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Correction to "An Abelian theorem for number-theoretic sums"*

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I am grateful to Professor P. T. Bateman for calling my attention to an error in the proof of the theorem. The theorem is true as it stands. But it seems that in order to prove it, the estimate on $N(x) = \sum_{k<x} k^{-1}\mu(k)$, borrowed from number theory, that $\sum k^{-1}|N(k)| < \infty$, is not enough. The error is in the inequality in the first line of page 177, which is incorrect for large values of k.

One way to repair the proof is to use the known result ([1], page 374) that $N(k) \leqslant H(\log k)^{-2}$, where H is a constant. To prove now that $\sum_{n\leqslant m} n^{-1}|N(m/n)|$ is uniformly bounded, it is enough to do it for $\sum_{n\leqslant m/2}$ since there is a trivial estimate for $\sum_{m/2<n\leqslant m}$. We then have

$$\sum_{n\leqslant m/2} n^{-1}|N(m/n)| \leqslant H \sum_{n\leqslant m/2} \left(\frac{1}{n/m}\right)\left(\frac{1}{\log(n/m)}\right)^2 \frac{1}{m}.$$

This last sum is an approximating Riemann sum to the convergent integral $\int_0^{1/2} (\log x)^{-2} x^{-1}\,dx$, and the result follows.

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* Acta Arithmetica, this volume, pp. 157 - 177.

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