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# Remarks on a paper of Tallini

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This paper is strictly connected with the paper [3] by G. Tallini. For definitions and notation used here see that paper; $u$ denotes always a positive integer.

In § 3 of [3] the following theorem is proved:

*If there exists a complete $K^3_{r,q}$ not contained in any cap of kind two then*

$$K = \frac{q-3+\sqrt{8q^{r+1}+q^2-6q+1}}{2(q-1)} \tag{1}$$

*is an integer and the equation*

$$qH^2-(2K+q-4)H+\Big(K^2-K-2\sum_{l=0}^{r-1}q^l\Big) = 0 \tag{2}$$

*has two positive integer roots $H_1$ and $H_2$.*

*Further there are given effectively $5^3_{3,2}$ and $11^3_{4,3}$ which are not contained in any cap of kind two.*

We find here all $K^3_{r,q}$ $(q = 2, 3, 4)$ which are not contained in any cap of kind two.

Suppose that $K^3_{r,2}$ is not contained in any cap of kind two. Then $2^{r+4}-7 = u^2$. T. Nagell proved in [2] that if $r \geqslant 3$, then $r = 3$, $u = 11$; $r = 11$, $u = 181$. For $q = 2$, $r = 3$, $K = 5$ equation (2) has two positive integer roots $H_1 = 3$, $H_2 = 1$ and for $q = 2$, $r = 11$, $K = 90$ it has no integer root.

Suppose that $K^3_{r,3}$ is not contained in any cap of kind two. Then $\frac{1}{2}(3^{r+1}-1) = (u/4)^2$. E. Fauquembergue proved in [1] that if $r \geqslant 3$, then $r = 4$, $u/4 = 11$, hence $K = 11$. Equation (2) has for $q = 3$, $r = 4$, $K = 11$ two positive integer roots $H_1 = 5$, $H_2 = 2$.

Suppose that $K^3_{r,4}$ is not contained in any cap of kind two. Then $2^{2r+5}-7 = u^2$ $(r \geqslant 3)$ and by result of Nagell quoted above $r = 5$, $u = 181$. Hence $K$ is not an integer.

In virtue of the results of Tallini quoted above the only $K^3_{r,q}$ $(q = 2, 3, 4)$ not contained in any cap of kind two are the sets projectively equivalent to $5^3_{3,2}$ and $11^3_{4,3}$ constructed in [3]. This result is stronger than some of the statements on page 25 of [3].

**References**

[1] E. Fauquembergue, Mathesis 4 (1894), pp. 169-170.

[2] T. Nagell, *Solution to problem* 2, Norsk Matematisk Tidsskrift 30 (1948), pp. 62-64.

[3] G. Tallini, *On caps of kind s in a Galois r-dimensional space*, Acta Arith. 7 (1961), pp. 19-28.

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