The number of powers of 2 in a representation of large even integers by sums of such powers and of two primes

by

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1. Main results. The Goldbach conjecture is that every integer not less than 6 is a sum of two odd primes. The conjecture still remains open. Let $E(x)$ denote the number of positive even integers not exceeding $x$ which cannot be written as a sum of two prime numbers. In 1975 Montgomery and Vaughan [9] proved that $E(x) \ll x^{1-\theta}$ for some small computable constant $\theta > 0$. In [4] the author proved that $E(x) \ll x^{0.921}$, and recently [5] he improved that to $E(x) \ll x^{0.914}$.

In 1951 and 1953, Linnik [6, 7] established the following “almost Goldbach” result.

Every large positive even integer $N$ is a sum of two primes $p_1, p_2$ and a bounded number of powers of 2, i.e.

$$N = p_1 + p_2 + 2^{\nu_1} + \ldots + 2^{\nu_k}. \quad (1.1)$$

Let $r''_k(N)$ denote the number of representations of $N$ in the form (1.1). In [8] Liu, Liu and Wang proved that for any $k \geq 54000$, there exists $N_k > 0$ depending on $k$ only such that if $N \geq N_k$ is an even integer then

$$r''_k(N) \gg N(\log N)^{k-2}. \quad (1.2)$$

In this paper we prove the following result.

**Theorem 1.** For any integer $k \geq 25000$, there exists $N_k > 0$ depending on $k$ only such that if $N \geq N_k$ is an even integer then

$$r''_k(N) \gg N(\log N)^{k-2}.$$
Let \( r'_k(n) \) denote the number of representations of an odd integer \( n \) in the form
\[
n = p + 2^{\nu_1} + \ldots + 2^{\nu_k}.
\]

The second purpose of this paper is to establish the following result.

**Theorem 2.** For any \( \varepsilon > 0 \), there exists a constant \( k_0 \) depending on \( \varepsilon \) only such that if \( k \geq k_0 \) and \( N \geq N_k \) then
\[
\sum_{2 \nmid n \leq N} (r'_k(n) - 2(\log_2 N)^k(\log N)^{-1})^2 \leq \varepsilon 2N(\log_2 N)^{2k}(\log N)^{-2}.
\]
In particular, for \( \varepsilon = 0.9893 \), one can take \( k_0 = 12500 \).

In what follows, \( \mathcal{L} \) always stands for \( \log PT \), and \( L(s, \chi) \) denotes the Dirichlet \( L \)-function. \( \delta \) denotes a positive constant which is arbitrarily small but not necessarily the same at each occurrence.

**2. Some lemmas.** Let \( N \) be a large integer, and set
\[
(2.1) \quad P := N^\theta, \quad T := P^3(\log N)^6, \quad Q := P^{-1}N(\log N)^{-3},
\]
where \( \theta \) is an absolute constant. Let \( \chi \pmod{q}, \chi_0 \pmod{q} \) be a character and a principal character mod \( q \) respectively.

**Lemma 1.** Let \( \chi \) be a non-principal character mod \( q \). Then for any \( \varepsilon > 0 \) there exists a \( \delta = \delta(\varepsilon) > 0 \) such that
\[
-\Re L'(s, \chi) \leq -\sum_{|1 + it - \rho| \leq \delta} \Re \frac{1}{s - \rho} + \left( \frac{3}{16} + \varepsilon \right) H
\]
uniformly for
\[
1 + \frac{1}{H \log H} \leq \sigma \leq 1 + \frac{\log H}{H}
\]
providing that \( q \) is sufficiently large, where \( H = \log q(|t| + 2) \) and \( s = \sigma + it \).

This is Lemma 2.4 of [3].

For a real number \( \alpha \), set \( \alpha^* = \alpha \mathcal{L}^{-1} \) and let
\[
\rho_j = 1 - \lambda_j^* + i\gamma_j^*, \quad j = 1, 2, \ldots,
\]
denote the non-trivial zeros of \( L(s, \chi) \) with \( |\gamma_j| \leq T\mathcal{L} \), where \( \lambda_j \) are in increasing order.

**Lemma 2.** Let \( N \) be sufficiently large. Then no function \( L(s, \chi) \) with \( \chi \) primitive mod \( q \leq P \), except for a possible exceptional one only, has a zero in the region
\[
\sigma \geq 1 - \frac{0.239}{\mathcal{L}}, \quad |t| \leq T.
\]
If the exceptional function, denoted by $L(s, \tilde{\chi})$, exists, then $\tilde{\chi}$ must be a real primitive character mod $\tilde{q}$, $\tilde{q} \leq P$, and $L(s, \tilde{\chi})$ has a real simple zero $\tilde{\beta}$; no other function $L(s, \chi)$ with $\chi$ primitive mod $q \leq P$ has a zero in the region
\[ \sigma \geq 1 - \frac{0.517}{\mathcal{L}}, \quad |t| \leq T. \]

**Proof.** If $q_1, q_2 \leq P$, $q_1 \neq q_2$, consider the zeros of $L(s, \chi_{q_1})$ and $L(s, \chi_{q_2})$ for non-principal characters $\chi_{q_1}$ and $\chi_{q_2}$. If $\varrho_1$ is a zero of $L(s, \chi_{q_1})$ and $\varrho_2$ is a zero of $L(s, \chi_{q_2})$, then as in Lemma 3.7 of [3] and setting $\eta = a\sqrt{N}/\mathcal{L}$, $\sigma = \eta + 1$, note that $T = P^3(\log N)^6$, $\log P^2T = (5/4 + \delta)\mathcal{L}$. Then for any positive constant $a$ we have
\[ G(0) - G\left(-\frac{\lambda_1}{a}\right) - G\left(-\frac{\lambda_2}{a}\right) + a\left(\frac{39}{64} + \varepsilon\right) \geq 0 \]
where
\[ G(z) = \int_0^{\infty} \exp\left\{-\frac{1}{4}x^2 + zx\right\} dx. \]

Take $a = 1.22$; then $\lambda_1 \leq 0.239$ implies $\lambda_2 > 0.63$. Take $a = 1.26$; then $\lambda_1 \leq 0.411$, implies $\lambda_2 > 0.411$. If $q_1 = q_2$, by Lemma 3.7 and Theorem 1.2 of [3] the lemma follows.

**Lemma 3.** Suppose $\chi$ is a real non-principal character mod $q \leq P$, and $\varrho_1$ is real. Then $\lambda_2 > 0.8$.

**Proof.** By Lemma 3.2 of [3] the assertion follows.

By Lemma 4 of [5] we have

**Lemma 4.** Let $\chi$ be a non-principal character mod $q \leq P$, and $\varrho_1$, $\varrho_2$, $\varrho_3$ be the zeros of $L(s, \chi)$. Then
\[ \lambda_2 > 0.575, \quad \lambda_3 > 0.618. \]

**Lemma 5.** Let $\chi \neq \chi_0$ be a character mod $q \leq P$. Let $n_0, n_1, n_2$ denote the numbers of zeros of $L(s, \chi)$ in the rectangles
\[ R_0 : 1 - \mathcal{L}^{-1} \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \quad |t - t_0| \leq 5.8\mathcal{L}^{-1}, \]
\[ R_1 : 1 - 5\mathcal{L}^{-1} \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \quad |t - t_1| \leq 23.4\mathcal{L}^{-1}, \]
\[ R_2 : 1 - \lambda_+\mathcal{L}^{-1} \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \quad |t - t_2| \leq 23.4\mathcal{L}^{-1}, \]
respectively, where $t_0$, $t_1$, $t_2$ are real numbers satisfying $|t_i| \leq T$, and $5 < \lambda_+ \leq \log \log L$. Then
\[ n_0 \leq 3, \quad n_1 \leq 10, \quad n_2 \leq 0.2292(\lambda_+ + 42.9). \]

**Proof.** It is well known that
\[ \frac{\zeta'}{\zeta}(\sigma) - \Re \frac{L'}{L}(s, \chi) \geq 0 \]
where $\sigma = \Re s$. 

**Representations by sums of powers, 231**
We consider the rectangle $R_0$. Let $\rho = \sigma + it_0$, $\sigma = 1 + 8.4L^{-1}$, and denote by $\rho = 1 - \lambda^* + i\gamma$ the zero of $L(s, \chi)$ in $R_0$. Hence $0.239 \leq \lambda \leq 1, |\gamma - t_0| \leq 5.8L^{-1}$. So we have

$$ -\Re \frac{1}{s - \rho} = -\mathcal{L} \frac{8.4 + \lambda}{(8.4 + \lambda)^2 + ((\gamma - t_0)\mathcal{L})^2} \leq -\mathcal{L} \frac{9.4}{9.4^2 + 5.8^2}. $$

By Lemma 1,

$$ -\Re \frac{L'}{L}(s, \chi) \leq -\sum_{|1 + it_0 - \rho| \leq \delta} \Re \frac{1}{s - \rho} + 0.18751\mathcal{L}. $$

If $|1 + it_0 - \rho| > \delta$ then $\Re \frac{1}{s - \rho} = O(1)$. So

$$ -\Re \frac{L'}{L}(s, \chi) \leq \mathcal{L} \left( 0.18751 - \frac{9.4n_0}{9.4^2 + 5.8^2} \right). $$

Since $-\zeta'(\sigma) \leq \frac{1}{\sigma - 1} + A$, where $A$ is an absolute constant, we have

$$ \frac{9.4n_0}{9.4^2 + 5.8^2} \leq \frac{1}{8.4} + 0.18752, \quad n_0 \leq 3. $$

Now as above, let $\sigma = 1 + 24L^{-1}$. Then $n_1 \leq 10$ and $n_2 \leq 0.2292(\lambda_+ + 42.9)$.

3. The zero density estimate of the Dirichlet $L$-function. In this section we use the notations of Section 3 of [8]. For $1 \leq j \leq 4$, let $h_j$ denote positive constants satisfying $h_1 < h_2 < h_3, h_2 + h_4 + 3/8 < h_3, 2h_4 + 3/8 < h_1$.

Let

$$ z_j := (P^2T)^{h_j}, \quad \alpha := 1 - \lambda L^{-1}, \quad \lambda \leq \log \log \mathcal{L}, $$

$$ D(\lambda, T) := D := \{ s = \sigma + it : \alpha \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \ |t| \leq T \}. $$

Let $N(\chi, \alpha, T)$ denote the number of zeros of $L(s, \chi)$ in $D$, and

$$ N^*(\alpha, P, T) = \sum_{q \leq P} \sum_{\chi (\text{mod } q)}^* N(\chi, \alpha, T), $$

where $\sum_{\chi (\text{mod } q)}^*$ indicates that the sum is over primitive characters mod $q$.

For positive $\delta_1, \delta_3$, let

$$ \kappa(s) := s^{-2} \{ (e^{-((1-\delta_1)(\log z_1))s} - e^{-(\log z_1)s})\delta_3(\log z_3) $$

$$ - e^{-(\log z_3)s} - e^{-(1+\delta_3)(\log z_3)s})\delta_1(\log z_1) \}. $$

For a zero $\rho_0 \in D$, let

$$ M(\rho_0) := \sum_{\chi} |\kappa(\rho(\chi) + \rho_0 - 2\alpha)|, $$

$$ D(\lambda_0, T) := D := \{ s = \sigma + it : \alpha_0 \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \ |t| \leq T \}. $$

Finally, let

$$ \kappa(s) := s^{-2} \{ (e^{-((1-\delta_1)(\log z_1))s} - e^{-(\log z_1)s})\delta_3(\log z_3) $$

$$ - e^{-(\log z_3)s} - e^{-(1+\delta_3)(\log z_3)s})\delta_1(\log z_1) \}. $$

For a zero $\rho_0 \in D$, let

$$ M(\rho_0) := \sum_{\chi} |\kappa(\rho(\chi) + \rho_0 - 2\alpha)|, $$

$$ D(\lambda_0, T) := D := \{ s = \sigma + it : \alpha_0 \leq \sigma \leq 1 - 0.239\mathcal{L}^{-1}, \ |t| \leq T \}. $$
where the sum is over the zeros of $L(s, \chi)$ in $D$. If $2h_4 + 3/8 < (1 - \delta_1)h_1$, then as in (3.17) of [8] we have

\begin{equation}
N^*(\alpha, P, T) \leq \frac{(1 + \delta) \max_{\rho_0} M(\rho_0)}{2(1 - \alpha)(h_2 - h_1)\delta_1\delta_3h_1h_3h_4(\log P^2T)^4} (P^2T)^{2h_3(1 - \alpha)}.
\end{equation}

(i) If $5 < \lambda \leq \log \log \mathcal{L}$, let $\Delta = 23.4\mathcal{L}^{-1}$. Then as in [8], by Lemma 5 we have

$M(\rho_0) \leq 0.2292(\lambda + 42.9)(\log P^2T)^3(1/2) \times \left\{ (\delta_1h_1(2\delta_3 + \delta_5^2)h_3^2 - \delta_3h_3(2\delta_1 - \delta_1^2)h_1^2) + (\pi/23.4)^2(\delta_1h_1 + \delta_3h_3) \right\}.$

Choose $h_1 = 0.58$, $h_2 = 0.669$, $h_3 = 1.08$, $h_4 = 0.0353$, $\delta_1h_1 = \delta_3h_3 = \pi/23.4$. By (3.6) we have

\begin{equation}
N^*(\alpha, P, T) \leq 268.6(P^2T)^{2.16(1 - \alpha)}.
\end{equation}

(ii) If $1 < \lambda \leq 5$, then as in [8], by Lemma 5 ($n_1 \leq 10$) we have

$M(\rho_0) \leq (10/2)(\log P^2T)^3 \times \left\{ (\delta_1h_1(2\delta_3 + \delta_5^2)h_3^2 - \delta_3h_3(2\delta_1 - \delta_1^2)h_1^2) + (\pi/23.4)^2(\delta_1h_1 + \delta_3h_3) \right\}.$

Choose $h_1 = 0.82$, $h_2 = 1.179$, $h_3 = 1.71$, $h_4 = 0.155$, $\delta_1h_1 = \delta_3h_3 = \pi/23.4$.

By (3.6) we have

\begin{equation}
N^*(\alpha, P, T) \leq (104.1/\lambda)(P^2T)^{3.42(1 - \alpha)}.
\end{equation}

(iii) If $0.618 < \lambda \leq 1$, for $a = 6.3$ we have

\[
\left( \frac{1}{a} - \frac{1}{a + 1} - \frac{2(a + 1)}{(a + 1)^2 + 5.8^2} + 0.1876 \right) \times \max \left\{ \frac{a + 1}{5.8^2} + \frac{1}{a + 1}, \frac{a + 0.618}{5.8^2} + \frac{1}{a + 0.618} \right\} \leq 0.014621.
\]

As in [8], by Lemma 5 we have

$M(\rho_0) \leq \{1.5(\delta_1h_1(2\delta_3 + \delta_5^2)h_3^2 - \delta_3h_3(2\delta_1 - \delta_1^2)h_1^2) + 2 \cdot 0.014621 \cdot (\delta_1h_1 + \delta_3h_3)\}(\log P^2T)^3.$

Choose $h_1 = 1.0065$, $h_2 = 1.599$, $h_3 = 2.25$, $h_4 = 0.2759$, $\delta_1 = 0.079$, $\delta_3 = 0.094$. By (3.6) we have

\begin{equation}
N^*(\alpha, P, T) \leq (14.3/\lambda)(P^2T)^{4.5(1 - \alpha)}.
\end{equation}

(iv) If $0.575 < \lambda \leq 0.618$, by Lemma 4 there are at most two zeros satisfying $\gamma = 1 - \beta/\mathcal{L} - i\gamma/\mathcal{L}$, $\beta < 0.618$. As in (v) of [8], we have

\begin{equation}
N^*(\alpha, P, T) \leq \frac{(1 + \delta)\tilde{M}}{2(1 - \alpha)(h_2 - h_1)h_4\log P^2T}(P^2T)^{2h_3(1 - \alpha)}.
\end{equation}
where
\[ \tilde{M} := \max_{\chi \mod q} \max_{1 \leq j \leq 2} \frac{1}{\log z_3} \int_{\log z_1} \left| \sum_{l=1}^{j} e^{-(g(l,\chi)-\alpha)x} \right|^2 \, dx, \]
and \( g(l,\chi) \) is a zero of \( L(s,\chi) \) in \( D \). We have
\[
\int_{\log z_1}^{\log z_3} \left| e^{-(g(l,\chi)-\alpha)x} \right|^2 \, dx \leq (h_3 - h_1) \log P^2 T, \\
\frac{1}{2} \int_{\log z_1}^{\log z_3} \left| \sum_{l=1}^{2} e^{-(g(l,\chi)-\alpha)x} \right|^2 \, dx \leq 2(h_3 - h_1) \log P^2 T.
\]

Choose \( h_1 = 0.9, h_2 = 1.4525, h_3 = 2.09, h_4 = 0.2624 \). By (3.10) we have
\[ N^*(\alpha, P, T) \leq (8.21/\lambda)(P^2 T)^{4.18(1-\alpha)}. \]

(v) If \( 0.411 < \lambda \leq 0.575 \), by Lemma 4 there is at most one zero satisfying \( \varrho = 1 - \beta/L - i\gamma/L, \beta < 0.575 \). As in (v) of [8], we have
\[
(3.11) \quad N^*(\alpha, P, T) \leq \frac{(1 + \delta)(h_3 - h_1)^2}{(h_2 - h_1)h_4}(P^2 T)^{2h_3(1-\alpha)}. \]
Choose \( h_1 = 1.01, h_2 = 1.4074, h_3 = 2.1, h_4 = 0.3174 \). By (3.11) we have
\[ N^*(\alpha, P, T) \leq 9.42(P^2 T)^{4.2(1-\alpha)}. \]

In conclusion we have

**Lemma 6.** If \( N^*(\alpha, P, T) \) and \( \alpha = 1 - \lambda L^{-1} \) are defined by (3.3), (3.1), then
\[ N^*(\alpha, P, T) \leq \begin{cases} 
2, & \lambda \leq 0.411, \\
9.42(P^2 T)^{4.2(1-\alpha)}, & 0.411 < \lambda \leq 0.575, \\
14.28(P^2 T)^{4.18(1-\alpha)}, & 0.575 < \lambda \leq 0.618, \\
23.14(P^2 T)^{4.5(1-\alpha)}, & 0.618 < \lambda \leq 1, \\
104.1(P^2 T)^{3.42(1-\alpha)}, & 1 < \lambda \leq 5, \\
268.6(P^2 T)^{2.16(1-\alpha)}, & 5 < \lambda \leq \log \log L.
\end{cases} \]

**4. The proof of the theorems.** By Dirichlet’s lemma on rational approximations, each \( \alpha \in [Q^{-1}, 1 + Q^{-1}] \) may be written in the form
\[
\alpha = a/q + \lambda, \quad |\lambda| \leq (qQ)^{-1},
\]
for some positive integers \( a, q \) with \( 1 \leq a \leq q, (a,q) = 1 \) and \( q \leq Q \). We denote by \( I(a,q) \) the set of \( \alpha \) satisfying (4.1), and put
\[
E_1 = \bigcup_{Q \leq P} \bigcup_{\alpha=1}^{q} I(a,q), \quad E_2 = [Q^{-1}, 1 + Q^{-1}] - E_1.
\]
When \( q \leq P \) we call \( I(a, q) \) a major arc. By (2.1), all major arcs are mutually disjoint. Let \( e(\alpha) = \exp(2\pi i \alpha) \) and \( S(\alpha) = \sum_{p \leq N} e(pa) \).

Let \( \sigma(n) \) denote the singular series in the Goldbach problem, i.e.
\[
\sigma(n) := \prod_{p|n} (1 + (p - 1)^{-1}) \prod_{p \nmid n} (1 - (p - 1)^{-2}) \gg 1
\]
for even \( n \). Let
\[
J(n) := \sum_{1 < n_1, n_2 \leq N, n_1 - n_2 = n} (\log n_1 \log n_2)^{-1}.
\]

For \( 0 < \theta < 1/30 \), define
\[
(4.2) \quad f(\theta) := \frac{268.6(1 - (6 + \delta)\theta)}{1 - (16.8 + \delta)\theta} \exp \left( -\frac{5 - (84 + \delta)\theta}{(4 + \delta)\theta} \right)
\]
\[
+ \frac{104.1(1 - (6 + \delta)\theta)}{1 - (23.1 + \delta)\theta} \times \left\{ \exp \left( -\frac{1 - (23.1 + \delta)\theta}{(4 + \delta)\theta} \right) - \exp \left( -\frac{5(1 - (23.1 + \delta)\theta)}{(4 + \delta)\theta} \right) \right\}
\]
\[
+ \frac{23.14(1 - (6 + \delta)\theta)}{1 - (28.5 + \delta)\theta} \times \left\{ \exp \left( -\frac{0.618(1 - (28.5 + \delta)\theta)}{(4 + \delta)\theta} \right) - \exp \left( -\frac{1 - (28.5 + \delta)\theta}{(4 + \delta)\theta} \right) \right\}
\]
\[
+ \frac{14.28(1 - (6 + \delta)\theta)}{1 - (26.9 + \delta)\theta} \times \left\{ \exp \left( -\frac{0.575(1 - (26.9 + \delta)\theta)}{(4 + \delta)\theta} \right) - \exp \left( -\frac{0.618(1 - (26.9 + \delta)\theta)}{(4 + \delta)\theta} \right) \right\}
\]
\[
+ \frac{9.42(1 - (6 + \delta)\theta)}{1 - (27 + \delta)\theta} \times \left\{ \exp \left( -\frac{0.411(1 - (27 + \delta)\theta)}{(4 + \delta)\theta} \right) - \exp \left( -\frac{0.575(1 - (27 + \delta)\theta)}{(4 + \delta)\theta} \right) \right\}
\]
\[
+ 2 \left\{ \exp \left( -\frac{0.239(1 - (6 + \delta)\theta)}{(4 + \delta)\theta} \right) - \exp \left( -\frac{0.411(1 - (6 + \delta)\theta)}{(4 + \delta)\theta} \right) \right\}
\]
and
\[
(4.3) \quad F(\theta) := \frac{5.094 \pi (1 + \delta)}{\sqrt{6}(1 - (6 + \delta)\theta)} f(\theta) + \frac{5.094 \pi^2 (1 + \delta)}{4\sqrt{6}(1 - (6 + \delta)\theta)^2} f^2(\theta).
\]
Theorem 3. Let \( n \) with \( |n| \leq N^2 \) be a non-zero integer, and \( P, Q \) satisfy (2.1). If \( \theta < 1/30 \), then for even \( n \) we have
\[
\int_{E_1} |S(\alpha)|^2 e(n\alpha) d\alpha = \sigma(n)J(n) + R,
\]
where
\[
|R| \leq |n|N(\phi(|n|)(\log N)^2)^{-1}\{F(\theta) + O(\tilde{\tau}(n, \tilde{\tau}))/\phi^2(\tilde{\tau})\},
\]
with the \( O \) term occurring only when there exists \( \tilde{\beta} \) in Lemma 2.

The proof of Theorem 3 is the same as in [8], but we use our Lemmas 1–4, Lemma 6 and the fact that \( \prod_{p \geq 5} (1 + 1/(p - 1)^2) \leq 1.132 \) (see page 6 of [1]) so we can replace 5.205 by 5.094.

For the proof of Theorems 1 and 2, as in Lemma 20 in Section 7 of [8], we define
\[
\Theta := \Theta(\eta) := \frac{1}{\log 2} \frac{\eta \csc^2(\pi/8) \log(\log 2 \eta \csc^2(\pi/8))}{\eta \csc^2(\pi/8)},
\]
\[
H(k) := \min_{9 \leq E \leq L} \left\{ 1.7811 \left( 1 - \frac{1}{E \csc^2(\pi/8)} \right)^{2k} \log E + 2.3270 \frac{1 + \log E}{E} \right\},
\]
where \( L = \log_2 N \).

Choose \( \theta = 1/98 \) and \( \eta = 1/7758 \), so \( \Theta(\eta) < \theta \). When \( k \geq 12500 \), choose \( E = 460 \) one has \( H(k) < 0.03989 \), for \( c_8 < 2.1967 \), \( c_9 < 17.2435 \) one has \( c_9(1 - \eta)^{2k - 2} < 0.6873 \), \( (c_8 + \delta)F(\theta) < 0.26202 \) and \( (c_9 + \delta)F(\theta) + H(k) \)
\( c_9(1 - \eta)^{2k - 2} < 0.9893 \). As in Section 7 of [8], Theorems 1 and 2 can be proved in the same way as Theorems 1 and 2 in [8].

References


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