Corrigendum to the paper
“The number of solutions of the Mordell equation”

by

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In Lemma 2 we produce an algebraic integer $\xi$ which satisfies some conditions. For our purpose $\xi$ must not be a rational integer. As Professors T. Wooley and M. Bennett pointed out to me this is not obvious by our arguments. So there is a gap in the proof. In this note we give a short proof of Lemma 2 by another method, which yields a significantly better estimate, and we considerably improve the estimates of our Theorems 1 and 2. For any positive integer $a$ we write $\log^* a$ for $\max\{1, \log a\}$ and $\omega(a)$ for the number of its prime divisors.

Lemma 2. Let $D$ be a rational integer with $|D| > 1$. Denote by $P(D)$ the product of distinct prime divisors $p$ of $D$ with $p > 3$. If $D$ has no prime divisors $> 3$ put $P(D) = 1$. Then the number of cubic fields (up to isomorphism) of discriminant $D$ is at most $225P(D)^{1/2}\log^* P(D)$.

Proof. If $D$ is a perfect square, then [1, Chapter 6, p. 333] implies that the number of cubic fields (up to isomorphism) of discriminant $D$ is $\leq 2^{\omega(D) - 1}$. Suppose now that $D$ is not a perfect square. Then $D = a(3^m b)^2$, where $a, b \in \mathbb{Z}$, $b$ is not divisible by 3, $a$ is square free and $m$ a nonnegative integer. It follows from [4, Théorème 2.5] that the number of cubic fields (up to isomorphism) of discriminant $D$ is $\leq 2^{\omega(b) - 1}h$, where $h$ is the class number of the quadratic field $\mathbb{Q}(\sqrt{-3a})$. By [2, pp. 620–625] we can take $|D| \geq 23$. Furthermore, [3] implies that $h < 5d^{1/2}\log^* d$, where $d$ is the discriminant of $\mathbb{Q}(\sqrt{-3a})$. Combining the above estimates yields the lemma.

Using the above version of Lemma 2, we obtain the following improved version for Theorem 2.

Theorem 2. Let $S$ be a finite set of rational primes with $2, 3 \in S$. 

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Denote by \( P(S) \) the product of primes \( p \) in \( S \) with \( p > 3 \). If \( S = \{2, 3\} \), put \( P(S) = 1 \). Then the number of \( \mathbb{Q} \)-isomorphism classes of elliptic curves over \( \mathbb{Q} \), with good reduction outside of \( S \), is
\[
< 10^{11 \sharp S + 23} P(S)^{1/2} \log^* P(S).
\]

As a consequence of Theorem 2, we get the following improved version for Theorem 1.

**Theorem 1.** Let \( k \) be a nonzero rational integer. Denote by \( P(k) \) the product of the prime divisors \( p \) of \( k \) with \( p > 3 \). If \( k \) has no prime divisors \( > 3 \), put \( P(k) = 1 \). Then the number of solutions \( (x, y) \in \mathbb{Z}^2 \) of the equation \( y^2 = x^3 + k \) is
\[
< 10^{11 \omega(k) + 45} P(k)^{1/2} \log^* P(k).
\]

**References**


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