ACTA ARITHMETICA
XCI.1 (1999)

A note on the Diophantine equation $a^x + b^y = c^z$

by

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1. Introduction. Let $\mathbb{Z}$, $\mathbb{N}$, $\mathbb{P}$ be the sets of integers, positive integers and primes respectively, and

$$\mathbb{P}^{\mathbb{N}} = \{p^n \mid p \in \mathbb{P} \text{ and } n \in \mathbb{N}\}.$$

Clearly, $\mathbb{P} \subseteq \mathbb{P}^{\mathbb{N}}$. In [13] Nagell first proved that if $\max(a, b, c) \le 7$, then all the solutions $(x, y, z) \in \mathbb{N}^3$ of the equation

$$a^x + b^y = c^z, \quad a, b, c \in \mathbb{P},\ a > b \tag{1}$$

are given by

$$(a, b, c) = (3, 2, 5) : (x, y, z) = (1, 1, 1), (2, 4, 2);$$
$$(a, b, c) = (5, 2, 3) : (x, y, z) = (2, 1, 3), (1, 2, 2);$$
$$(a, b, c) = (5, 3, 2) : (x, y, z) = (1, 1, 3), (1, 3, 5), (3, 1, 7);$$
$$(a, b, c) = (3, 2, 7) : (x, y, z) = (1, 2, 1);$$
$$(a, b, c) = (7, 2, 3) : (x, y, z) = (1, 1, 2), (2, 5, 4);$$
$$(a, b, c) = (7, 3, 2) : (x, y, z) = (1, 2, 4);$$
$$(a, b, c) = (5, 2, 7) : (x, y, z) = (1, 1, 1);$$
$$(a, b, c) = (7, 5, 2) : (x, y, z) = (1, 2, 5).$$

Later, Mąkowski [11], Hadano [7], Uchiyama [23], Qi Sun and Xiaoming Zhou [16], and Xiaozhuo Yang [24] determined all solutions $(x, y, z) \in \mathbb{N}^3$ of equation (1), when $11 \le \max(a, b, c) \le 23$. In [1] we have given all solutions $(x, y, z) \in \mathbb{N}^3$ of equation (1), when $29 \le \max(a, b, c) \le 97$ (60 solutions in total), and we have proved the following:

THEOREM A. *If* $\max(a, b, c) > 13$, *then equation* (1) *has at most one solution* $(x, y, z) \in \mathbb{N}^3$ *with* $z > 1$.

1991 *Mathematics Subject Classification*: Primary 11D61.
Key words and phrases: exponential Diophantine equation.
Supported by the National Natural Science Foundation of China and the Heilongjiang Provincial Natural Science Foundation.

A more general result was given in [2]. Let $A, B \in \mathbb{N}$, $A > B > 1$ and $\gcd(A, B) = 1$. If the equation

$$(2) \qquad X^2 + ABY^2 = p^Z, \qquad X, Y, Z \in \mathbb{N},\ p \in \mathbb{P},\ \text{and}\ \gcd(X, Y) = 1,$$

has a solution (X, Y, Z), then there exists a unique solution (X_p, Y_p, Z_p) which satisfies $Z_p \le Z$, where Z runs over all solutions of (2). That (X_p, Y_p, Z_p) is called the *least solution* of (2). From [2] we have

Theorem B. *If* $x, y \in \mathbb{N}$ *satisfy the equation*

$$(3) \qquad Ax^2 + By^2 = 2^z, \qquad z > 2,\ x \mid^* A,\ y \mid^* B,$$

where the symbol $x \mid^* A$ *means that every prime divisor of* x *divides* A, *then*

$$|Ax^2 - By^2| = 2X_2, \qquad xy = Y_2,\ 2z - 2 = Z_2,$$

except for $(A, B, x, y, z) = (5, 3, 1, 3, 5), (5, 3, 5, 1, 7)$ *and* $(13, 3, 1, 9, 8)$.

Theorem C. *If* $x, y \in \mathbb{N}$ *satisfy the equation*

$$(4) \qquad Ax^2 + By^2 = p^z, \qquad p \in \mathbb{P},\ x \mid^* A,\ y \mid^* B,$$

then

$$|Ax^2 - By^2| = X_p, \qquad 2xy = Y_p,\ 2z = Z_p,$$

or

$$|Ax^2 - By^2| = X_p|X_p^2 - 3ABY_p^2|, \qquad 2xy = Y_p|3X_p^2 - ABY_p^2|,\ 2z = 3Z_p,$$

the latter occurring only for

$$Ax^2 + By^2 = 3^{4s+3}\left(\frac{3^{2s} - 1}{8}\right) + \left(\frac{3^{2s+2} - 1}{8}\right) = \left(\frac{3^{2s+1} - 1}{2}\right)^3 = p^z,$$

where $s \in \mathbb{N}$.

From Theorems B and C, we have (cf. Lemma 6 of [15])

Theorem D. *The equation*

$$a^x + b^y = c^z, \qquad \gcd(a, b) = 1,\ c \in \mathbb{P},\ a > b > 1,$$

has at most one solution when the parities of x *and* y *are fixed*, *except for* $(a, b, c) = (5, 3, 2), (13, 3, 2), (10, 3, 13)$. *The solutions in case* $(5, 3, 2)$ *are given by* $(x, y, z) = (1, 1, 3), (1, 3, 5), (3, 1, 7)$, *in case* $(13, 3, 2)$ *by* $(1, 1, 4)$ *and* $(1, 5, 8)$, *and in case* $(10, 3, 13)$ *by* $(1, 1, 1)$ *and* $(1, 7, 3)$ (cf. [14], [3]).

In [3], we obtained further results when the right sides of equation (3) and equation (4) are replaced by $4k^z$ and k^z respectively, where $k \in \mathbb{N}$.

Recently, N. Terai [20, 21] conjectured that if $a, b, c, p, q, r \in \mathbb{N}$ are fixed, and $a^p + b^q = c^r$, where $p, q, r \ge 2$, and $\gcd(a, b) = 1$, then the Diophantine equation

$$(5) \qquad a^x + b^y = c^z, \qquad x, y, z \in \mathbb{N},$$

has only the solution $(x, y, z) = (p, q, r)$. The conjecture is clearly false. For example, from Nagell's result [13], we see that the equation $3^x + 2^y = 5^z$ has two solutions $(x, y, z) = (1, 1, 1), (2, 4, 2)$, and the equation $7^x + 2^y = 3^z$ also has two solutions $(x, y, z) = (1, 1, 2), (2, 5, 4)$. Furthermore, if $a = 1$ or $b = 1$, then the conjecture is also false. So, the condition $\max(a, b, c) > 7$ should be added to the hypotheses of the conjecture.

For $p = q = r = 2$ the above statement was conjectured previously by Jeśmanowicz. We shall use the term Terai–Jeśmanowicz conjecture for the above conjecture with the added condition that $\max(a, b, c) > 7$. Some recent results on the Terai–Jeśmanowicz conjecture are as follows:

(a) If $p = q = r = 2$, we may assume without loss of generality that $2 \mid a$. Then we have

$$a = 2st, \quad b = s^2 - t^2, \quad c = s^2 + t^2,$$

where $s, t \in \mathbb{N}$, $s > t$, $\gcd(s, t) = 1$ and $2 \mid st$. In 1982, we [4] proved that

(Cao.1) if $2 \parallel s, t \equiv 1 \pmod 4$, or $2 \parallel s, t \equiv 3 \pmod 4$ and $s + t$ has a prime factor of the form $4k - 1$, then the Terai–Jeśmanowicz conjecture holds;

(Cao.2) if $(s, t) \equiv (1, 6), (5, 2) \pmod 8$, or $(s, t) \equiv (3, 4) \pmod 4$ and $s + t$ has a prime factor of the form $4k - 1$, then the Terai–Jeśmanowicz conjecture holds (also see [5], pp. 366–367).

Maohua Le [8, 9, 6] proved that

(Le.1) if $2 \parallel s, t \equiv 3 \pmod 4$ and $s \geq 81t$, then the Terai–Jeśmanowicz conjecture holds;

(Le.2) if $2^2 \parallel a$ and $c \in \mathbb{P}^{\mathbb{N}}$, then the Terai–Jeśmanowicz conjecture holds;

(Le.3) if $t = 3, s$ is even, and $s \leq 6000$, then the Terai–Jeśmanowicz conjecture holds.

K. Takakuwa and Y. Asaeda [17–19] considered the case $s = 2s'$, $t = 3, 7, 11, 15$. For example, they proved that if $2 \nmid s'$, then the Terai–Jeśmanowicz conjecture holds.

(b) N. Terai [20–22] considered the cases $(p, q, r) = (2, 2, 3); (2, 2, 5); (2, 2, r)$, where $r \in \mathbb{P}$. He proved that

(Terai.1) if $a = m(m^2 - 3), b = 3m^2 - 1, c = m^2 + 1$ with m even and b is a prime, and there is a prime l such that $m^2 - 3 \equiv 0 \pmod l$ and $e \equiv 0 \pmod 3$, where e is the order of 2 modulo l, then equation (5) has only the solution $(x, y, z) = (2, 2, 3)$;

(Terai.2) if $a = m|m^4 - 10m^2 + 5|, b = 5m^4 - 10m^2 + 1, c = m^2 + 1$ with m even and b is a prime, and there is an odd prime l such that $ab \equiv 0 \pmod l$ and $e \equiv 0 \pmod 5$, where e is the order of c modulo l, then equation (5) has only the solution $(x, y, z) = (2, 2, 5)$.

In addition, Scott [15] proved a result which implies the following.

THEOREM E. *If $c = 2$, then the Terai–Jeśmanowicz conjecture is true. If c is an odd prime, then there is at most one other solution to the Terai–Jeśmanowicz conjecture.*

In this note, we deal with the Terai–Jeśmanowicz conjecture for the special case $p = q = 2, r$ odd. We have

THEOREM. *If $p = q = 2$, $2 \nmid r$, $c \equiv 5 \pmod 8$, $b \equiv 3 \pmod 4$ and $c \in \mathbb{P}^{\mathbb{N}}$, then the Terai–Jeśmanowicz conjecture holds.*

COROLLARY 1 TO THEOREM. *Let*

$$a = m|m^2 - 3n^2|, \quad b = n|3m^2 - n^2|, \quad c = m^2 + n^2,$$

where $m, n \in \mathbb{N}$, $\gcd(m, n) = 1$. If $m \equiv 2 \pmod 4$, $n \equiv 1 \pmod 4$ and $m^2 + n^2 \in \mathbb{P}^{\mathbb{N}}$, then equation (5) has only the solution $(x, y, z) = (2, 2, 3)$.

COROLLARY 2 TO THEOREM. *Let*

$$a = m|m^4-10m^2n^2+5n^4|, \quad b = n|5m^4-10m^2n^2+n^4|, \quad c = m^2+n^2, \tag{6}$$

where $m, n \in \mathbb{N}$ with $\gcd(m, n) = 1$, $m^2 + n^2 \in \mathbb{P}^{\mathbb{N}}$ and $m \equiv 2 \pmod 4$. If one of the following cases holds, then equation (5) has only the solution $(x, y, z) = (2, 2, 5)$:

CASE 1: $m > \sqrt{2}n$ *and* $n \equiv 3 \pmod 4$;
CASE 2: $m > \sqrt{10}n$;
CASE 3: $n = 1$.

From Corollary 1, we see that if $m \equiv 2 \pmod 4$ and $m^2 + 1 \in \mathbb{P}$, then the equation

$$(m(m^2 - 3))^x + (3m^2 - 1)^y = (m^2 + 1)^z, \quad x, y, z \in \mathbb{N},$$

has only the solution $(x, y, z) = (2, 2, 3)$.

2. Preliminaries.

We will use the following lemmas.

LEMMA 1. *If $2 \nmid r$ and $r > 1$, then all solutions (X, Y, Z) of the equation*

$$X^2 + Y^2 = Z^r, \quad X, Y, Z \in \mathbb{Z}, \ \gcd(X, Y) = 1,$$

are given by

$$X + Y\sqrt{-1} = \lambda_1(X_1 + \lambda_2 Y_1\sqrt{-1})^r, \quad Z = X_1^2 + Y_1^2,$$

where $\lambda_1, \lambda_2 \in \{-1, 1\}$, $X_1, Y_1 \in \mathbb{N}$ and $\gcd(X_1, Y_1) = 1$.

Lemma 1 follows directly from a theorem in the book of Mordell [12], pp. 122–123.

LEMMA 2. *For any $k \in \mathbb{N}$ and any complex numbers α, β, we have*

$$\alpha^k + \beta^k = \sum_{j=0}^{[k/2]} \begin{bmatrix} k \\ j \end{bmatrix} (\alpha + \beta)^{k-2j} (-\alpha\beta)^j,$$

where

$$\begin{bmatrix} k \\ j \end{bmatrix} = \frac{(k-j-1)!k}{(k-2j)!\,j!} \in \mathbb{N} \quad (j = 0, 1, \ldots, [k/2]).$$

It is Formula 1.76 in [10].

LEMMA 3. *Let $a, b, c, p, q, r \in \mathbb{N}$ satisfy the hypotheses of the Terai–Jeśmanowicz conjecture. If $p = q = 2$, $2 \nmid r$, and if $c \equiv 5 \pmod 8$, $2 \mid a$, then*

$$\left(\frac{a}{c}\right) = -1, \qquad \left(\frac{b}{c}\right) = 1,$$

and so $2 \mid x$ in equation (5). *Here $\left(\frac{*}{c}\right)$ denotes the Legendre–Jacobi symbol.*

Proof. Since $p = q = 2$, $2 \nmid r$, we have

$$a^2 + b^2 = c^r, \qquad a, b, c \in \mathbb{N},\ \gcd(a, b) = 1. \tag{7}$$

By Lemma 1, we deduce from (7) that

$$a + b\sqrt{-1} = \lambda_1(m + \lambda_2 n\sqrt{-1})^r, \qquad c = m^2 + n^2, \tag{8}$$

where $\lambda_1, \lambda_2 \in \{-1, 1\}$, $m, n \in \mathbb{N}$ and $\gcd(m, n) = 1$. From (8), we have

$$2\lambda_1 a = (m + \lambda_2 n\sqrt{-1})^r + (m - \lambda_2 n\sqrt{-1})^r,$$
$$2\lambda_1 b\sqrt{-1} = (m + \lambda_2 n\sqrt{-1})^r - (m - \lambda_2 n\sqrt{-1})^r.$$

Hence, by Lemma 2 we have

$$\lambda_1 a = \frac{1}{2} \sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (2m)^{r-2j} (-m^2 - n^2)^j \tag{9}$$
$$= m \sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (4m^2)^{(r-1)/2-j} (-m^2 - n^2)^j,$$

$$\lambda_1 b = \frac{1}{2\sqrt{-1}} \sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (2\lambda_2 n\sqrt{-1})^{r-2j} (m^2 + n^2)^j \tag{10}$$
$$= \lambda_2 n \sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (-4n^2)^{(r-1)/2-j} (m^2 + n^2)^j.$$

Since $c \equiv 5 \pmod 8$, and $2 \mid a$, we see from (8) and (9) that $2 \parallel m$, $2 \nmid n$.

So by (9) and (10), we have

$$\left(\frac{a}{c}\right) = \left(\frac{\lambda_1 a}{c}\right) = \left(\frac{m(4m^2)^{(r-1)/2}}{m^2+n^2}\right) = \left(\frac{m}{m^2+n^2}\right) = -\left(\frac{m/2}{m^2+n^2}\right) = -1,$$

$$\left(\frac{b}{c}\right) = \left(\frac{\lambda_1 b}{c}\right) = \left(\frac{\lambda_2 n(-4n^2)^{(r-1)/2}}{m^2+n^2}\right) = \left(\frac{n}{m^2+n^2}\right) = 1.$$

This completes the proof of Lemma 3.

3. Proof of the Theorem and its corollaries

Proof of Theorem. Since $c \equiv 5 \pmod 8$ and $b \equiv 3 \pmod 4$, we have $2 \mid a$. By Lemma 3, we find that $2 \mid x$. From (5), we have $3^y \equiv 1 \pmod 4$. Hence $2 \mid y$. Then by Theorem D, we deduce that equation (5) has at most one solution (x, y, z), except for

$$(a, b, c) = (5, 3, 2),\ (13, 3, 2),\ (10, 3, 13).$$

Clearly, $(a, b, c) \neq (5, 3, 2)$ since $\max(a, b, c) > 7$, and the equations $13^p + 3^q = 2^r$ and $10^p + 3^q = 13^r$ are all impossible since $p, q, r \geq 2$ (see [23] and [14]). Thus, (5) has only the solution $(x, y, z) = (2, 2, r)$. The Theorem is proved.

Proof of Corollary 1. If $m > n/\sqrt{3}$, then we find that $b = n(3m^2 - n^2) \equiv 3 \pmod 4$. By the Theorem, Corollary 1 holds.

If $m < n/\sqrt{3}$, then

$$a = m(3n^2 - m^2), \quad b = n(n^2 - 3m^2), \quad c = m^2 + n^2.$$

By Lemma 3 and Theorem D, if $2 \mid y$, then Corollary 1 holds. Now assume that $2 \nmid y$. From (5),

$$\left(\frac{n(n^2-3m^2)}{3n^2-m^2}\right) = \left(\frac{b}{3n^2-m^2}\right)^y = \left(\frac{m^2+n^2}{3n^2-m^2}\right)^z. \tag{11}$$

Since $m \equiv 2 \pmod 4$ and $n \equiv 1 \pmod 4$, we have

$$\left(\frac{n(n^2-3m^2)}{3n^2-m^2}\right) = \left(\frac{n^2-3m^2}{3n^2-m^2}\right) = \left(\frac{3n^2-m^2}{n^2-3m^2}\right) = \left(\frac{8m^2}{n^2-3m^2}\right) = -1, \tag{12}$$

$$\left(\frac{m^2+n^2}{3n^2-m^2}\right) = \left(\frac{3n^2-m^2}{n^2+m^2}\right) = \left(\frac{-4m^2}{n^2+m^2}\right) = 1. \tag{13}$$

From (11)–(13), we get $-1 = 1$, a contradiction. This proves the corollary.

Proof of Corollary 2. From the Theorem, it suffices to prove Cases 2 and 3 of Corollary 2. Now we assume that

$$a = m|m^4 - 10m^2n^2 + 5n^4|, \quad b = 5m^4 - 10m^2n^2 + n^4, \quad c = m^2 + n^2,$$

$m > \sqrt{10}n$ and $n \equiv 1 \pmod 4$. Clearly, $m^4 - 10m^2n^2 + 5n^4 \in \mathbb{N}$. If $5 \nmid n$, then we have

$$\begin{aligned}\left(\frac{b}{m^4-10m^2n^2+5n^4}\right) &= \left(\frac{m^4-10m^2n^2+5n^4}{5m^4-10m^2n^2+n^4}\right)\\ &= \left(\frac{5}{5m^4-10m^2n^2+n^4}\right)\left(\frac{5m^4-50m^2n^2+25n^4}{5m^4-10m^2n^2+n^4}\right)\\ &= \left(\frac{-40m^2n^2+24n^4}{5m^4-10m^2n^2+n^4}\right) = \left(\frac{5m^2-3n^2}{5m^4-10m^2n^2+n^4}\right)\\ &= \left(\frac{5m^4-10m^2n^2+n^4}{5m^2-3n^2}\right) = \left(\frac{-7m^2+n^2}{5m^2-3n^2}\right)\\ &= \left(\frac{5}{5m^2-3n^2}\right)\left(\frac{-7(5m^2-3n^2)-16n^2}{5m^2-3n^2}\right) = \left(\frac{5m^2-3n^2}{5}\right) = -1.\end{aligned}$$

If $5 \mid n$ then we also have

$$\left(\frac{b}{m^4-10m^2n^2+5n^4}\right) = -1$$

by a similar method. Moreover

$$\left(\frac{c}{m^4-10m^2n^2+5n^4}\right) = \left(\frac{m^4-10m^2n^2+5n^4}{m^2+n^2}\right) = \left(\frac{16}{m^2+n^2}\right) = 1.$$

Hence, from (5) we have $2 \mid y$. From Lemma 3 we deduce similarly that $2 \mid x$. Then Theorem D implies Case 2. For Case 3 the only remaining case is $m = 2$. Then $a = 38, b = 41, c = 5$. As above we find $2 \mid x$, $2 \mid y$, and Case 3 follows from Theorem D.

4. An open problem. Let $r > 1$ be a given odd number, and let

$$\begin{aligned}a &= m\left|\sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (4m^2)^{(r-1)/2-j}(-m^2-1)^j\right|,\\ b &= \left|\sum_{j=0}^{(r-1)/2} \begin{bmatrix} r \\ j \end{bmatrix} (-4)^{(r-1)/2-j}(m^2+1)^j\right|,\\ c &= m^2+1,\end{aligned}$$

where $m \in \mathbb{N}$ with $m^2 + 1 \in \mathbb{P}$ and $m \equiv 2 \pmod 4$. Clearly, (a, b, c) is a solution of equation (7). Is it possible to prove the Terai–Jeśmanowicz conjecture by the method of this paper under the above condition? When $r = 3, 5$, the answer to the question is "yes".

Acknowledgements. The author would like to thank the referee for his valuable suggestions.

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Received on 21.9.1998
and in revised form on 7.5.1999 (3472)