	3	2	4
	Expansion bellows	6	6	6	5
	Header piping system	5	3	6	4
	Vacuum system and pump	3	2	1	2
	Alumnium oxide powder	2	2	2	3
	Nickel powder	4	3	6	4
	PTFE(teflon)	3	3	3	2
	Large electrical switching yard	3	6	5	5
	Large heat increase in air or water	6	3	6	4
	Larger specific power consumption	4	3	5	6
	Larger cooling requirements (towers)	3	1	2	1
$D(F_w)$ (Mean)		3.5	3.17	4.08	3.58
$D(F_w)$ (Norm)		3	3	4	4
$D_i(A)$ (max-mean-norm)		4.9	3.98	5.26	5.31
$D_i(A)$ (max-mean-mean)		4.94	4.01	5.26	5.27
$D_i(A)$ (max-H-norm)		5.08	4	5.17	5.33
$D_i(A)$ (max-H-mean)		5.13	4.01	5.17	5.29
$D(A)$ (max-mean-norm)		4.64			
$D(A)$ (max-mean-mean)		4.74			
$D(A)$ (max-H-norm)		4.76			
$D(A)$ (max-H-mean)		4.77			

importance vector EW of E_i $(i = 1, \ldots, 4)$ is taken as (s_3, s_5, s_4, s_2). Here for convenience and without loss of generality, the input values are all considered as integers, although they may not be integer values from the expert's assessment.

Here $D(F_s)(\max)$, $D(F_m)(\text{mean})$ and $D(F_w)$ (mean) stand for the aggregation results in each class by using Max, Mean and Mean, respectively. The others have a similar meaning. $D_i(A)$ (max-mean-norm) means the weighted aggregation of the results obtained from Step 2, where Max, Mean and Norm are applied to the aggregation of strong, medium and weak indicators, respectively. The others have similar meanings. $D(A)$ (max-mean-norm) is the corresponding weighted aggregation result from Step 3. Finally, we can see that the assessment of "conducting a specific process Gaseous diffusion enrichment" is close to s_5, i.e., close to *very high*.

All the results in Table 6 are based on the formulation from Steps 1–4. The calculations were made by hand. Software with a huge amount of data becomes necessary due to many factors and a lot of indicators involved in each process.

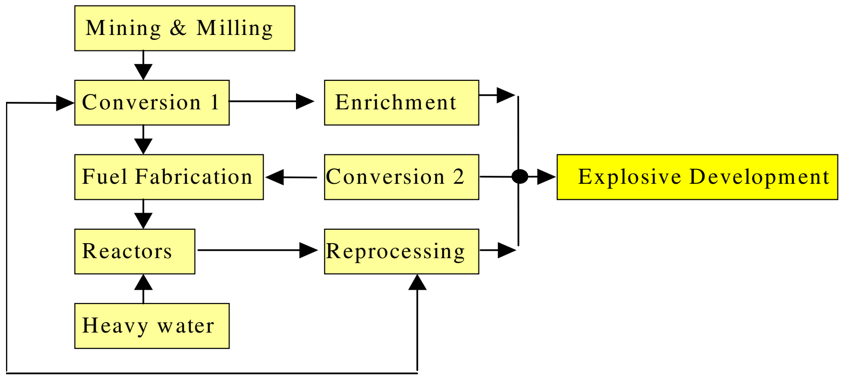

Fig. 5. General lay-out of the evaluation screen.

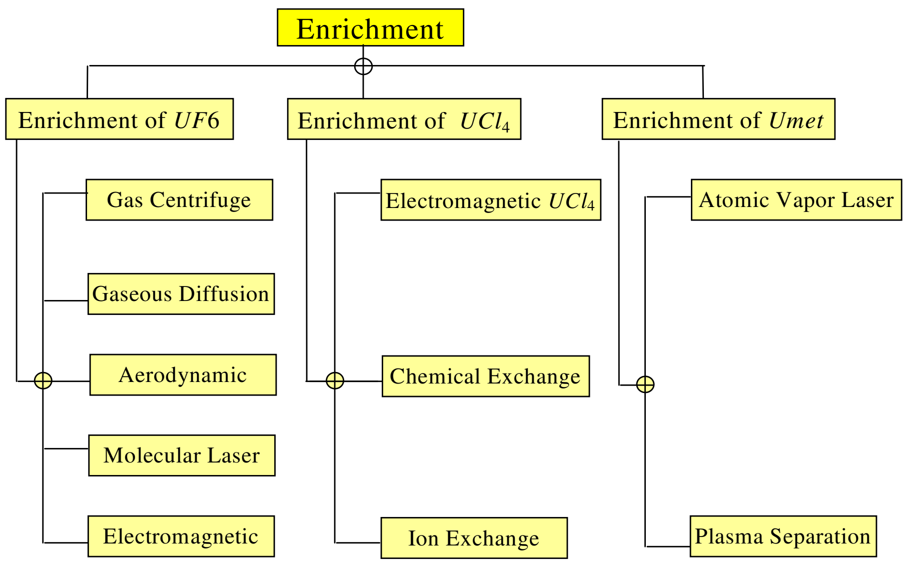

Fig. 6. 'Enrichment" screen.

All the evaluation principles explained in the previous section are implemented in Microsoft EXCEL. It uses several sheets corresponding to the different fuel cycle items that can be addressed from the title page by clicking on the fuel cycle item under investigation, as shown in Fig. 5.

Moreover, when clicking on the appropriate term in the overall sheet, the subsequent level of the term is activated into another sheet, e.g., clicking on "Enrichment" gives the screen of Fig. 6. Note that the symbol $\oplus$ represents the "or" relationship, and the aggregation operator is proposed to take "Max."

If a sub-factor on the second level, e.g., gas diffusion in enrichment, is selected, then a detailed template used to evaluate this process is open and you can use this template file (like Table 6) to input some necessary data to evaluate the overall result on the capacity of "conducting the gas diffusion process at a State." The values for D could be automatically calculated in the approaches mentioned above and be compared with the State Declaration, yielding inconsistency values (or "warning" signals).

4. Conclusions

A mathematical formulation was developed towards decision-making based on information that can be vague, incomplete, conflicting, etc. Computing with words was applied for that purpose.

To manipulate the linguistic information, we worked with aggregation operators for combining the linguistic unweighted and weighted values by direct computation on labels. Based on the above analysis, we presented a multi-criteria, multi-expert evaluation method to get the overall linguistic assurance value for a given process, taking into account the particular nature of the indicators and the specific differences among the experts' activities through the aggregation process. The approach is computationally simple and fast. A case study on the application of these aggregation operators to the fusion of safeguards relevant information is given. A sensitivity study is made to detect in what sense the overall assessment is influenced by the choice of the aggregation operators.

By using this evaluation model of States' nuclear activities, we can assess, on a qualitative level, the States' capabilities on processing nuclear materials. If we focus on the indicators of undeclared nuclear activities, then we can get an assurance of undeclared nuclear activities or misuse of declared facilities in a State. Some relevant works are (Carchon *et al.*, 2000; 2001; Liu *et al.*, 2001a; 2001b; 2001c; Ruan *et al.*, 1999).

References

Bellman R.E. and Zadeh L.A. (1970): *Decision making in a fuzzy environment*. — Manag. Sci., Vol. 17, No. 2, pp. 141–164.

Carchon R., Ruan D., Liu J. and Bruggeman M. (2000): *Application of logical computing methods*. — Proc. 22nd ESARDA Symp. *Safeguards and Nuclear Material Management*, Dresden, Germany, pp. 231–241.

Carchon R., Ruan D. and Liu J. (2001): *A linguistic evaluation model for strength safeguard relevant information*. — Proc. 23rd ESARDA Symp. *Safeguards and Nuclear Material Management*, Bruges, Belgium, pp. 380–387.

Delgado M., Verdegay J.L. and Vila M.A. (1993): *On aggregation operations of linguistic labels*. — Int. J. Intell. Syst., Vol. 8, pp. 351–370.

Dubois D. and Prade H. (1985): *A review of fuzzy sets aggregation connectives*. — Inf. Sci., Vol. 36, No. 1, pp. 85–121.

Herrera F. and Herrera-Viedma E. (1997): *Aggregation operators for linguistic weighted information*. — IEEE Trans. Syst. Man Cybern., Part A, Vol. 27, No. 5, pp. 646–656.

Herrera F. and Martinez L. (2000): *A 2-tuple fuzzy linguistic representation model for computing with words*. — IEEE Trans. Fuzzy Syst., Vol. 8, No. 6, pp. 746–752.

Herrera F. and Martinez L. (1999): *A 2-tuple fuzzy linguistic representation model based on a symbolic translation*. — Proc. EUROFUSE-SIC'99 Conf., Budapest, Hungary, pp. 25–28.

Herrera F., Herrera-Viedma E., and Martnez L. (2000): *A fusion approach for managing multi-granularity linguistic terms sets in decision making*. — Fuzzy Sets Syst., Vol. 114, No. 1, pp. 43–58.

Herrera F. and Herrera-Viedma E. (2000): *Linguistic decision analysis: Steps for solving decision problems under linguistic information*. — Fuzzy Sets Syst., Vol. 115, No. 1, pp. 67–82.

IAEA (1999): *Physical model*. — Int. Atomic Energy Agency, Rep. STR–314, Vienna.

Liu J., Ruan D. and Carchon R. (2001): *A new decision model for nuclear safeguards applications based on linguistic expressions*. — SCK•CEN Rep. No. BLG–873.

Liu J., Ruan D. and Carchon R. (2001): *Synthesis and evaluation analysis of the physical model indicator information if considered in combination*. — SCK•CEN Rep. No. R–3463.

Liu J., Ruan D. and Carchon R. (2001): *A fuzzy evaluation approach for strengthened safeguards relevant information*. — Proc. Joint 9th IFSA World Congress and 20th NAFIPS Int. Conf. *Fuzziness and Soft Computing in the New Millenium*, Vancouver, Canada, pp. 651–656.

Ruan D., Carchon R. and Kerre E.E. (1999): *Aggregation operators: Properties and applications*. — SCK•CEN Int. Rep. No. R–3331.

Yager R.R. (1981): *A new methodology for ordinal multiple aspect decision based on fuzzy sets*. — Dec. Sci., Vol. 12, pp. 589–600.

Yager R.R. (1992): *Applications and extension of OWA aggregation*. — Int. J. Man Mach. Stud., Vol. 37, No. 1, pp. 103–132.

Yager R.R. (1993): *An approach to ordinal decision making*. — Int. J. Approx. Reas., Vol. 12, No. 3, pp. 237–261.

Zadeh L.A. (1975): *The concept of a linguistic variable and its applications to approximate reasoning, Parts I, II, III*. — Inf. Sci., Vol. 8, No. 1, pp. 199–249; No. 2, pp. 301–357; Vol. 9, No. 1, pp. 42–80.