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On a problem by H. Steinhaus *

by

K. Chikawa, K. Iséki, and T. Kusakabe (Kôbe)

One of the present authors [1] considered a problem by H. Steinhaus. The original problem (for $k = 2$) was formulated in his recent book, *Sto zadań* (100 problems, in Polish). For any natural number

$$a = 10^{n-1}a_n + 10^{n-2}a_n + \ldots + 10^2 a_3 + 10a_2 + a_1$$

expressed in the decimal system, we make the sum a_1 of the kth power of each digit a_i $(i = 1, 2, \ldots, n)$ of a:

$$a_1 = a_n^k + a_{n-1}^k + \ldots + a_3^k + a_2^k + a_1^k$$

and for a_1, we make the sum a_2 of kth power of each digit of a_1. We repeat such a calculation, then we have a sequence $a_1, a_2, \ldots, a_n, \ldots$ for a given number a. For $k = 3$, $a = 2$, we have

$$8,\ 512,\ 134,\ 92,\ 737,\ 713,\ 371,\ 371,\ \ldots$$

For $k = 3$, $a = 28$, we have

$$520,\ 133,\ 55,\ 250,\ 133,\ 55,\ \ldots$$

Therefore, we have a cyclic part 55, 250, 133 with length 3. For any k, we proved [2] that each sequence $\{a_n\}$ contains only a finite number of different numbers. K. Iséki [1] calculated all cyclic parts for $k = 3$, and H. Steinhaus gave a complete solution for $k = 2$.

For $k = 4$, we found the following cyclic parts by using calculating punch (IBM 602A) of the Kôbe University. There are four cyclic parts of length 1:

$$1;\quad 1634;\quad 8208;\quad 9474,$$

there is a cyclic part of length 2:

$$2178,\ 6514$$

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and there is a cyclic part of length 7:

13139, 6725, 4338, 4514, 1138, 4179, 9219.

Therefore *there are six different cyclic parts for* $k = 4$.

We are now calculating the cyclic parts for $k = 5$ by an automatic computer FACOM as well as IBM 602 A.

References

[1] K. Iséki, *A problem of number theory,* Proceedings of Japan Academy 36 (1960), 578-583.

[2] K. Iséki, *Necessary results for computation of cyclic parts in Steinhaus problem,* Proceedings of Japan Academy 36 (1960), 650-651.

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Computation of cyclic parts of Steinhaus problem for power 5 *

by

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This paper is concerned with an arithmetic problem. As the problem was appeared in H. Steinhaus, *Sto zadań* (100 problems), we call it Steinhaus problem. (For the related terminologies, see [1], [2]). Some of the present writers found all cyclic parts of Steinhaus problem for powers 3 and 4. (For power 4, see [1]). In this paper, we shall decide all cyclic parts for power 5.

For the purpose, we calculated the cyclic parts of natural numbers less than 3×10^5, as stated in [2]. For the numerical calculation, we used two different types of the automatic computers: Fuji Automatic Computer (FACOM) 128 B and IBM punched cards system 602 A. The calculation from 2 to 10^5 and from 2×10^5 to 3×10^5 was automatically done by FACOM 128 B, the other by IBM 602. We found the following cyclic parts:

length 1:						
	1					(1)
	54748					(247)
	93084					(3489)
	92727					(22779)
	194979					(147999)
length 2:						
	145	4150				(145)
	76438	58618				(199)
	157596	89883				(38889)
length 4:						
	50062	10933	59536	73318		(4)
length 6:						
	44155	8299	150898	127711	33649	
	68335					(16)
length 10:						
	83633	41273	18107	49577	96812	
	99626	133682	41063	9044	61097	(5)
	92873	108899	183635	44156	12950	
	62207	24647	26663	23603	8294	(17)

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