Daher folgt für diese K mit $\eta := z^{-1} y \log y$ aus (8.2) and (8.5)

$$S \ll K^{-1} MN + (K^2 \eta^2 M^3 N^4)^{1/5} + (K^9 \eta^9 MN^3)^{1/10} + M^{1/2} N \log y + (K\eta N)^{1/2}.$$

Setzt man $K := (\eta^{-2} M^2 N)^{1/7}$, so ist $0 < K \ll M^{3/7} \ll y$ und $K^{-1} MN = (K^2 \eta^2 M^3 N^4)^{1/5}$, und mit $A := (\eta^2 M^5 N^6)^{1/14}$, $B := M^{1/2} N \log y$ und $C := (\eta^5 M^2 N^8)^{1/14}$ gilt

$$(8.6) \qquad S \ll A^2 + A\eta^{1/2} + B + C \quad \text{für } K > 1.$$

Ferner gilt

$$(8.7) \qquad S \ll A^2 \quad \text{für } K \leqslant 1,$$

da $S \ll MN$ und $A^2 = K^{-1} MN$ ist. Ich zeige nun

$$(8.8) \qquad A\eta^{1/2} \gg C, \quad A^2 \gg B, \quad A \gg \eta^{1/2} \quad \text{für } K > 1.$$

$A\eta^{1/2} \gg C$ ist zu $\eta^4 M^3 \gg N^2$ äquivalent und daher erfüllt. $A^2 \gg B$ ist zu $(y/z)^4 (M/N)^2 M \gg \log^{10} y$ äquivalent. Aus (8.1) folgt aber $(y/z)^4 (M/N)^2 M > (y/z)^{1/\omega} \cdot 1^2 \cdot z^{1/\omega} = y^{1/\omega} \gg \log^{10} y$. Schließlich ist $A \gg \eta^{1/2}$ zu $M^5 N^6 \gg \eta^5$ äquivalent. Wegen $K > 1$ ist $M^2 N > \eta^2$, und daher $M^5 N^6 \gg (M^2 N)^{5/2} > \eta^5$. (8.6)–(8.8) ergeben

$$S \ll A^2 = \{y^2 z^{11/\omega - 2} (N/M)^{6 - 11\sigma/\omega} \log^2 y\}^{1/7},$$

und Hilfssatz 5 ist bewiesen.

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On linear recurrence relations satisfied by Pisot sequences Addenda and errata

by

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The proof of Theorem 4 of [1] contains the incorrect inequality $t! < t^t e^{1-t}$. If this is replaced by the correct $t! < t^{t+1} e^{-t}$ the method of proof yields a weaker theorem, namely that θ is recurrent if $\theta > 11 d \log d$. We present here a corrected proof of a stronger assertion, giving more details than presented in [1]. I would like to thank Mustapha Ben Amri Bettaieb for pointing out this error.

THEOREM 4. *Let θ be a Pisot or Salem number of degree d. If $\theta > 5d \log d$ then θ is recurrent. In fact, as $d \to \infty$, the assumption that $\theta > (2+o(1)) d \log d$ implies that θ is recurrent. Furthermore the set of K admissible for θ is finite and effectively determinable.*

Proof. By Theorem 2 of [1], θ is recurrent if $\theta > 2^{d-2}$. Since $2^{d-2} < 5d \log d$ for $d \leqslant 8$ this implies Theorem 4 for these values of d, so we may assume $d \geqslant 9$. By Lemma 2 and Theorem 1 (a), θ will be recurrent if we can determine positive integers t and L so that $L^{t-d} > t!\, 3^d \theta^t$ and $L < (\theta - 1)^2$.

Choosing $L = [(\theta-1)^2]$ the second inequality holds and, since $L > (\theta-1)^2 - 1 = \theta(\theta-2)$, it suffices for the first inequality that $(\theta-2)^{t-d} > t!\, 3^d \theta^d$. Taking logarithms, it suffices to have $F(\theta, t, d) > 0$, where

$$F(\theta, t, d) = (t-d)\log(\theta-2) - d\log\theta - \log(t!) - d\log 3.$$

Differentiating F with respect to θ we find that $F_\theta > 0$ if $\theta > 2$ and $t \geqslant 2d$. Thus, for each $t \geqslant 2d$ there is a unique solution θ_0 of $F(\theta, t, d) = 0$ and $\theta > \theta_0$ implies $F(\theta, t, d) > 0$. For given d, it is natural to choose t so as to minimize θ_0. For large d, t and θ, $F \approx (t-2d)\log\theta - t\log t$, so that $\theta = t \log t/(t-2d)$. An elementary calculation shows that the minimum of θ occurs for $t = (2+o(1)) d \log d$ as $d \to \infty$ with the corresponding value of $\theta_0 = (2+o(1)) d \log d$. This suggests the second assertion of the Theorem. A more precise analysis is given below.

To obtain results valid for all $d \geqslant 9$, we first examine $F(\theta, t, d)$ numerically for small d. For example, we find that, for $d = 9$, the minimum value of θ_0 occurs for $t = 95$ giving $\theta_0 = 97.2978 = 4.920\, d \log d$. For $d = 10$, the minimum occurs at $t = 108$, giving $\theta_0 = 110.0895 = 4.781\, d \log d$. These prove the first statement of the Theorem for $d = 9$ and 10, and suggest that a reasonable choice of t is $t = kd \log d$, for some $k \approx 5$ and that $\theta_0 > t$ for this choice of t.

Assuming then that $t = kd \log d$, where k will be specified shortly, it suffices to prove that $F(t, t, d) > 0$. We assume that $t \geqslant 95$ so that $\log((t-2)/t) > \log(93/95)$ and use the inequality $t^{t+1} e^{-t} > t!$. We then have

$$F(t, t, d) \geqslant -(2d+1)\log t + t - d\log 3 + (t-d)\log(93/95),$$

which is an increasing function of t for the set of t and d under consideration. Thus $F(t, t, d) > g(d)$ where

$$g(d) = (Ak-2)\, d\log d - (2d+1)\log\log d - (2\log k + B)\, d - \log d - \log k,$$

where $A = 1 + \log(93/95)$ and $B = \log 3 + \log(93/95)$. If we choose $k = 4.94$, we find that g is increasing for $d \geqslant 9$ and that $g(10) > 0$, and thus that $F(kd\log d, kd\log d, d) > 0$ if $d \geqslant 10$ and $k \geqslant 4.94$. To insure that t is an integer, we take $t = [5d\log d] = kd\log d$ with $k \geqslant 5 - 1/d\log d \geqslant 4.94$, provided $d\log d \geqslant 50/3$ which is true if $d \geqslant 9$. This proves the first claim of the Theorem for $d \geqslant 10$.

To prove the second statement, we again examine $F(t, t, d)$ for $t = kd\log d$, where now $k = 2 + c(\log\log d/\log d)$. Since now $A = 1 + O(1/\log d)$, it is easy to verify that $F(t, t, d) > 0$ for a suitable choice of the constant c. This proves the second assertion of the Theorem.

The final statement of the Theorem follows as in [1].

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