(iv) The series $\sum_{n=1}^{\infty} a_n(1-xe^{2\pi in})^{-1}$, where $|x| < 1$ and $a$ are real, obviously converges if $\sum |a_n|$ converges. Can one give a less stringent condition? See [194].

(v) A non-homogeneous cubic congruence $f(x, y, z) \equiv 0 \pmod{p}$ has $p^3+O(p)$ solutions, apart from certain exceptions. (This has been substantially proved by Davenport and Lewis, using a result of Dwork; see Quart. J. of Math. (2), 14 (1963), 154-159.)

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Further developments in the comparative prime-number theory I

by

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1. In a report dated from June 19, 1871, which was written to support the designation of Chebyshev as foreign member of the Academy in Berlin and signed among others by Kronecker, Kummer, and Weierstrass, one reads the following passage (see [1]).

"...Endlich ist Herr Tschebyscheff der erste Mathematiker, welcher für die Anzahl der Primzahlen bis zu einer hohen Grenze den Ueberschluss der Primzahlen der Form $4n+3$ über diejenigen von der Form $4n+1$ constatirt und für den asymptotischen Ausdruck $\sqrt{x}/\log x$ angegeben hat."

What was behind these lines? Chebyshev wrote in a letter in 1853, i.e. a few years after Dirichlet proved that for $(k, l_1) = (k, l_2) = 1$ in a weak sense the number of primes $= l(\mod k)$ is asymptotically equal to that of the primes $= l_1(\mod k)$, that he is in possession of a theorem which can be popularly expressed so that there are more primes of the form $4n+3$ than of $4n+1$. He meant by that (according to his letter, which is printed in [2]) that

$$\lim_{x \to +\infty} \sum_{p \leq x} (-1)^{(p-1)/2} e^{-px} = -\infty$$

where $p$ denotes always primes and stated also the existence of a sequence

$$x_1 < x_2 < \ldots \to \infty$$

such that for $\nu \to \infty$

$$\frac{n(x_4, 4, 3) - n(x_1, 4, 1)}{(\nu x/\log x)} \to 1.$$ 

(Here and later $n(x, k, l)$ stands for the number of primes not exceeding $x$ which are $\equiv l(\mod k)$, $(k, l) = 1$, $c_1, c_2, \ldots$ positive, explicitly calculable constants.) Most probably in Germany nobody read the original