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On an equation with prime numbers

by

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1. Introduction. B. I. Segal ([13], [14]) was the first to consider in 1933 additive problems with non-integer degrees. He studied the inequality

$$|x_1^c + x_2^c + \ldots + x_k^c - N| < \varepsilon \tag{1}$$

and the equation

$$[x_1^c] + [x_2^c] + \ldots + [x_k^c] = N, \tag{2}$$

where $c > 1$ is not an integer, and proved in both cases that there exists $k_0(c)$ such that the corresponding problem has solutions if $k \geq k_0$ and N is sufficiently large. Later Deshouillers [4] and Arkhipov and Zhitkov [1] improved Segal's result on (2). One may also mention the papers of Deshouillers [5] and Gritsenko [7], where the equation (2) in two variables was considered.

In 1952 I. I. Piatetski-Shapiro [12] considered (1) with $x_1, \ldots, x_k$ restricted to prime numbers. Let $H(c)$ denote the least k such that the inequality (1) with fixed $\varepsilon > 0$ has solutions in prime numbers for every sufficiently large real N. Piatetski-Shapiro proved that

$$\limsup_{c\to\infty} \frac{H(c)}{c \log c} \leq 4.$$

He also proved that $H(c) \leq 5$ for $1 < c < 3/2$. The theorem of Goldbach–Vinogradov [16] motivates the conjecture that for c close to 1, $H(c) \leq 3$. This was proved by D. I. Tolev [15]. He showed that if $1 < c < 15/14$ and $\varepsilon = N^{-(1/c)(15/14-c)} \log^9 N$ then the quantity

$$D(N) := \sum_{|p_1^c + p_2^c + p_3^c - N| < \varepsilon} \log p_1 \log p_2 \log p_3$$

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is positive for a sufficiently large N. Recently Y. C. Cai [3] improved the upper bound for c to $13/12$.

In [10] Laporta and Tolev considered the corresponding equation of the type (2). For $1 < c < 17/16$ they proved an asymptotic formula for the sum

$$R(N) := \sum_{[p_1^c]+[p_2^c]+[p_3^c]=N} \log p_1 \log p_2 \log p_3 .$$

In the present paper we improve the range of c they obtained.

Theorem 1. *Assume that $1 < c < 12/11$ and $\delta > 0$ is arbitrary small. Then for any sufficiently large integer N we have the asymptotic formula*

$$R(N) = \frac{\Gamma^3(1+1/c)}{\Gamma(3/c)} N^{3/c-1} + \mathcal{O}(N^{3/c-1} \exp(-(\log N)^{1/3-\delta})).$$

We also improve the result from [3]. We obtain an asymptotic formula for the sum $D(N)$. Since the proof is similar to the proof of Theorem 1, we omit it.

Theorem 2. *Assume that $1 < c < 11/10$ and $\delta > 0$ is arbitrary small. Then for any sufficiently large real N and $\varepsilon \ge N^{-(1/c)(11/10-c)+\nu}$ for some $\nu > 0$ we have the asymptotic formula*

$$D(N) = 2\varepsilon \frac{\Gamma^3(1+1/c)}{\Gamma(3/c)} N^{3/c-1} + \mathcal{O}(\varepsilon N^{3/c-1} \exp(-(\log N)^{1/3-\delta})).$$

The range of c in both problems depends on the estimate of an exponential sum over primes. In [10] and [15] Vaughan's identity and the exponent pair $(1/2, 1/2)$ are used. We derive Theorem 1 from a more precise estimate of this sum (Lemma 5 below). To prove it we use the identity of Heath-Brown [8], van der Corput's method as described in Chapters 2 and 3 of [6] and the estimate of a double exponential sum due to Kolesnik [9].

2. Notation. Since for $1 < c < 17/16$ Theorem 1 is proved in [10], we can assume that $17/16 \le c < 12/11$. In this paper $\eta > 0$ is a fixed small number depending only on c; $P = N^{1/c}$; $\omega = P^{1-c-\eta}$; $p, p_1, \ldots$ are primes; $\alpha \in (0,1)$; ε is an arbitrary small positive number, not necessarily the same in different appearances. We use $[x]$, $\{x\}$ and $\|x\|$ for the integral part of x, fractional part of x and the distance from x to the nearest integer respectively. $\Lambda(n)$ is von Mangoldt's function. Moreover,

- $e(x) = \exp(2\pi i x)$;
- $f(x) \ll g(x)$ means that $f(x) = \mathcal{O}(g(x))$;
- $f(x) \asymp g(x)$ means that $f(x) \ll g(x) \ll f(x)$;
- $x \sim X$ means that x runs through a subinterval of $[X, 2X]$;
- $f(x_1, \ldots, x_n) \sim_\Delta g(x_1, \ldots, x_n)$ means that

$$\frac{\partial^{j_1+\ldots+j_n}}{\partial x_1^{j_1}\ldots\partial x_n^{j_n}} f(x_1,\ldots,x_n) = \frac{\partial^{j_1+\ldots+j_n}}{\partial x_1^{j_1}\ldots\partial x_n^{j_n}} g(x_1,\ldots,x_n)(1+\mathcal{O}(\Delta))$$

for all n-tuples $(j_1,\ldots,j_n)$ for which it makes sense.

We use sums of two types, which we define in the following way:

- *type* I *sums*:

$$\sum_{\substack{m\sim M,\, n\sim L\\ mn\sim X}}\sum a_m F(mn),$$

- *type* II *sums*:

$$\sum_{\substack{m\sim M,\, n\sim L\\ mn\sim X}}\sum a_m b_n F(mn),$$

where the coefficients satisfy the conditions $a_m \ll m^{\varepsilon}$, $b_n \ll n^{\varepsilon}$.

We define

$$\sigma = \exp((\log N)^{1/3-\delta}).$$

We also set

$$S(\alpha) = \sum_{p\le P} \log p\cdot e(\alpha[p^c]),$$

$$R_i = \int\limits_{\Omega_i} S^3(\alpha)e(-\alpha N)\, d\alpha \qquad (i=1,2)$$

where $\Omega_1 = (-\omega,\omega)$ and $\Omega_2 = (\omega, 1-\omega)$.

3. Some preliminary results

LEMMA 1. *Let $\mathcal{D}$ be a subdomain of the rectangle $\{(x,y)\mid X\le x\le 2X$, $Y\le y\le 2Y\}$ $(X\ge Y)$ such that any line parallel to any coordinate axis intersects it in $\mathcal{O}(1)$ line segments. Let α, β be real numbers, $\alpha\beta\ne 0$, $\alpha+\beta\ne 1$, $\alpha+\beta\ne 2$, and let $f(x,y)$ be a real sufficiently many times differentiable function such that $f(x,y)\sim_{\Delta} Ax^{\alpha}y^{\beta}$ throughout $\mathcal{D}$. Setting $N=XY$, $F=AX^{\alpha}Y^{\beta}$, we have*

$$\Big|\sum_{(x,y)\in\mathcal{D}} e(f(x,y))\Big| \ll (NF)^{\varepsilon}(F^{1/3}N^{1/2} + NY^{-1/2} + N^{5/6} + NF^{-1/4} + NF^{-1/8}X^{-1/8} + \Delta^{2/5}F^{1/5}N^{9/10}X^{-2/5} + \Delta^{1/4}NX^{-1/4}).$$

Proof. This is a version of Theorem 1 of [9]. The proof may be found in [11].

LEMMA 2. *Let $3<U<V<Z<X$ and suppose that $Z-1/2\in\mathbb{N}$, $X\ge 64Z^2U$, $Z\ge 4U^2$, $V^3\ge 32N$. Assume further that $F(n)$ is a complex-*

valued function such that $|F(n)| \le 1$. *Then the sum*

$$\sum_{n\sim X} \Lambda(n)F(n)$$

may be decomposed into $\mathcal{O}(\log^{10} X)$ *sums, each either of type* I *with* $L > Z$, *or of type* II *with* $U < L < V$.

Proof. This is Lemma 3 of [8].

Lemma 3. *Let* x *not be an integer*, $\alpha \in (0,1)$, $H \ge 3$. *Then*

$$e(-\alpha\{x\}) = \sum_{|h|\le H} c_h(\alpha)\, e(hx) + \mathcal{O}\bigg(\min\bigg(1, \frac{1}{H\|x\|}\bigg)\bigg)$$

where

$$c_h(\alpha) = \frac{1-e(-\alpha)}{2\pi i(h+\alpha)}.$$

Proof. See Lemma 12 of [2].

In the following lemma we estimate the number $\mathcal{N}(\Delta)$ of quadruples (h_1, h_2, n_1, n_2) for which $h_1, h_2 \sim H$, $n_1, n_2 \sim N$ and

$$|(h_1+\alpha)n_1^c - (h_2+\alpha)n_2^c| \le \Delta.$$

Lemma 4. *Suppose that* $c \ne 0$, $\alpha \in (0,1)$, $\Delta > 0$, $H \ge 3$ *and* N *is large. Then*

$$\mathcal{N}(\Delta) \ll \Delta H N^{2-c} + H^{3/2} N \log(2HN).$$

Proof. We follow the approach of D. R. Heath-Brown [8]. We define the quantity

$$\mathcal{N}(\Delta; a, b) = \#\{(h_1, h_2, n_1, n_2) \mid h_1, h_2 \sim H,\ (h_1, h_2) = a,\ n_1, n_2 \sim N,\\ (n_1, n_2) = b,\ |(h_1+\alpha)n_1^c - (h_2+\alpha)n_2^c| \le \Delta\}$$

which we are going to estimate. If $h_1, h_2 \sim H$, $n_1, n_2 \sim N$ and $|(h_1 + \alpha)n_1^c - (h_2+\alpha)n_2^c| \le \Delta$ we have

$$\bigg|\bigg(\frac{n_1}{n_2}\bigg)^c - \frac{h_2+\alpha}{h_1+\alpha}\bigg| \ll \frac{\Delta}{HN^c}, \qquad \bigg|\frac{h_2}{h_1} - \frac{h_2+\alpha}{h_1+\alpha}\bigg| \ll \frac{1}{H},$$

hence

$$\bigg|\frac{h_2}{h_1} - \bigg(\frac{n_1}{n_2}\bigg)^c\bigg| \ll \frac{1}{H} + \frac{\Delta}{HN^c}. \tag{3}$$

We also have

$$\bigg|\frac{n_1}{n_2} - \bigg(\frac{h_2+\alpha}{h_1+\alpha}\bigg)^{1/c}\bigg| \ll \frac{\Delta}{HN^c}. \tag{4}$$

From (3) and (4), arguing as on pp. 256–257 of [8], we obtain

$$\mathcal{N}(\Delta; a, b) \ll \frac{\Delta}{HN^c} \cdot \frac{H^2N^2}{a^2b^2} + \min\left(\frac{H^2}{a^2}, \frac{N^2}{b^2} + \frac{HN^2}{a^2b^2}\right).$$

Since

$$\mathcal{N}(\Delta) \le \sum_{a \le 2H} \sum_{b \le 2N} \mathcal{N}(\Delta; a, b),$$

the proof of the lemma is complete.

4. The main lemma

LEMMA 5. *Suppose that $X > P^{9/10}$, $H = \sigma X^{c-1}$ and $c_h(\alpha)$ are complex numbers such that $|c_h(\alpha)| \ll (1 + |h|)^{-1}$. Then, uniformly with respect to $\alpha \in (\omega, 1-\omega)$, we have*

$$T(\alpha) = \sum_{|h| \le H} c_h(\alpha) \sum_{n \sim X} \Lambda(n)\, e((h+\alpha)n^c) \ll X^{2-c-\varrho}$$

for some sufficiently small $\varrho > 0$, depending only on c.

Proof. We use Lemma 2 with $F(n) = e((h+\alpha)n^c)$ to reduce the estimation of $T(\alpha)$ to the estimation of the sums

$$T_i(\alpha) = \sum_{|h| \le H} c_h(\alpha) \sum\nolimits_i \qquad (i = 1, 2)$$

where $\sum_1$, $\sum_2$ are type I and type II sums, respectively. We choose the parameters U, V, Z as follows:

$$U = X^{2c-2+2\varrho}/256, \quad V = 4X^{1/3}$$

and

$$Z = \begin{cases} [X^{(16c-16)/3+3\varrho}] + 1/2 & \text{if } 17/16 \le c < 14/13, \\ [X^{(13c-13)/3+3\varrho}] + 1/2 & \text{if } 14/13 \le c < 13/12, \\ [X^{(20c-21)/2+5\varrho}] + 1/2 & \text{if } 13/12 \le c < 12/11\ . \end{cases}$$

Let us consider $T_2(\alpha)$. We have

$$T_2(\alpha) \ll \max_{\omega \le \lambda \le 2} |T_2^{(1)}(\lambda)| + (\log X) \max_{2 \le J \le H} |T_2^{(2)}(\alpha; J)| \tag{5}$$

where

$$T_2^{(1)}(\lambda) = \sum_{m \sim M} \sum_{n \sim L} a_m b_n e(\lambda(mn)^c),$$

$$T_2^{(2)}(\alpha; J) = \sum_{h \sim J} c_h(\alpha) \sum_{m \sim M} \sum_{n \sim L} a_m b_n e((h+\alpha)(mn)^c).$$

First we estimate $T_2^{(2)}(\alpha;J)$. We obtain

$$T_2^{(2)}(\alpha;J) \ll \frac{X^\varepsilon}{J} \sum_{m\sim M} \sum_{q\le Q} \Big| \sum_{(h,n)\in\mathcal{I}_q} d(h,n)\, e((h+\alpha)(mn)^c)\Big|$$

where $|d(h,n)|\le 1$, $Q>1$ is a parameter to be defined later and for $q\le Q$,

$$\mathcal{I}_q = \{(h,n) \mid h\sim J,\ n\sim L,\ 5(q-1)JL^c < Q(h+\alpha)n^c \le 5qJL^c\}.$$

So, using the Cauchy inequality, we get

$$|T_2^{(2)}(\alpha;J)|^2 \ll \frac{X^\varepsilon MQ}{J^2} \sum_{\substack{h_1,h_2\sim J\\ n_1,n_2\sim L\\ |\lambda|\le 5JL^c/Q}}\sum \Big| \sum_{m\sim M} e(\lambda m^c)\Big|$$

where $\lambda=(h_1+\alpha)n_1^c-(h_2+\alpha)n_2^c$. We estimate the innermost sum trivially if $|\lambda|\le M^{-c}$, and using the exponent pair $(13/40, 11/20)$ otherwise. From Lemma 4 we now obtain

$$\begin{aligned}
&|T_2^{(2)}(\alpha;J)|^2\\
&\quad\ll \frac{X^\varepsilon MQ}{J^2}(M\,\mathcal{N}(M^{-c})\\
&\qquad + \max_{M^{-c}\le\Delta\le 5JL^c/Q}(\Delta^{13/40}M^{(9+13c)/40} + \Delta^{-1}M^{1-c})\mathcal{N}(\Delta))\\
&\quad\ll X^\varepsilon(J^{-1/2}M^2LQ + J^{13/40}M^{(49+13c)/40}L^{(80+13c)/40}Q^{-13/40}\\
&\qquad + J^{-1}M^{2-c}L^{2-c}Q + J^{-7/40}M^{(49+13c)/40}L^{(40+13c)/40}Q^{27/40}).
\end{aligned}$$

We choose Q via Lemma 2.4 of [6] and the conditions on J, M and L imply

$$\max_{2\le J\le H}|T_2^{(2)}(\alpha;J)| \ll X^{2-c-\varrho+\varepsilon}. \tag{6}$$

Let us now estimate $T_2^{(1)}(\lambda)$. Using the Cauchy inequality and Lemma 2.5 of [6] we get

$$|T_2^{(1)}(\lambda)|^2 \ll X^\varepsilon\left(\frac{M^2L^2}{Q} + \frac{ML}{Q}\sum_{q\le Q}\sum_{n\sim L}\Big|\sum_{m\sim M} e(\lambda((n+q)^c-n^c)m^c)\Big|\right)$$

where $Q\ll L$ is a positive integer. We apply the exponent pair $(13/40, 11/20)$ to the innermost sum and choose Q via Lemma 2.4 of [6] to obtain

$$\begin{aligned}
|T_2^{(1)}(\lambda)|^2 \ll X^\varepsilon(&M^2L + \lambda^{13/40}M^{(49+13c)/40}L^{(67+13c)/40}\\
&+\lambda^{13/53}M^{(75+13c)/53}L^{(93+13c)/53})
\end{aligned}$$

and using the conditions on M, L and λ we get

$$\max_{\omega\le\lambda\le 2}|T_2^{(1)}(\lambda)| \ll X^{2-c-\varrho+\varepsilon}. \tag{7}$$

The needed estimate for $T_2(\alpha)$ follows from (5)–(7).

Let us now consider $T_1(\alpha)$. We have

$$T_1(\alpha) \ll X^{\varepsilon} \max_{|\lambda|\in(\omega,H+1)} \sum_{m\sim M} \Big| \sum_{n\sim L} e(\lambda(mn)^c)\Big|. \tag{8}$$

If $L \geq X^{(57c-49)/23+3\varrho}$ we estimate the sum over n using the exponent pair $(8/41, 26/41)$ to obtain

$$|T_1(\alpha)| \ll X^{2-c-\varrho+\varepsilon}. \tag{9}$$

Otherwise we first use the Cauchy inequality and Lemma 2.5 of [6] to the sum on the right-hand side of (8) and obtain

$$|T_1|^2 \ll X^{\varepsilon}\bigg(\frac{M^2L^2}{Q} + \frac{ML}{Q}\sum_{q\sim J}\sum_{n\sim L}\sum_{m\sim M} e(f(m,n,q))\bigg)$$

where $f(m,n,q) = \lambda((n+q)^c - n^c)m^c$, $J \leq Q/2$ and $Q \ll L$ is a parameter to be chosen later. Then we apply the Poisson summation formula (Lemma 3.6 of [6]) to the sums over m and n successively and Abel's transformation:

$$\begin{aligned}
&\sum_{q}\sum_{m,n} e(f(m,n,q))\\
&\quad = \sum_{q,n}\sum_{\mu}\bigg(\frac{\partial^2 f(m_\mu,n,q)}{\partial m^2}\bigg)^{-1/2} e(1/8 + f(m_\mu,n,q) - \mu m_\mu)\\
&\qquad + \mathcal{O}(MLJF^{-1/2} + LJ\log X)\\
&\quad \ll MF^{-1/2}\Big|\sum_{q,\mu}\sum_{n} e(f_1(\mu,q,n))\Big| + XJF^{-1/2} + LJ\log X\\
&\quad \ll MF^{-1/2}\Big|\sum_{q,\mu}\sum_{\nu}\bigg(\frac{\partial^2 f_1(\mu,q,n_\nu)}{\partial n^2}\bigg)^{-1/2} e(1/8 + f_1(\mu,q,n_\nu) - \nu n_\nu)\Big|\\
&\qquad + MF^{-1/2}JFM^{-1}(LF^{-1/2} + \log X) + XJF^{-1/2} + LJ\log X\\
&\quad \ll MLF^{-1}\Big|\sum_{q,\mu,\nu} e(g(\mu,\nu,q))\Big| + F^{1/2}J\log X + LJ\log X + XJF^{-1/2}
\end{aligned}$$

where $F = \lambda JM^cL^{c-1}$, $f_1(\mu,q,n) = f(m_\mu,n,q) - \mu m_\mu$,

$$g(\mu,\nu,q) = f_1(\mu,q,n_\nu) - \nu n_\nu \sim_{\varDelta} c_0(\lambda q)^{1/(2-2c)}\nu^{1/2}\mu^{c/(2c-2)} \asymp F,$$

c_0 is a constant depending only on c, $\varDelta = J/L$, $\nu \asymp FL^{-1}$, $\mu \asymp FM^{-1}$.

Hence

$$\begin{aligned}
X^{-\varepsilon}|T_1|^2 \ll X^2Q^{-1} + X^2F^{-1}Q^{-1}\sum_{q\sim J}\Big|\sum_{\mu\asymp FM^{-1}}\sum_{\nu\asymp FL^{-1}} e(g(\mu,\nu,q))\Big|\\
+ X^2F^{-1/2} + XL + XF^{1/2}.
\end{aligned} \tag{10}$$

If $X^{1/2} \le L < X^{(57c-49)/23+3\varrho}$ we estimate the sum over μ,ν in (10) using Lemma 1 with $X = FM^{-1}$, $Y = FL^{-1}$ and $f(x,y) = g(\mu,\nu,q)$. We get

$$\begin{aligned} X^{-\varepsilon}|T_1|^2 \ll{}& X^2Q^{-1} + F^{1/3}X^{3/2} + XF^{1/2}L^{1/2} \\ &+ X^{7/6}F^{2/3} + X^{3/2}F^{3/5}J^{2/5}L^{-4/5} + XF^{3/4}M^{1/8} \\ &+ J^{1/4}X^{5/4}F^{3/4}L^{-1/2} + X^2F^{-1/2} + XL. \end{aligned}$$

Now we substitute the expression for F in the last estimate and choose Q via Lemma 2.4 of [6]. We obtain (9).

If $Z \le L < X^{1/2}$ we interchange the roles of μ and ν and prove (9) again.

This completes the proof of the lemma.

5. Proof of Theorem 1. It is easy to see that

$$R(N) = \int_0^1 S^3(\alpha)\, e(-\alpha N)\, d\alpha = R_1 + R_2.$$

The integral R_1 is studied by Laporta and Tolev [10], pp. 928–929. They proved that if $1 < c < 17/16$ then

$$R_1 = \frac{\Gamma^3(1+1/c)}{\Gamma(3/c)} N^{3/c-1} + \mathcal{O}(\sigma^{-1}N^{3/c-1})$$

but the same argument shows that this asymptotic formula holds for $1 < c < 3/2$. Hence the theorem follows from the estimate

$$R_2 \ll \sigma^{-1}P^{3-c}. \tag{11}$$

It is not difficult to prove that

$$R_2 \ll P\log P \max_{\alpha\in\Omega_2} |S(\alpha)|.$$

To prove (11) it remains to show that

$$\max_{\alpha\in\Omega_2} |S(\alpha)| \ll \sigma^{-1}P^{2-c}.$$

We have

$$S(\alpha) = \sum_{n\le P} \Lambda(n)\, e(\alpha n^c)\, e(-\alpha\{n^c\}) + \mathcal{O}(P^{1/2}).$$

So, it is sufficient to prove that for X satisfying $P^{9/10} < X \le P$,

$$S_1(\alpha) = \sum_{n\sim X} \Lambda(n)\, e(\alpha n^c)\, e(-\alpha\{n^c\}) \ll \sigma^{-1}X^{2-c}.$$

Using Lemma 3 with $x = n^c$ and $H = \sigma X^{c-1}$ we obtain

$$S_1(\alpha) = \sum_{|h|\le H} c_h(\alpha) \sum_{n\sim X} \Lambda(n)\, e((h+\alpha)n^c) + \mathcal{O}\bigg(\log X \sum_{n\sim X} \min\Big(1, \frac{1}{H\|n^c\|}\Big)\bigg).$$

The estimation of the error term in the above equality is standard (see [8], pp. 245–246). Hence (11) follows from Lemma 5.

The proof of Theorem 1 is complete.

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