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MODELING AND CONTROL OF INDUCTION MOTORS†

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This paper is devoted to the modeling and control of the induction motor. The well-established field oriented control is recalled and two recent control strategies are exposed, namely the passivity-based control and the flatness-based control.

Keywords: induction motors, field oriented control, passivity based control, flatness based control, nonlinear control

1. Introduction

Induction motors constitute a theoretically interesting and practically important class of nonlinear systems. They are described by a fifth-order nonlinear differential equation with two inputs and only three state variables available for measurement. The control task is further complicated by the fact that induction motors are subject to unknown (load) disturbances and the parameters are of great uncertainty. We are faced then with the challenging problem of controlling a highly nonlinear system, with unknown time-varying parameters, where the regulated output, besides being unmeasurable, is perturbed by an unknown additive signal.

Existing solutions to this problem, in particular the *de facto* industry standard field-oriented control (FOC), were not theoretically well understood. Consequently, no guidelines were available for the designer who had to rely on trial-and-error analysis and intuition for commissioning and high performance applications. These compelling factors, together with the recent development of powerful theoretical tools for analysis and synthesis of nonlinear systems, motivated some control researchers to tackle this problem.

The main purpose of this paper is to review some of the main developments in the field, with particular emphasis on applications of passivity and flatness ideas. We start

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by presenting the physical model of the motor adopting an innovative perspective that underscores the control aspects—*in lieu* of the classical electrical engineering viewpoint. We then review, again from a control theory perspective, the well-known FOC. Connections between this classical technique and passivity ideas have been revealed in the literature; in particular, it has been shown that passivity-based control schemes exactly reduce to FOC under some simplifying modeling assumptions. After reviewing these results, we present the recent developments which rely on the property of flatness of the motor.

The control of induction drives gave rise to a large number of publications which are not possible to be reported in total here. An overview of various aspects can be found in the papers (Bodson *et al.*, 1994; 1995; Chiasson, 1993; 1995; De Luca, 1989; Marino *et al.*, 1993; 1996; 1998; Ortega *et al.*, 1993b; Taylor, 1994).

The paper is organized as follows: Section 2 is devoted to the physical modeling of the induction motor on standard assumptions but in the perspective of the control of the machine. Section 3 gives a novel presentation of the well-established field-oriented control. Section 4 exposes the passivity-based control. Finally, Section 5 introduces the flatness-based approach to the control of the induction motor.

2. Modeling

2.1. Physical Modeling[1]

The induction machine considered here has a three-phase stator and a squirrel-cage rotor which can be represented by a short-circuited three-phase rotor winding (see Fig. 1(a)). We adopt the classical assumptions: linearity of the materials (no saturation), sinusoidal distribution of the field in the air-gap, balanced structure. The vectors relative to stator variables are denoted[2] by $x_{abcs} = (x_{as},\, x_{bs},\, x_{cs})^t \triangleq x_{abc} = (x_a,\, x_b,\, x_c)^t$ and vectors relative to rotor variables are denoted the $x_{abcr} = (x_{ar},\, x_{br},\, x_{cr})^t \triangleq x_{ABC} = (x_A,\, x_B,\, x_C)^t$. Fluxes, currents and voltage are denoted respectively by ψ_{abc}, ψ_{ABC}, i_{abc}, i_{ABC}, v_{abc} and v_{ABC}. The fundamental physical equations of the machine are the relations between fluxes and currents:

$$\psi_{abc} = \boldsymbol{l_s}\, i_{abc} + \boldsymbol{m_{sr}}(\theta)\, i_{ABC}, \tag{1a}$$

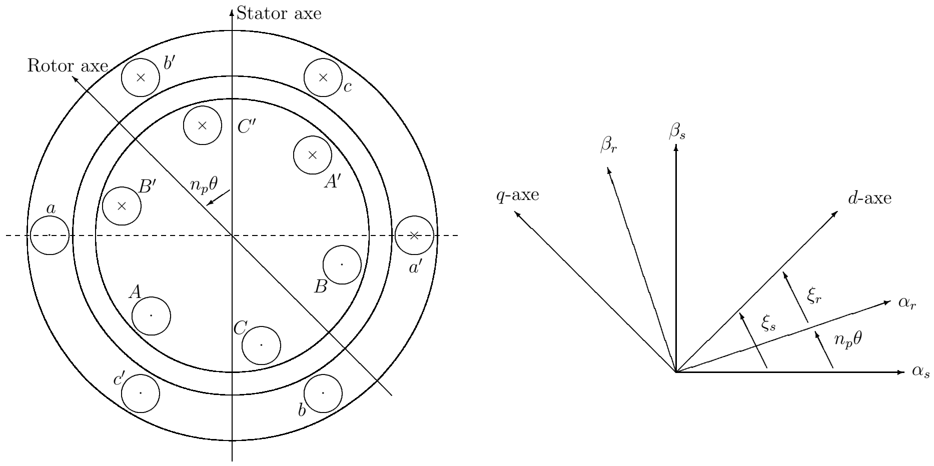
$$\psi_{ABC} = \boldsymbol{m_{rs}}(\theta)\, i_{abc} + \boldsymbol{l_r}\, i_{ABC}, \tag{1b}$$

with

$$\boldsymbol{l_s} = \begin{pmatrix} l_s & m_s & m_s \\ m_s & l_s & m_s \\ m_s & m_s & l_s \end{pmatrix}, \quad \boldsymbol{l_r} = \begin{pmatrix} l_r & m_r & m_r \\ m_r & l_r & m_r \\ m_r & m_r & l_r \end{pmatrix},$$

[1] See (Chatelain, 1983; Semail *et al.*, 1999) for details.

[2] Recall that v^t is the transpose of a vector v.



(a) (b)

Fig. 1. (a) Transverse section; (b) Park transformation.

$$\boldsymbol{m_{sr}}(\theta) = \boldsymbol{m_{rs}}(\theta)^t = M_o \begin{pmatrix} \cos(n_p\theta) & \cos(n_p\theta+\gamma) & \cos(n_p\theta-\gamma) \\ \cos(n_p\theta-\gamma) & \cos(n_p\theta) & \cos(n_p\theta+\gamma) \\ \cos(n_p\theta+\gamma) & \cos(n_p\theta-\gamma) & \cos(n_p\theta) \end{pmatrix},$$

where n_p is the number of pairs of poles, θ signifies the mechanical position of the rotor and $\gamma = 2\pi/3$; the other parameters (inductances) are constant. The second system of equations is composed of the voltage equations:

$$v_{abc} = R_s\, i_{abc} + \frac{\mathrm{d}\psi_{abc}}{\mathrm{d}t}, \tag{2a}$$

$$0 = R_r\, i_{ABC} + \frac{\mathrm{d}\psi_{ABC}}{\mathrm{d}t}. \tag{2b}$$

The final equation is given by the expression of the electromagnetic torque

$$\tau_{\mathrm{em}} = i_{abc}^t \frac{\partial \boldsymbol{m_{sr}}(\theta)}{\partial\theta}\, i_{ABC}. \tag{3}$$

2.2. Algebraic Properties of the Coupling Matrix $\boldsymbol{m_{sr}(\theta)}$

The most important term in eqns. (1) and (3) is the coupling matrix $\boldsymbol{m_{sr}}(\theta)$ which describes the electromechanical conversion. We must detail some of its algebraic properties. Its eigenvalues are 0, $(3/2)M_o e^{\jmath n_p\theta}$ and $(3/2)M_o e^{-\jmath n_p\theta}$. This matrix is diagonalizable, but we prefer first to eliminate the terms relative to the "zero-sequence" component (associated with the eigenvalue equal to zero) because they are almost always zero and do not participate in the energy conversion. Then we will use a real transformation. For this, define the planar rotation matrix P and the *Concordia sub-matrix* T_{32} as follows:

$$P(\xi) = \begin{pmatrix} \cos(\xi) & -\sin(\xi) \\ \sin(\xi) & \cos(\xi) \end{pmatrix}, \quad T_{32} = \sqrt{\frac{2}{3}} \begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & +\frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}^t. \tag{4}$$

We have[3] $T_{32}^t\, T_{32} = I_2$ and the "factorization" $\boldsymbol{m_{sr}} = M\, T_{32}\, P(n_p\, \theta)\, T_{32}^t$, where $M = (3/2)M_o$. Furthermore, T_{32} diagonalizes the matrices like $\boldsymbol{l_s}$ and $\boldsymbol{l_r}$: $\boldsymbol{l_s}\, T_{32} = L_s\, T_{32}$ and $\boldsymbol{l_r}\, T_{32} = L_r\, T_{32}$, with: $L_s = l_s - m_s$ and $L_r = l_r - m_r$. Thus the fundamental equations (1) and (3) can be rewritten as

$$\psi_{abc} = L_s\, i_{abc} + M\, T_{32}\, P\,(n_p\, \theta)\; T_{32}^t\, i_{ABC}, \tag{5a}$$

$$\psi_{ABC} = M\, T_{32}\, P\,(-n_p\, \theta)\; T_{32}^t\, i_{abc} + L_r\, i_{ABC}, \tag{5b}$$

$$\tau_{\text{em}} = n_p\, M\, i_{abc}^t\, T_{32}\, P\left(n_p\, \theta + \frac{\pi}{2}\right) T_{32}^t\, i_{ABC}. \tag{6}$$

2.3. Concordia Transformation

Examining eqns. (5) and (6), we conclude that we do not have six unknown variables, but only four are given by $T_{32}^t\, i_{abc}$ and $T_{32}^t\, i_{ABC}$. Therefore, it is useful to define the *Concordia transformation* applied to all electric variables (voltages v, fluxes ψ, currents i) by $x_{\alpha\beta s} \triangleq (x_{\alpha s},\, x_{\beta s})^t = T_{32}^t\, x_{abc}$ and $x_{\alpha\beta r} \triangleq (x_{\alpha r},\, x_{\beta r})^t = T_{32}^t\, x_{ABC}$. Then the equations of fluxes and torque (5) and (6) can be rewritten as

$$\psi_{\alpha\beta s} = L_s\, i_{\alpha\beta s} + M\, P(+n_p\theta)\, i_{\alpha\beta r}, \tag{7a}$$

$$\psi_{\alpha\beta r} = M\, P(-n_p\theta)\, i_{\alpha\beta s} + L_r\, i_{\alpha\beta r}, \tag{7b}$$

$$\tau_{\text{em}} = n_p\, M\, i_{\alpha\beta s}^t\, P\left(n_p\theta + \frac{\pi}{2}\right) i_{\alpha\beta r}. \tag{8}$$

Furthermore, we have for the voltages:

$$v_{\alpha\beta s} \;=\; R_s\, i_{\alpha\beta s} + \frac{\mathrm{d}\psi_{\alpha\beta s}}{\mathrm{d}t}, \tag{9a}$$

$$0 \;=\; R_r\, i_{\alpha\beta r} + \frac{\mathrm{d}\psi_{\alpha\beta r}}{\mathrm{d}t}. \tag{9b}$$

[3] Recall that I_n denotes the n-dimensional identity matrix.

2.4. Choice of "Useful Variables"

Equations (7) and (8) show that we now have four unknown variables to determine the components of the stator and rotor currents, and we will have to consider only two equations relative to the torque $\tau_{\rm em}$ and to the magnitude of the rotor flux ψ_r. For convenience, we decide to use not exactly the rotor flux, but the so-called magnetizing current $i_{\mu r}$. This new variable, its magnitude i_μ, and its polar angle ξ_r are defined by the transformation:

$$\psi_{\alpha\beta r} \triangleq M\, i_{\mu\alpha\beta r} = M \begin{pmatrix} i_{\mu r}\ \cos(\xi_r) \\ i_{\mu r}\ \sin(\xi_r) \end{pmatrix} = M\, i_{\mu r}\, P(\xi_r)\,(1,\,0)^t. \tag{10}$$

Then it appears that it will be natural to choose the following "useful variables" (physical signification of the future state variables which will be defined in a later section): the stator currents, which are measurable, and the magnetizing current (magnitude and polar angle). We can rewrite the other variables (torque, stator fluxes, rotor currents) with the help of these two vector variables:

$$\tau_{\rm em} \;=\; n_p\, L_m\, i_{\mu r}\, i^t_{\alpha\beta s}\, P\left(n_p\theta + \xi_r + \frac{\pi}{2}\right)\,(1,\,0)^t, \tag{11a}$$

$$\psi_{\alpha\beta s} \;=\; N_1\, i_{\alpha\beta s} + L_m\, i_{\mu r}\, P(n_p\theta + \xi_r)\,(1,\,0)^t, \tag{11b}$$

$$i_{\alpha\beta r} \;=\; \frac{M}{L_r} i_{\mu\alpha\beta r} - P(-n_p\theta)\, i_{\alpha\beta s}, \tag{11c}$$

with the definition of the following new parameters: the *dispersion coefficient*: $\sigma = 1 - M^2/L_s\,L_r$, the *leakage inductance* $N_1 = \sigma\,L_s = \big(L_s - M^2/L_r\big)$, and the *magnetizing inductance* $L_m = (1-\sigma)L_s = M^2/L_r$.

2.5. Park Transformation

Examination of (11a) indicates that it will be much simpler to write it if we make the following transformation for the stator variables (voltages, currents, fluxes):

$$P(-\xi_s)\ (x_{\alpha s},\, x_{\beta s})^t \;=\; (x_d,\, x_q)^t\,, \tag{12a}$$

$$P(-\xi_r)\ (x_{\alpha r},\, x_{\beta r})^t \;=\; (x_D,\, x_Q)^t\,, \tag{12b}$$

$$\xi_s \;=\; \xi_r + n_p\,\theta. \tag{12c}$$

This leads to the following torque equation which has the simplest form

$$\tau_{\rm em} = n_p\, L_m\, i_{\mu r}\, i_q. \tag{13}$$

This transformation is, in fact, a rotation of the axes and the d-axe is given by the direction of the rotor flux. It is known as the *Park transformation* (see Fig. 1(b)).

2.6. State Variables and State Equations

The most practical state variables are the stator currents in the Cartesian representation i_d and i_q (cf. (12a)), and the magnetizing current in the polar form (10). The state equations which determine these state variables are respectively given for the equations deduced from the stator voltages:

$$\frac{\mathrm{d}i_d}{\mathrm{d}t} = \frac{1}{N_1}(v_d - e_d), \tag{14a}$$

$$\frac{\mathrm{d}i_q}{\mathrm{d}t} = \frac{1}{N_1}(v_q - e_q), \tag{14b}$$

with the following definitions for the *back electromotive forces* e_d and e_q:

$$e_d = R_s\, i_d + \frac{L_m}{T_r}(i_d - i_{\mu r}) - \left(n_p\,\omega + \frac{1}{T_r}\frac{i_q}{i_{\mu r}}\right) N_1\, i_q, \tag{15a}$$

$$e_q = R_s\, i_q + \left(n_p\,\omega + \frac{1}{T_r}\frac{i_q}{i_{\mu r}}\right)(N_1\, i_d + L_m\, i_{\mu r}), \tag{15b}$$

where $T_r = L_r/R_r$, and for the equations deduced from the rotor voltages:

$$\frac{\mathrm{d}i_{\mu r}}{\mathrm{d}t} = \frac{1}{T_r}(i_d - i_{\mu r}), \tag{16a}$$

$$\frac{\mathrm{d}\xi_r}{\mathrm{d}t} = \frac{1}{T_r}\frac{i_q}{i_{\mu r}}. \tag{16b}$$

3. Field-Oriented Control[4]

The designers have chosen the following two criteria to have a good control of the IM:

1. Controlling the electromagnetic torque τ_{em}.
2. Controlling the magnitude of the rotor flux ψ_{ABC} : ψ_r.

The problem has now two aspects:

(a) What are the currents needed to impose the torque and the magnitude of the rotor flux? That is, how to "inverse" eqns. (1) and (3) which are algebraic equations? Examining (1) and (3), we see that we have 6 unknown variables (the currents i_{abc} and i_{ABC}) and only 2 equations given by τ_{em} and ψ_r.

(b) What are the voltages which can create the appropriate currents? That is, how to inverse (2) which are differential equations?

Furthermore, we must protect the motor against excessive magnitudes of stator currents.

[4] The classical field orientation is presented in (Blaschke, 1972; Caron and Hautier, 1995; De Fornel, 1990; Grellet and Clerc, 1997; Leonhard, 1997; Vas, 1990). Extensions are given in (Délémontey, 1995; Graf von Westerholt, 1994; Jacquot, 1995; Jelassi, 1991; Mendes, 1993; Robyns, 1993). Our approach ("inversion of models") is mainly deduced from the point of view of (Grenier *et al.*, 1998; Louis and Bergmann, 1999).

3.1. Estimator

If we want to impose a dynamic with the help of a state feedback, it is necessary to determine the variables which are not measurable, i.e., ξ_s, ξ_r and $i_{\mu r}$, with the help of the measurable variables, i.e., stator currents, $i_{\alpha\beta s} = T_{32}^t \, i_{abc}$, and mechanical variables, ω and θ. The equations of the estimator are deduced from eqns. (16a) and (16b):

$$\widehat{\imath}_{dq} = P(-\widehat{\xi}_s) \, T_{32}^t \, i_{abc}, \tag{17a}$$

$$\frac{\mathrm{d}\widehat{\imath}_{\mu r}}{\mathrm{d}t} = \frac{1}{T_r} \left(\widehat{\imath}_d - \widehat{\imath}_{\mu r}\right), \tag{17b}$$

$$\frac{\mathrm{d}\widehat{\xi}_r}{\mathrm{d}t} = \frac{1}{T_r} \frac{\widehat{\imath}_q}{\widehat{\imath}_{\mu r}}, \tag{17c}$$

$$\widehat{\xi}_s = n_p \, \theta + \widehat{\xi}_r, \tag{17d}$$

$$\widehat{\tau_{\mathrm{em}}} = n_p \, L_m \, \widehat{\imath}_{\mu r} \, \widehat{\imath}_q. \tag{17e}$$

The symbol $\widehat{\ }$ denotes the corresponding estimated variables.

3.2. Closed-Loop Control and Introduction of Physical Constraints

For the design of controllers, we have to solve two problems:

1. We want to impose the dynamics and the steady-state behaviors of the two variables of interest: the torque, τ_{em}, and the amplitude of the magnetizing current, $i_{\mu r}$. For this we will apply input-output linearization by state-feedback using the differential equations

$$\frac{\mathrm{d}\tau_{\mathrm{em}}}{\mathrm{d}t} = n_p \, L_m \left(\frac{\mathrm{d}i_{\mu r}}{\mathrm{d}t} \, i_q + \frac{\mathrm{d}i_q}{\mathrm{d}t} \, i_{\mu r} \right), \tag{18a}$$

$$N_1 \, \frac{\mathrm{d}i_{\mu r}}{\mathrm{d}t} + N_1 \, T_r \, \frac{\mathrm{d}^2 i_{\mu r}}{\mathrm{d}t^2} = v_d - e_d, \tag{18b}$$

$$i_{\mu r} + T_r \, \frac{\mathrm{d}i_{\mu r}}{\mathrm{d}t} = i_d. \tag{18c}$$

We choose arbitrarily the following dynamic models which have the lowest order physically realizable (the first for the torque, and the second for the magnetizing current):

$$\frac{\mathrm{d}\tau_{\mathrm{em}}}{\mathrm{d}t} = \frac{1}{\tau_c} \left((\tau_{\mathrm{em}})_{\mathrm{ref}} - \tau_{\mathrm{em}} \right) \triangleq \frac{1}{\tau_c} \, n_p \, L_m \, i_{\mu r} \, \left(I_{q_{\mathrm{ref}}} - i_q \right), \tag{19a}$$

$$i_{\mu r} + \frac{2\,\xi}{\omega_n} \frac{\mathrm{d}i_{\mu r}}{\mathrm{d}t} + \frac{1}{\omega_n^2} \, \frac{\mathrm{d}^2 i_{\mu r}}{\mathrm{d}t^2} = i_{\mu r_{\mathrm{ref}}} \tag{19b}$$

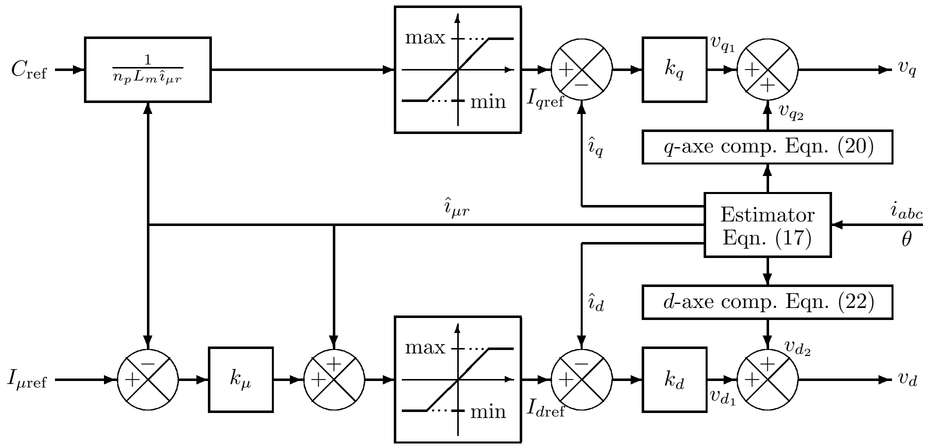


Fig. 2. Control scheme.

2. We have to limit the variations of the stator currents for security during excessive movements. Thus, we want to have a control structure which contains internal loops on the stator currents, as indicated in Fig. 2. Algebraic computations give the following results for the control law:

   (a) On the q-axis, we control the torque and the magnitude of the component i_q of the stator current:

$$v_q = k_q\,(I_{q_{\rm ref}} - \widehat{\imath}_q) + e_q - \frac{\widehat{N}_1}{T_r}\,\frac{\widehat{\imath}_q}{\widehat{\imath}_{\mu r}}\,(\widehat{\imath}_d - \imath_{\mu r}),\quad \text{with } k_q = \frac{\widehat{N}_1}{\tau_c}. \tag{20}$$

   (b) On the d-axis we control the rotor flux $\psi_r = M\,i_{\mu r}$ and the magnitude of the component i_d of the stator current: first, the two-loop structure is given by

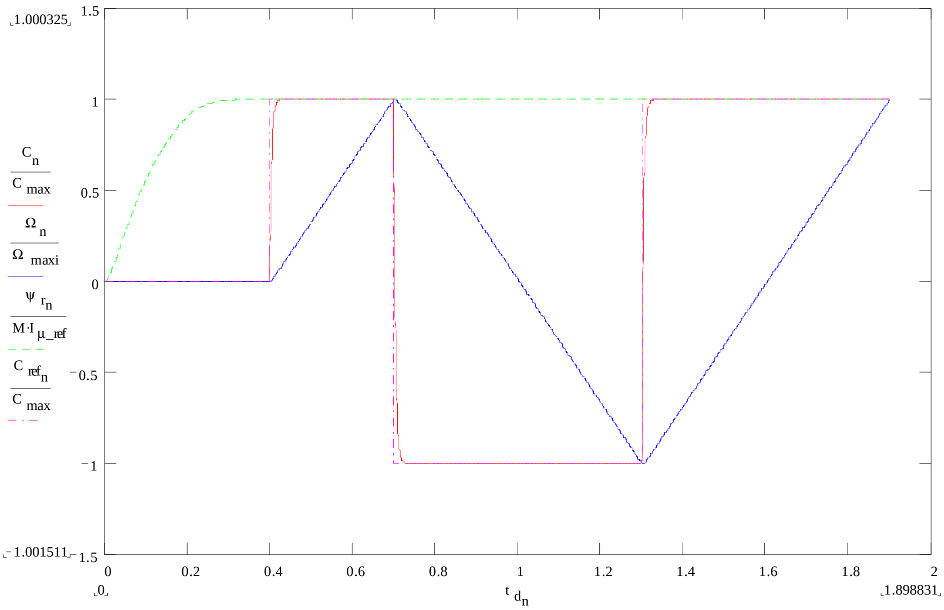
$$I_{d_{\rm ref}} \;=\; k_\mu\,(I_{\mu r_{\rm ref}} - i_{\mu r}) + i_{\mu r},\quad \text{with}\quad k_\mu = \frac{(\omega_n T_r)^2}{2\xi\omega_n T_r - 1}, \tag{21}$$

$$v_d \;=\; k_d\,(I_{d_{\rm ref}} - i_d) + e_d,\quad \text{with}\quad k_d = \frac{N_1}{T_r}\,(2\xi\omega_n T_r - 1)\,. \tag{22}$$

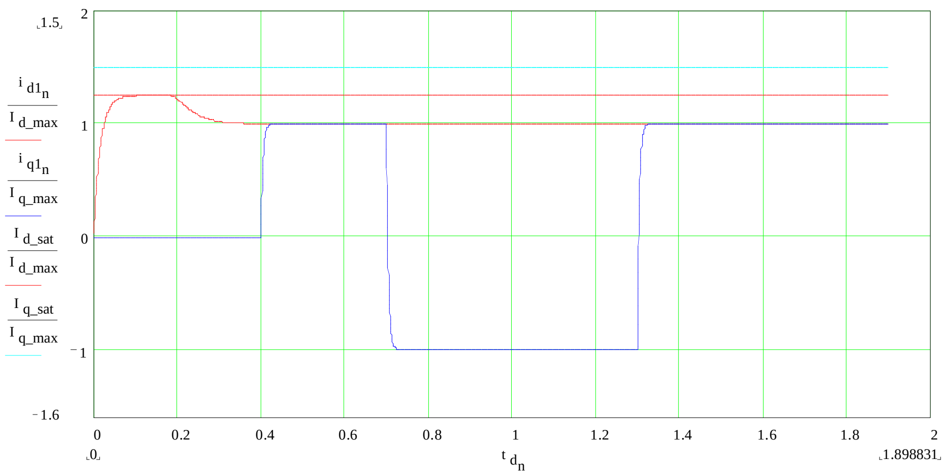
We observe that this structure makes use of proportional controllers like $k_q,\ k_\mu,\ k_d$, and additive compensators like $v_{d_2},\ v_{q_2}$ and $I_{d_{\rm ref2}} = i_{\mu r}$.

3.3. Examples of Transients

Figures 3(a) and (b) show transients which prove that this approach gives a complete inversion of the dynamical model. Figure 3(a) shows three responses: First, the magnetization, i.e., the response of the stator flux to a step reference (dynamics of the second order, given by (19b)). Figure 3(b) shows that the i_d component of the



(a)



(b)

Fig. 3. (a) Flux, torque and speed responses; (b) Responses of i_d and i_q.

stator current reaches a maximum value (denoted by $i_{d_{\mathrm{sat}}}$), but without overshoot: this is a protection effect due to the internal loop. After the magnetization we see the response of the torque to the steps reference: the dynamics is of the first order, given by (19a). The speed ω has quasi-linear responses (the torque is well controlled). We observe that the decoupling between the two axes is perfect: the magnitude of the flux remains constant when the torque has large variations. The inversion of the model is completed.

4. Passivity-Based Control

4.1. Control Properties of the Induction Motor Model

In this section we establish some properties of the model (input-output and geometrical) that will be instrumental for further developments.

4.1.1. Model

In Section 2.3 we have derived the standard two phase $\alpha\beta$-model of an n_p pole pair squirrel cage induction motor with uniform air-gap. For convenience in the sequel, set $\psi = [\psi^t_{\alpha\beta s}, \psi^t_{\alpha\beta r}]^t$ and $i = [i^t_{\alpha\beta s}, i^t_{\alpha\beta r}]^t$.

Thus, the relation between the flux and the currents has the form

$$\psi = L(\theta) i, \quad L(\theta) = \begin{bmatrix} L_s I_2 & M \mathrm{e}^{\mathcal{J} n_p \theta} \\ M \mathrm{e}^{-\mathcal{J} n_p \theta} & L_r I_2 \end{bmatrix}, \tag{23}$$

$L(\theta) = L^t(\theta) > 0$ being the 4×4 inductance matrix of the windings, where we have defined the (skew-symmetric) rotation matrices $\mathcal{J} = -\mathcal{J}^t = P(\pi/2)$ and $\mathrm{e}^{\mathcal{J} n_p \theta} = P(n_p \theta)$ (see (4)). The electrical dynamics are defined by the voltage balance equation

$$\dot{\psi} + Ri = Nu, \quad R = \begin{bmatrix} R_s I_2 & 0 \\ 0 & R_r I_2 \end{bmatrix}, \quad N = \begin{bmatrix} I_2 \\ 0 \end{bmatrix}, \tag{24}$$

with $R_s, R_r > 0$ being the stator and rotor resistances, respectively. Particularly useful for further developments is the following relationship between rotor fluxes and rotor currents:

$$\dot{\psi}_r + R_r i_r = 0 \tag{25}$$

The model is completed by computing the electromagnetic torque as

$$\tau_{\mathrm{em}} = \frac{1}{2} i^t \frac{\partial L(\theta)}{\partial \theta} i = -\frac{1}{2} \psi^t \frac{\partial L^{-1}(\theta)}{\partial \theta} \psi \tag{26}$$

and replacing it in the mechanical dynamics

$$J\ddot{\theta} = \tau_{\mathrm{em}} - \tau_L, \tag{27}$$

where $J > 0$ is the inertia of the rotor and τ_L signifies a term of load torque which we will assume to be constant but unknown. For simplicity, we neglect the effect of friction but, as shown in Ortega *et al.* (1998), it can be easily accommodated into our analysis.

Remark. We established in (Espinosa and Ortega, 1995) that PBC is coordinate independent, i.e., it can be derived in any reference frame chosen for model representation. For instance, in (Espinosa and Ortega, 1994) the *ab*-model was used, while the developments of Ortega and Espinosa (1993) and Ortega *et al.* (Ortega *et al.*, 1993a) relied on the *dq*-model. Finally, the work of Nicklasson *et al.* (1997) was carried-out in the original $\alpha\beta$ frame.

4.1.2. Input-Output Properties

The *cornerstone* of the passivity-based design philosophy is to reveal the passivity property of the system and identify, as a by-product, its *workless forces*. This is easily established from the system total energy, which for the induction motor is given as

$$\mathcal{H}(\psi,\dot\theta,\theta) = \underbrace{\frac{1}{2}\psi^t L^{-1}(\theta)\psi}_{\mathcal{H}_e} + \underbrace{\frac{1}{2}\mathcal{J}\dot\theta^2}_{\mathcal{H}_m},$$

where $\mathcal{H}_e(i,\theta)$ and $\mathcal{H}_m(\dot\theta)$ denote the electrical energy and the mechanical kinetic co-energy, respectively. We have neglected the capacitive effects in the windings of the motor and considered a rigid shaft, hence the potential energy of the motor is zero.

The rate of change in the energy (the *system work*) is given by

$$\dot{\mathcal{H}} = i_s^t u - \dot\theta\tau_L - i_s^t R i_s.$$

From the integration of the equation above we obtain the energy balance

$$\underbrace{\mathcal{H}(t)-\mathcal{H}(0)}_{\text{stored energy}} = \underbrace{-\int_0^t i^t(s)\ R\, i(s)\,\mathrm{d}s}_{\text{dissipated}} + \underbrace{\int_0^t \Big[i_s^t(s)u(s) - \dot\theta(s)\tau_L\Big]\,\mathrm{d}s}_{\text{supplied}-\text{extacted}} \tag{28}$$

which proves that the mapping $[u^t, -\tau_L]^t \mapsto [i_s^t, \dot\theta]^t$ is passive, with storage function $\mathcal{H}$.

Furthermore, as shown by Espinosa and Ortega (1994), the motor model can be decomposed as a feedback interconnection of two passive operators with storage functions $\mathcal{H}_e$ and $\mathcal{H}_m$, respectively. These passivity properties, and their corresponding storage functions, are the basis for two different PBCs studied in (Ortega *et al.*, 1998).

4.1.3. Geometric Properties

We now exhibit an invertibility property of the induction motor model which is essential for obtaining an explicit expression of the PBC. From (26) and (23), we see that the torque can be written as

$$\tau_{\mathrm{em}} = n_p M i_s^t \mathcal{J} e^{\mathcal{J} n_p \theta} i_r, \tag{29}$$

where the fact that $\mathcal{J}$ and $\mathrm{e}^{\mathcal{J}n_p\theta}$ commute ($\mathcal{J}\mathrm{e}^{\mathcal{J}n_p\theta} = \mathrm{e}^{\mathcal{J}n_p\theta}\mathcal{J}$), and that the skew-symmetry of $\mathcal{J}$ ($\mathcal{J}^t = -\mathcal{J} \to x^t\mathcal{J}x = 0,\ \forall\ x \in \mathbb{R}^2$) have been used. Now, solving (25) for i_r and substituting it into (29) gives

$$\tau_{\mathrm{em}} = n_p \frac{M}{L_r} i_s^t \mathcal{J} e^{\mathcal{J}n_p\theta}\psi_r. \tag{30}$$

Finally, (23) can be solved for i_s as

$$i_s = \frac{1}{M} e^{\mathcal{J}n_p\theta}(\psi_r - L_r i_r)$$

and then substituted into (30) to give

$$\tau_{\mathrm{em}} = \frac{n_p}{R_r}\dot{\psi}_r^t \mathcal{J}\psi_r, \tag{31}$$

where (25) has been used again. This is a key expression that allows us to invert the systems dynamics, that is, explicitly solve this equation as

$$\dot{\psi}_r = \frac{\tau_{\mathrm{em}}}{\|\psi_r\|}\frac{R_r}{n_p}\mathcal{J}\psi_r, \tag{32}$$

where $\|\cdot\|$ is the Euclidean norm.

The two equations above will be instrumental in the next section for the derivation of the PBC. In (Nicklasson *et al.*, 1997), where we study the model of the generalized rotating machine, we assume that the machine is Blondel-Park transformable to ensure this invertibility property. The underlying fundamental assumption for the machine to be Blondel-Park transformable is that the windings are sinusoidally distributed, giving a sinusoidal air-gap magnetomotive force and sinusoidally varying elements in the inductance matrix $L(\theta)$. For a practical machine, this means that the magnetomotive force can be suitably approximated with the first harmonic in a Fourier approximation. Examples of machines in which higher-order harmonics must be taken into account are square-wave brushless DC motors and machines with significant saliency in the air gap. For this class of machines the application of PBC is still an open issue.

Equation (32) also shows that the zero dynamics of the motor with outputs τ_{em} and $\|\psi_r\|$ are periodic. This fact becomes clearer if we evaluate the angular speed of the rotor flux vector with respect to the rotor fixed frame (the *slip speed*) as

$$\begin{aligned}\dot{\rho} &= \frac{\mathrm{d}}{\mathrm{d}t}\arctan(\psi_{r2}/\psi_{r1}) = \frac{1}{1+(\psi_{r2}/\psi_{r1})^2}\frac{\dot{\psi}_{r2}\psi_{r1} - \psi_{r2}\dot{\psi}_{r1}}{\psi_{r1}^2} \\ &= \frac{1}{\|\psi_r\|^2}\dot{\psi}_r^t\mathcal{J}\psi_r = \frac{R_r}{n_p\|\psi_r\|^2}\tau_{\mathrm{em}}.\end{aligned} \tag{33}$$

From this equation we conclude that if τ_{em} and $\|\psi_r\|$ are fixed to constant values, the rotor flux rotates at a constant speed. This expression also shows that the torque can be controlled by controlling the rotor flux norm and slip speed, as is well-known in the drives community.

4.2. Nested-Loop Passivity-Based Control

It is shown in Ortega *et al.*, 1998 that for electromechanical systems the PBC approach can be applied in at least *two different ways*, leading to different controllers. In the first, more direct form, a PBC is designed for the whole electromechanical system using as the storage function the total energy of the whole system. This is the way PBCs are typically defined for mechanical and electrical systems, and it is usually referred to as *PBC with total energy shaping*.

Another route stems from the application of a passive subsystems decomposition to the electromechanical system. Namely, we show that (under some reasonable assumptions) we can decompose the system into its electrical and mechanical dynamics, where the latter can be treated as a "passive disturbance." We then design a PBC *for the electrical subsystem* using as the storage function only the electrical part of the system's total energy. An outer-loop controller (which can also be a PBC, but here is a simple pole-placement) is then added to regulate the mechanical dynamics. The so-designed controller will be called the *nested-loop PBC*. There are at least three motivations for this approach: firstly, using this feedback-decomposition leads to simpler controllers which in general do not require observers. Secondly, there is typically a time-scale separation between the electrical and the mechanical dynamics. Finally, since the nested-loop configuration is the prevailing structure in practical applications, we can in some important cases establish a clear connection between our PBC and current practice.

Although for both controllers we can prove global asymptotic speed/position tracking, for the sake of brevity we present here only the torque tracking version of the nested-loop PBC.

4.2.1. Controller Structure

In this section we solve the speed-position tracking problem adopting a nested-loop (i.e., cascaded) scheme, where $\mathcal{C}_{il}$ is an inner-loop torque tracking PBC, and $\mathcal{C}_{ol}$ is an outer-loop speed controller which generates the desired torque[5] $\tau_{\mathrm{em}d}$. We will show in this section that $\mathcal{C}_{ol}$ may be taken as an LTI system that asymptotically stabilizes the mechanical dynamics. The main technical obstacle for its design stems from the fact that $\mathcal{C}_{il}$ requires the knowledge of $\dot{\tau}_{\mathrm{em}d}$, and this in turn implies measurement of acceleration. To overcome this obstacle, we proceed as in (Ortega *et al.*, 1998) for a robotics problem, and replace the acceleration by its approximate differentiation, while preserving the global stabilization property. (In simple applications, of course, $\mathcal{C}_{ol}$ is just a PI around speed error. We go here through these additional complications to provide a complete proof of stability.)

A very interesting property of the resulting scheme, which is further elaborated below, is that if the inverter can be modeled as a current source and the desired speed and rotor flux norm are constant, the controller exactly reduces to the well-known

[5] We will adopt throughout the following notation convention. If a signal is explicitly given as an external reference, we denote it by $(\cdot)_*$. If, instead, it is generated by the controller, we use the notation $(\cdot)_d$.

indirect field-oriented control, hence providing a solid theoretical foundation to this popular control strategy.

4.2.2. Torque Tracking PBC

Implicit and explicit forms. In this subsection we derive a torque tracking PBC from the perspective of a system's inversion. For that purpose, using (23), we rewrite (24) and (26) as

$$\dot{\psi} + RL^{-1}(\theta)\psi \;=\; Nu, \tag{34}$$

$$\tau_{\mathrm{em}} \;=\; \frac{n_p}{R_r}\dot{\psi}_r^t \mathcal{J}\psi_r, \tag{35}$$

where, for ease of reference, we have repeated (31).

Typically, the PBC is a "copy" of the electrical dynamics of the motor (34), (35) with an additional *damping injection* term that improves the transient performance. To simplify the presentation, we will omit the damping injection here, and refer the reader to Ortega *et al.*, 1998. Thus, we define the PBC in an *implicit* form as

$$Nu \;=\; \dot{\psi}_d + RL^{-1}(\theta)\psi_d, \tag{36}$$

$$\tau_{\mathrm{em}*} \;=\; \frac{n_p}{R_r}\dot{\psi}_{rd}^t \mathcal{J}\psi_{rd}, \tag{37}$$

where $\tau_{\mathrm{em}*}$ is the torque reference and $\psi_d = [\psi_{sd}^t, \psi_{rd}^t]^\top$ defines the desired values for the fluxes.

An *explicit* realization of the PBC above is obtained by "inversion" of (37) as

$$\dot{\psi}_{rd} = \frac{1}{\beta_*^2(t)}\left(\frac{R_r}{n_p}\tau_{\mathrm{em}*}\mathcal{J} + \dot{\beta}_*(t)\beta_*(t)I_2\right)\psi_{rd},$$

where $\psi_{rd}(0) = [\beta_*(0), 0^\top]$, and $\beta_*(t)$ is a (time-varying) reference for $\|\psi_r\|$. The last equation can actually be solved as

$$\psi_{rd} \;=\; e^{\mathcal{J}\rho_d}\begin{bmatrix}\beta_*(t)\\ 0\end{bmatrix}, \tag{38}$$

$$\dot{\rho}_d \;=\; \frac{R_r}{n_p\beta_*^2(t)}\tau_{\mathrm{em}*}, \quad \rho_d(0) = 0, \tag{39}$$

The description of the controller is completed by the replacement of ψ_{rd} and $\dot{\psi}_{rd}$ in the last two equations of (36) to get ψ_{sd}. After differentiation we get $\dot{\psi}_{sd}$ which can be replaced in the first two equations of (36) to get[6]

$$u = \dot{\psi}_{sd} + \begin{bmatrix} I_2 & 0 \end{bmatrix} R_s L^{-1}(\theta)\psi_d.$$

[6] An explicit state space description is given in Proposition 1.

The expression above causes a difficulty in the implementation of the nested-loop scheme, as the control law depends on $\dot{\psi}_{sd}$, which in turn depends on $\dot{\tau}_{\mathrm{em}_*}$. On the other hand, the signal $\tau_{\mathrm{em}*}$ will now be generated by an outer-loop controller $\mathcal{C}_{ol}$, which will generally depend on $\dot{\theta}$. We will see in Proposition 1 how to overcome this obstacle with the use of a linear filter.

Stability. Let us now analyze the *stability* of the closed loop. The error equation for the fluxes is obtained from (34) and (36) as

$$\dot{\tilde{\psi}} + RL^{-1}(\theta)\tilde{\psi} = 0,$$

where $\tilde{\psi} \triangleq \psi - \psi_d$ are the flux errors. Global convergence can be easily established by considering the storage function[7]

$$H_\psi = \frac{1}{2}\tilde{\psi}^t R^{-1}\tilde{\psi} \geq 0$$

whose derivative satisfies

$$\dot{H}_\psi = -\tilde{\psi}^t L^{-1}(\theta)\tilde{\psi} \leq -\alpha H_\psi$$

for some $\alpha > 0$. Hence, $\tilde{\psi} \to 0$ exponentially fast.

To illustrate the second difficulty in the stability analysis of the nested-loop scheme, let us turn our attention to the torque tracking error $\tilde{\tau}_{\mathrm{em}} \triangleq \tau_{\mathrm{em}} - \tau_{\mathrm{em}*}$. After some simple operations, from (35) and (37) we get

$$\tilde{\tau}_{\mathrm{em}} = \frac{n_p}{R_r}\left\{\dot{\tilde{\psi}}_r^t \mathcal{J}\tilde{\psi}_r + \dot{\tilde{\psi}}_r^t \mathcal{J}\psi_{rd} + \dot{\psi}_{rd}\mathcal{J}\tilde{\psi}_r\right\}.$$

We have shown above that $\tilde{\psi} \to 0$ (exp.), and consequently, $\dot{\tilde{\psi}} \to 0$. Also, ψ_{rd} is bounded by construction, see (38). Unfortunately, we cannot prove that $\dot{\psi}_{rd}$ is bounded, unless $\tau_{\mathrm{em}*}$ is bounded. In position-speed control, $\tau_{\mathrm{em}*}$ is not *a priori* bounded, since it will be generated by $\mathcal{C}_{ol}$. Therefore, $\mathcal{C}_{ol}$ must be chosen with care and a new argument should be invoked to complete the proof. The proposition below shows that $\mathcal{C}_{ol}$ can be taken as a linear filter.

4.2.3. Speed Tracking PBC

Main result. A globally stable speed tracking PBC is presented in the proposition below, whose proof can be found in Ortega *et al.*, 1998.

Proposition 1. *The nonlinear dynamic output feedback nested-loop controller*

$$u = \underbrace{L_s\dot{i}_{sd} + M\mathrm{e}^{\mathcal{J}n_p\theta}\dot{i}_{rd} + n_pM\mathcal{J}\mathrm{e}^{\mathcal{J}n_p\theta}\dot{\theta}i_{rd}}_{\dot{\psi}_{sd}} + R_si_{sd} \tag{40}$$

[7] This function was used in (Martin and Rouchon, 1996a) to give an "implicit observer" interpretation of the PBC controller.

with

$$i_d = \begin{bmatrix} \dfrac{1}{M}\left[\left(1+\dfrac{L_r\dot{\beta}_*}{R_r\beta_*}\right)I_2 + \dfrac{L_r}{n_p\beta_*^2}\tau_{\mathrm{em}\,d}\mathcal{J}\right]\mathrm{e}^{\mathcal{J}n_p\theta}\psi_{rd} \\ -\left(\dfrac{\tau_{\mathrm{em}\,d}}{n_p\beta_*^2}\mathcal{J} + \dfrac{\dot{\beta}_*}{R_r\beta_*}I_2\right)\psi_{rd} \end{bmatrix}, \tag{41}$$

where

$$\tilde{i} = \begin{bmatrix} \tilde{i}_s \\ \tilde{i}_r \end{bmatrix} = \begin{bmatrix} i_s - i_{sd} \\ i_r - i_{rd} \end{bmatrix},$$

$$\tau_{\mathrm{em}\,d} = J\ddot{\theta} * -z + \tau_L,$$

and controller state equations

$$\dot{\psi}_{rd} = \left(\frac{R_r}{n_p\beta_*^2}\tau_{\mathrm{em}\,d}\mathcal{J} + \frac{\dot{\beta}_*}{\beta_*}I_2\right)\psi_{rd}, \tag{42}$$

$$\dot{z} = -az + b\dot{\tilde{\theta}} \tag{43}$$

with $\dot{\tilde{\theta}} \triangleq \dot{\theta} - \dot{\theta}_*$ *and* $a, b > 0$, *provides a solution to the speed and rotor flux norm tracking problem. That is, when placed in the closed loop with (23) and (24), eqns. (26) and (27) ensure*

$$\lim_{t\to\infty}\dot{\tilde{\theta}} = 0, \quad \lim_{t\to\infty} \left|\, \|\psi_r\| - \beta_*(t)\right| = 0$$

for all initial conditions and with all internal signals uniformly bounded.

Extensions.

- *Position control.* It is easy to see that choosing the desired torque in the controller above as

$$\tau_{\mathrm{em}\,d} = J\ddot{\theta}_* - z - f\tilde{\theta} + \tau_L \tag{44}$$

  yields global asymptotic *position* tracking for all positive values of a, b, f. The proof of global asymptotic rotor flux norm and position tracking follows *verbatim* from the proof of the main result above.

- *Adaptation of load torque.* We can extend the result given in Proposition 1 to the case of unknown but linearly parameterized load,

$$\tau_L = \eta^t\phi(\theta, \dot{\theta}),$$

  where $\eta \in \mathbb{R}^q$ is a vector of unknown constant parameters and $\phi(\theta, \dot{\theta})$ is a measurable regressor.

- *Integral action in stator currents.* It is common in applications to add an integral loop around the stator current errors to the input voltages. The experimental evidence presented Ortega *et al.*, 1998 shows that, indeed, this robustifies the PBC by compensating for unmodeled dynamics. It is interesting to note that the global tracking result above still holds in this case.

5. Flatness-Based Control

5.1. Structural Properties of the Model

5.1.1. Complex Form of the Model

For simplicity, we prefer to work with a complex[8] model instead of the real one given by (7)–(9). To this end, for any variable x introduce the notation

$$\underline{x}_\bullet = x_{\alpha\bullet} + \jmath x_{\beta\bullet}, \tag{45}$$

where we set $\bullet = s$ or $\bullet = r$ depending on whether one considers a stator or a rotor variable, respectively. To simplify the proof of flatness, it is useful to consider some variables in the frame rotating at the speed $n_p\omega$ which is the natural frame to consider variables of the rotor. In order to distinguish the value of a given variable between being referenced in the fixed frame and in the rotating frame, we mark the variable with a tilde when it is given in the rotating frame; otherwise, it is referenced in the fixed frame which is the natural frame for considering the variable of the stator.

Therefore, eqns. (7a) and (7b) take on the forms

$$\underline{\psi}_s = L_s\,\underline{i}_s + M\,\mathrm{e}^{\jmath n_p\theta}\,\tilde{\underline{i}}_r, \tag{46a}$$

$$\tilde{\underline{\psi}}_r = M\,\mathrm{e}^{-\jmath n_p\theta}\,\underline{i}_s + L_r\,\tilde{\underline{i}}_r, \tag{46b}$$

Equations (9) become

$$\frac{\mathrm{d}\underline{\psi}_s}{\mathrm{d}t} + R_s\,\underline{i}_s = \underline{u}_s, \tag{47a}$$

$$\frac{\mathrm{d}\tilde{\underline{\psi}}_r}{\mathrm{d}t} + R_r\,\tilde{\underline{i}}_r = 0, \tag{47b}$$

respectively. The advantage of considering complex variables will clearly appear in the sequel. It reduces the number of equations, and changes of frames are simply accomplished by multiplying complex variables by an appropriate complex exponential. We thus have $\underline{\psi}_r = \tilde{\underline{\psi}}_r\mathrm{e}^{\jmath n_p\theta}$, $\underline{\psi}_s = \tilde{\underline{\psi}}_s\mathrm{e}^{\jmath n_p\theta}$, $\underline{i}_r = \tilde{\underline{i}}_r\mathrm{e}^{\jmath n_p\theta}$ and $\underline{i}_s = \tilde{\underline{i}}_s\mathrm{e}^{\jmath n_p\theta}$.

In this notation the expression for the electromagnetic torque (8) becomes

$$\tau_{\mathrm{em}} = \frac{n_p M}{L_r}\Im\mathrm{m}\left(\underline{i}_s\underline{\psi}_r^*\right). \tag{48}$$

[8] We denote by $\jmath$ the pure imaginary number satisfying $\jmath^2 = -1$. The real part, the imaginary part and the conjugate of a complex quantity x are respectively denoted by $\Re\mathrm{e}\,(x)$, $\Im\mathrm{m}\,(x)$, and x^*.

5.1.2. Flatness of the Model

The concept of (differential) flatness was introduced in 1992 and we refer to (Fliess *et al.*, 1995) for an introduction to this subject. Recall that a (nonlinear) control system $\dot{x} = f(x, u)$, where x is the n-dimentional state and u the m-dimentional input, is *(differentially) flat* if there exists a set of variables $y = (y_1, \ldots, y_m)$ such that:

1. $y = \mathcal{A}(x, u, \dot{u}, \ldots, u^{(q)})$ for an appropriate integer q;
2. $x = \mathcal{B}(y, \dot{y}, \ldots, y^{(r)})$ and $u = \mathcal{C}(y, \dot{y}, \ldots, y^{(r+1)})$ for an appropriate integer r;
3. the components of y are differentially independent.

A set of variables y with these properties is called the *flat output*. A strong interest in flatness stems from the fact that it allows a straightforward solution to the motion planning problem: in practice, the flat output has a clear physical meaning with respect to the control objective. This leads to a huge collection of industrial applications (Fliess *et al.*, 1995). See (Boichot *et al.*, 1999) and (Hagenmeyer *et al.*, 2000) for flatness-based control regarding other kinds of motors.

The flatness of the model of the induction motor was established in (Martin and Rouchon, 1996b). We recall the proof in the present notation. Set $\rho = |\underline{\psi}_r|$ and define δ as the angle such that $\underline{\psi}_r = \rho e^{\jmath\delta}$; therefore δ is the angle of the rotor flux with respect to a fixed frame. Set $\alpha = \delta - n_p\theta$.

A flat output of the induction motor is $y = (\theta, \alpha)$. As usual, this flat output has a physical meaning which will simplify the control design: θ (or its first derivative ω) is the variable to be controlled, and α is the angle of the rotor flux with respect to a frame rotating at speed $n_p\omega$ (recall that $n_p\omega$ is called the synchronous speed). Notice that $\dot{\alpha} = \dot{\delta} - n_p\omega$ is the *slip speed*, usually only defined on constant speed operations.

We thus have $\tilde{\underline{\psi}}_r = \underline{\psi}_r e^{-\jmath n_p\theta} = \rho e^{\jmath\alpha}$. It is useful to express the electromagnetic torque produced by the motor in terms of ρ and α: using (46b) and (47b) leads to

$$\tilde{\underline{i}}_s = \frac{1}{M}(\tilde{\underline{\psi}}_r + \frac{L_r}{R_r}\frac{\mathrm{d}}{\mathrm{d}t}(\tilde{\underline{\psi}}_r)).$$

Thus,

$$\underline{i}_s\,\underline{\psi}_r^{\,*} = \tilde{\underline{i}}_s\,\tilde{\underline{\psi}}_r^{\,*} = \frac{1}{M}(|\tilde{\underline{\psi}}_r|^2 + \frac{L_r}{R_r}\frac{\mathrm{d}}{\mathrm{d}t}(\tilde{\underline{\psi}}_r)\,\tilde{\underline{\psi}}_r^{\,*}),$$

and finally, $\tau_{\mathrm{em}} = n_p\rho^2\dot{\alpha}/R_r$. So the mechanical equation of the induction motor becomes

$$\dot{\omega} = \frac{n_p}{JR_r}\rho^2\dot{\alpha} - \frac{f}{J}\omega - \frac{1}{J}\tau_L. \tag{49}$$

Hypothesis: *The torque load is an unknown function of time which can possibly depend on θ or its derivatives, but not on other variables ($\underline{i}_s$, $\underline{i}_r$, $\underline{\psi}_r$, ...).*

It is obvious that

$$\omega = \dot{\theta}, \tag{50a}$$

$$\delta = n_p\theta + \alpha. \tag{50b}$$

From (49), ρ satisfies

$$\rho = \sqrt{\frac{R_r(J\ddot{\theta} + f\dot{\theta} + \tau_L)}{n_p\dot{\alpha}}} = a(\dot{\theta}, \ddot{\theta}, \dot{\alpha}, \tau_L) \tag{50c}$$

which is a function of the flat output and its two first derivatives. Consequently,

$$\tilde{\underline{\psi}}_r = \rho e^{\jmath\alpha} = b(\dot{\theta}, \ddot{\theta}, \alpha, \dot{\alpha}, \tau_L). \tag{50d}$$

Continuing the calculations using successively (46b), and (47a), we obtain

$$\tilde{\underline{i}}_r = -\frac{1}{R_r}\frac{\mathrm{d}}{\mathrm{d}t}(\tilde{\underline{\psi}}_r) = c(\ddot{\theta}, \theta^{(3)}, \dot{\alpha}, \ddot{\alpha}, \tau_L, \dot{\tau_L}), \tag{50e}$$

$$\underline{i}_s = \frac{e^{\jmath n_p\theta}}{M}\left(\tilde{\underline{\psi}}_r - L_r\tilde{\underline{i}}_r\right) = d(\theta, \ldots, \theta^{(3)}, \alpha, \ldots, \ddot{\alpha}, \tau_L, \dot{\tau_L}), \tag{50f}$$

$$\underline{\psi}_s = L_s\,\underline{i}_s + M\,e^{\jmath n_p\theta}\,\underline{i}_r = e(\theta, \ldots, \theta^{(3)}, \alpha, \ldots, \ddot{\alpha}, \tau_L, \dot{\tau_L}), \tag{50g}$$

$$\underline{u}_s = R_s\underline{i}_s + \frac{\mathrm{d}}{\mathrm{d}t}(\underline{\psi}_s) = f(\theta, \ldots, \theta^{(4)}, \alpha, \ldots, \alpha^{(3)}, \tau_L, \ldots, \ddot{\tau_L}). \tag{50h}$$

Accordingly, $y = (\theta, \alpha)$ is a flat output of the induction motor.

5.1.3. Stationary Operation

The most useful frame to study stationary operations of the motor is certainly the frame of the flux, usually called the *dq-frame*. We denote by x^{dq} the value of the variable x in this frame, i.e., $x^{\mathrm{dq}} = xe^{-\jmath\delta} = \tilde{x}e^{-\jmath\alpha}$. Then the state-variable complex model reads as follows:

$$\dot{\omega} = \frac{n_p}{JR_r}\rho^2\dot{\alpha} - \frac{f}{J}\omega - \frac{1}{J}\tau_L, \tag{51a}$$

$$\dot{\rho} + \jmath\dot{\alpha}\rho = -\frac{1}{T_r}\rho + \frac{M}{T_r}\underline{i}_s{}^{\mathrm{dq}}, \tag{51b}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{i}_s{}^{\mathrm{dq}}) = \frac{M}{\sigma L_s L_r}(\frac{1}{T_r} - \jmath n_p\omega)\underline{\psi}_r^{\mathrm{dq}} - \left(a + \jmath\dot{\delta}\right)\underline{i}_s{}^{\mathrm{dq}} + \frac{1}{\sigma L_s}\underline{u}_s{}^{\mathrm{dq}}, \tag{51c}$$

with

$$a = \frac{1}{\sigma L_s}\left(R_s + \frac{M^2R_r}{L_r^2}\right).$$

Notice that as ρ and α are real variables, eqn. (51b) can be splitted into

$$\dot{\rho} = -\frac{1}{T_r}\rho + \frac{M}{T_r}\Re\mathrm{e}\left(\underline{i}_s{}^{\mathrm{dq}}\right) \quad \text{and} \quad \dot{\alpha} = \frac{M}{T_r\,\rho}\Im\mathrm{m}\left(\underline{i}_s{}^{\mathrm{dq}}\right).$$

A stationary operation at constant speed ω_o with constant load $\tau_L = \tau_{Lo}$ is obtained when $\theta = \omega_o t + \theta_o$, $\alpha = \alpha_1 t + \alpha_o$, where α_1 and α_o are constant (i.e., the slip speed is constant). In this case $\dot{\delta} = n_p\omega_o + \alpha_1 = \delta_1$ is constant.

As a consequence, (51a) implies that ρ is constant $\rho = \rho_o$ and thus, $\underline{\psi}_r^{\mathrm{dq}} = \rho_o$. In turn, (51b) implies that $\underline{i}_s{}^{\mathrm{dq}} = \underline{i}_{so}{}^{\mathrm{dq}}$ is constant, and finally, with (51c), so is $\underline{u}_s{}^{\mathrm{dq}} = \underline{u}_{so}{}^{\mathrm{dq}}$.

In conclusion, $\underline{\psi}_r$, $\underline{i}_s$ and $\underline{u}_s$ are periodical functions of time with pulsation $\delta_1 = n_p\omega_o + \alpha_1$. To run at a constant speed with constant load, the induction motor has to be fed by sinusoidal voltages.

Notice that usually the stationary operation is analyzed by imposing $\underline{u}_s$ to be sinusoidal under constant load and deducing that all electric and magnetic quantities are periodic and finally, that the speed is constant. Here, with the flatness properties, we are able to make the reverse analysis, i.e., beginning with the variable to be controlled and deducing the control.

5.2. Trajectory Generation

By (50a)–(50h) we obtained the expressions for all system variables in terms of the flat output components and the disturbance τ_L. In particular, these expressions allow us to calculate the control $\underline{u}_s$, at least when $\tau_L = 0$, for a known mean value τ_{Lo} of τ_L or for an estimated value $\widehat{\tau_L}$.

As $\theta^{(4)}$ and $\alpha^{(3)}$ appear in the expression (50h) of the control $\underline{u}_s$, the induction motor can only follow trajectories such that $t \mapsto \theta$ is everywhere 4-times left- and right-differentiable and $t \mapsto \alpha$ is everywhere 3-times left- and right-differentiable. The choice of the reference trajectories of θ and α is made in order to fulfill the constraints on all system variables.

For the first component θ of the flat output, the trajectory is often designed with respect to the control objective. This corresponds to a known function of time $t \mapsto \theta^d$ on a given time interval $[t_i, t_f]$.

For the second component α of the flat output, the choice of the desired trajectory $t \mapsto \alpha^d$ is not so obvious because the value of α does not correspond to a clear control objective. However, this variable gives a degree of freedom in order to perform a complementary control task. For example, it is possible to minimize the copper losses in the stator at every constant speed with an appropriate choice of the value of $\dot{\alpha}$: We get $\underline{\psi}_r = \rho \mathrm{e}^{\jmath\delta} = \rho \mathrm{e}^{\jmath(n_p\theta+\alpha)}$. So, using (51b), we have

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{\psi}_r) = (-\frac{1}{T_r} + \jmath n_p\omega)\underline{\psi}_r + \frac{M}{T_r}\underline{i}_s.$$

At a constant speed $\omega = \omega_o$, $\rho = \rho_o$ and $\dot{\alpha} = \alpha_1$ are both constant and therefore

$$\underline{i}_s = \frac{T_r}{M}\rho_o\left(\frac{1}{T_r} + \jmath\alpha_1\right)\mathrm{e}^{\jmath(n_p\theta+\alpha)},$$

$$|\underline{i}_s|^2 = \frac{{T_r}^2}{M^2}\rho_o^2\left(\frac{1}{{T_r}^2} + \alpha_1^2\right).$$

Thus,

$$|\underline{i}_s|^2 = \frac{{T_r}^2}{M^2}\frac{R_r(f\omega_o + \tau_{Lo})}{n_p}\left(\frac{1}{\alpha_1 T_r} + \alpha_1 T_r\right).$$

The magnitude of $\underline{i}_s$ is minimum if $\alpha_1 = 1/T_r$ (see (Chelouah *et al.*, 1996) for more details).

Between two time intervals on which ω is constant, $t \mapsto \alpha^d$ can be chosen as a function of ω. For example, we refer to Chelouah *et al.*, 1996 for a detailed planning of the reference trajectories of θ and α in order to start the motor from rest to a nominal speed without any singularity[9] ($\underline{u}_s$, $\underline{i}_s$ and $\underline{i}_r$ remain bounded everywhere).

5.3. Stabilization around Desired Trajectories

In this section, we present a tracking feedback law which is designed by studying the stationary operation of the system. We use a singular perturbation approach due to the good separation of the time scales.

Coupling $\tilde{\underline{\psi}}_r = \underline{\psi}_r \mathrm{e}^{-\jmath n_p\theta}$, $\tilde{\underline{i}}_r = \underline{i}_r \mathrm{e}^{-\jmath n_p\theta}$ and (47b), it is possible to write

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{\psi}_r \mathrm{e}^{-\jmath n_p\theta}) + R_r\underline{i}_r \mathrm{e}^{\jmath n_p\theta} = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{\psi}_r)\mathrm{e}^{-\jmath n_p\theta} - \jmath n_p\omega\underline{\psi}_r \mathrm{e}^{\jmath n_p\theta} + R_r\underline{i}_r \mathrm{e}^{\jmath n_p\theta} = 0,$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\underline{\psi}_r) - \jmath n_p\omega\underline{\psi}_r + R_r\underline{i}_r = 0, \tag{52}$$

which is the expression of the electrical equation of the rotor in the fixed frame. The stationary modes of eqns. (47a) and (52) are

$$\jmath\dot{\delta}\underline{\psi}_s + R_s\underline{i}_s = \underline{u}_s, \tag{53a}$$

$$\jmath(\dot{\delta} - n_p\omega)\underline{\psi}_r + R_r\underline{i}_r = 0, \tag{53b}$$

respectively. As $\underline{\psi}_s = L_s\,\underline{i}_s + M\,\underline{i}_r$ and $\underline{\psi}_r = M\,\underline{i}_s + L_r\,\underline{i}_r$, we have

$$\underline{i}_r = \frac{1}{L_r}\left(\underline{\psi}_r - M\underline{i}_s\right), \tag{54a}$$

$$\underline{\psi}_s = L_s\underline{i}_s + \frac{M}{L_r}\left(\underline{\psi}_r - M\underline{i}_s\right). \tag{54b}$$

[9] This is an important industrial problem as mentioned by Bartos (1998).

Equations (53) and (54) lead to

$$\begin{aligned} \left(R_s + \jmath\sigma\dot{\delta}L_s\right) \underline{i}_s + \jmath\dot{\delta}\frac{M}{L_s}\underline{\psi}_r &= \underline{u}_s, \\ -M\frac{R_r}{L_r}\underline{i}_s + \left(\frac{R_r}{L_r} + \jmath(\dot{\delta} - n_p\omega)\right)\underline{\psi}_s &= 0. \end{aligned}$$

Thus

$$\underline{i}_s = Z_s(\omega, \dot{\delta})\,\underline{u}_s, \tag{55a}$$

$$\underline{\psi}_r = Z_r(\omega, \dot{\delta})\,\underline{u}_s, \tag{55b}$$

where

$$Z_s(\omega, \dot{\delta}) = \frac{R_r/L_r + \jmath(\dot{\delta} - n_p\omega)}{Z}, \quad Z_r(\omega, \dot{\delta}) = \frac{MR_r/L_r}{Z}$$

and

$$Z = \left(R_s + \jmath\sigma\dot{\delta}L_s\right)\left(\frac{R_r}{L_r} + \jmath(\dot{\delta} - n_p\omega)\right) + M\frac{R_r}{L_r}\jmath\dot{\delta}\frac{M}{L_s}.$$

Finally, the control law is given by

$$|\underline{u}_s|^2 = \frac{JL_r}{n_p M \Im\mathrm{m}\left(Z_r^*(\omega, \dot{\delta}^d)\, Z_s(\omega, \dot{\delta}^d)\right)}\left(\dot{\omega}^d - \kappa(\omega - \omega^d)\right),$$

where $\omega^d = \dot{\theta}^d$ is the reference trajectory of the angular speed and $\delta^d = \alpha^d - n_p\omega^d$ is the reference trajectory of the slip speed. This control does not necessitate a flux observer.

5.4. Experimental Results

We conclude with the presentation of some experimental results[10] of a flatness-based control scheme. The first experiment (Fig. 4(a)) consists in starting the motor from rest to its nominal speed. We observe a good tracking for the acceleration motion (the experimental trajectory is hardly distinguishable from the reference one) and a small overshoot. Figure 4(b) shows the braking of the motor from its nominal speed to rest.

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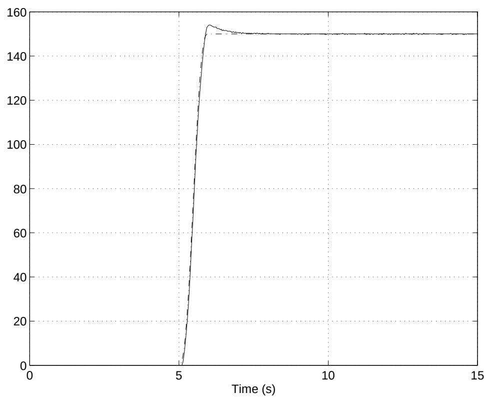
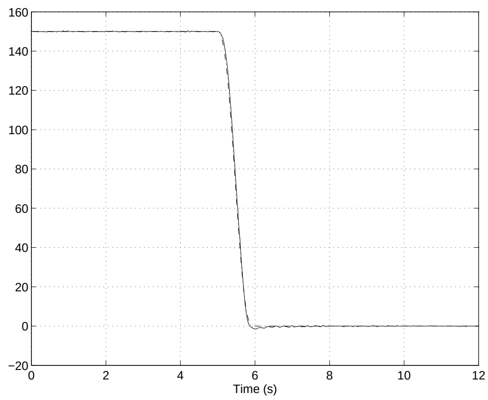
[10] Implemented on the experimental setup of the *GDR Automatique*, IRCyN, Nantes, France.



(a)



(b)

Fig. 4. Mechanical speed w [rad/s] (solid line) and the reference trajectory (dash-dot line) for $T_{5\%} = 0.84$ s (a) and $T_{5\%} = 0.76$ s (b).

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