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Corrigendum to the paper "On the distribution of *s*-dimensional Kronecker sequences"

Acta Arith. 51 (1988), 335–347

by

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In the paper "On the distribution of s-dimensional Kronecker sequences" (Acta Arith. 51 (1988), 335–347) there are some inaccuracies in the proofs and also in the statement of some results. In the following I will give a correction of these errors. I want to thank very much G. Turnwald in Tübingen who has pointed out these inaccuracies in Math. Reviews 90f:11065.

First of all, on page 336, p_j and θ_j should be defined in the form

$$\alpha_j = \frac{p_j}{q} + \frac{\theta_j}{q \cdot q_{i+1}^{1/s}} \qquad \text{for } j = 1, \ldots, s \text{ with } |\theta_j| \le 1\,,$$

and on page 337, Γ_i should be defined as the lattice spanned by $(\frac{p_1}{q}, \ldots, \frac{p_s}{q})$ and by $\boldsymbol{Z}^s$.

In the proof of Lemma 2 the assumption $(p_1, q) = 1$ actually is a restriction of generality, so that I give another proof.

Proof of Lemma 2. We have $\det(\Gamma_i) = 1/q$. Let $\mathcal{F}$ be a covering of $\boldsymbol{R}^s$ by fundamental regions F of Γ_i. Let B be a convex set in I^s. The area of the set of all $F \in \mathcal{F}$ for which the intersection with the boundary of B is not empty, is at most $c(s)\lambda_s$, with an absolute constant $c(s)$. Because to every F in the interior of B we can attach exactly one point $\overline{w}_q$ on the boundary of F, and since $\lambda(F) = 1/q$, we have $\overline{J}_q \le c_3(s)\lambda_s$.

As a lower bound for $\overline{J}_q$ we get quite analogously to the method in [2], Beispiel c, applied to the lattice Γ_i:

$$\frac{c_4(s)}{q\lambda_1\lambda_2\ldots\lambda_{s-1}} \le \overline{J}_q.$$

By the Theorem of Minkowski on successive minima and because of $\overline{M}_{q'} \le \lambda_1 \le s^{1/2}\overline{M}_{q'}$ the result follows.

Since Davenport and Mahler [1] actually have shown that for every pair (α_1, α_2) of reals, for every $\varepsilon > 0$, there are infinitely many q, p_1, $p_2 \in \boldsymbol{Z}$ with

$$\alpha_i = \frac{p_i}{q} + \frac{\theta_i}{q^{3/2}}, \quad i = 1, 2, \quad \text{and} \quad \theta_1^2 + \theta_2^2 \le \frac{2}{23^{1/2}} + \varepsilon,$$

Lemma 8 in [3] has to be stated in the following form:

LEMMA 8. *For all* $(\alpha_1, \alpha_2) \in \boldsymbol{R}^2$ *we have*

$$\limsup_{N\to\infty} N^{1/2} J_N \ge \frac{1}{2}\left(1 - \frac{2}{23^{1/4}}\right) = 0.0433\ldots$$

For the "only if" part of Theorem 1 we need a Lemma 7a instead of Lemma 7.

LEMMA 7a. *Let* $i \in \boldsymbol{N}$ *and* $q := q_i$ *be such that*

$$4s^{1/2} q M_q \lambda_1 \ldots \lambda_{s-1} \le 1.$$

Then with an absolute constant $c(s)$ *we have for* $N = Bq$ *with* $B := [1/(4s^{1/2} q M_q \lambda_1 \ldots \lambda_{s-1})]$:

$$N J_N \ge \frac{c(s)}{q M_q (\lambda_1 \ldots \lambda_{s-1})^2}.$$

Proof follows directly from the proof of Lemma 7 in [3].

Proof of the "only if" part of Theorem 1. If L is not extremal, then for every $\varepsilon > 0$ there is a q with $q^{1/s} M_q < \varepsilon$.

By Minkowski's Theorem on successive minima we have

$$\lambda_1 \ldots \lambda_{s-1} q^{1-1/s} < c_1 \quad \text{for every } q \ \ (c_1 := c_1(s) > 0).$$

Let $\varepsilon < (4s^{1/2} c_1)^{-1}$. Then for q as above we have $4s^{1/2} q M_q \lambda_1 \ldots \lambda_{s-1} \le 1$. Therefore Lemma 7a holds and with $N = Bq$ we have

$$N^{1/s} J_N \ge \frac{1}{(Bq)^{1-1/s}} \cdot \frac{c(s)}{q M_q (\lambda_1 \ldots \lambda_{s-1})^2} \ge \frac{c_2(s)}{\varepsilon^{1/s}}$$

and the result follows.

A corrected form of Theorem 2(a) is the following (Theorem 2(b) is not true in the stated form):

THEOREM 2a. *If for a* $c_1 > 0$ *and a* $\sigma \ge 1/2$ *we have* $q_{i+1}^{\sigma} M_{q_i} \ge c_1$ *for all* i, *then for all* N *we have*

$$N^{1-\sigma(s-1)} J_N \le c_2 (\max_{i \le i(N)} a_i)^{1-\sigma(s-1)}.$$

(*Here* $i(N)$ *is such that* $q_{i(N)} \le N < q_{i(N)+1}$.)

Proof. By Lemma 6 we have, with $\tau := \sigma(s-1)$,

$$N^{1-\tau} J_N \le c_8 \sum_{i=1}^{r} \frac{b_i}{b_r^\tau} \cdot \frac{q_i^\tau}{q_r^\tau (q_i^\sigma M_{q_{i-1}})^{s-1}}$$
$$\le c_9 \left(b_r^{1-\tau} + b_{r-1}^{1-\tau} + b_{r-2}^{1-\tau} \left(\frac{q_{r-1}}{q_r} \right)^\tau + \ldots \right)$$
$$\le c_9 (\max_{i \le i(N)} a_i)^{1-\tau}.$$

Finally, from the new form of Theorem 2a we now have

THEOREM 3. *For $s \ge 2$ and for almost all $(\alpha_1, \ldots, \alpha_s)$ in $\mathbf{R}^s$ in the sense of Lebesgue measure, we have for every $\varepsilon > 0$*

$$J_N = O(N^{-1/s} (\log N)^{(1/s)+\varepsilon}).$$

References

[1] H. Davenport and K. Mahler, *Simultaneous Diophantine approximation*, Duke Math. J. 13 (1946), 105–111.

[2] G. Larcher, *Über die isotrope Diskrepanz von Folgen*, Arch. Math. (Basel) 46 (1986), 240–249.

[3] —, *On the distribution of s-dimensional Kronecker sequences*, Acta Arith. 51 (1988), 335–347.

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