An analogous argument using Propositions 4.2, 6.1, 6.3 and 7.3 proves the theorem in the case that \( m = 2k+2 \).

Now we consider the case where \( m = 2k+1 \). Take \( e \) and \( q \) as in Proposition 6.4. Then for \( x \in \varphi L \) with \( Q(x) \notin \varphi^3 N(L) \),
\[
Qx \in \varphi L = \varphi^3 N(L) = \varphi^3 L_{q^0} = \varphi^3 L = L_{q^0},
\]
(Recall that by Proposition 2.1, \( L_{q^0} \) is \( N(L) \) over \( \varphi^3 \)-modular.) Hence \( Qx \) is anisotropic in the space \( \varphi^{-e} L_{q^0} / \varphi^{-e-1} L_{q^0} \), and so
\[
\text{disc}(\varphi^{-e} L_{q^0}/\varphi^{-e-1} L_{q^0}) = (-1)^{m-1/2} Q(e x) / Q(\varphi x) / Q(\varphi^3 x).
\]

Now Propositions 6.4 and 7.3 yield the result of the theorem.

**Remark.** In the case that \( m = 2k+2 \), we could use the methods used above to write \( \Theta(L, \tau) | T(\varphi) \) as a linear combination of theta series \( \Theta(L^{-1}, \tau) \) is a \( \varphi \)-sublattice of \( L \) and \( \Theta(L^{-1}, \tau) \). However, these theta series do not lie in the same space of modular forms as \( \Theta(L, \tau) | T(\varphi) \).

**References**

and if the numbers $1, \alpha_1, \ldots, \alpha_n$ are $Q$-linearly independent, then there exist infinitely many integer points $\mathcal{G}$ such that

$$\mathcal{G} \in \Psi \quad \text{and} \quad |L(\mathcal{G})| \leq \delta \{G\}^{2-n}.$$  

Theorem 2. Let $x$ be real, $1/n < x \leq 1/(n-1)$. Suppose the numbers $1, \alpha_1, \ldots, \alpha_n$ are $Q$-linearly independent and the inequality

$$\max_{1 \leq k \leq n} |x_k - q_k| \leq y^{-x}$$

has only finitely many solutions in integers $y, q_1, \ldots, q_n$. If

$$y < y_0 = u_0(n, x) = \frac{1}{2n}\left(n(n-1)^2 + 1 + \sqrt{(n(n-1)^2 + 1)^2 + 4n^2x^2(n-1)^2}\right),$$

then there exist infinitely many integer points $\mathcal{G}$ such that

$$\mathcal{G} \in \Psi \quad \text{and} \quad |L(\mathcal{G})| \leq \{G\}^{-n}.$$  

(iii) Remarks. 1. We have $u_0(n, 1/n) = n$ and $u_0(n, 1/(n-1)) = v(n)$.

2. Theorems 1 and 2 will be proved with $\Phi(w)$ and $\Psi(A)$ replaced by more general sets. Let $A: \mathbb{R}^n \rightarrow \mathbb{R}^n$, $(\xi_1, \ldots, \xi_n) \rightarrow (x_1, \ldots, x_n)$ be any transformation of the form

$$x_v = \sum_{j=1}^n k_{ij} \xi_j, \quad v = 1, \ldots, n,$$

where the numbers $k_{ij}$ are real and such that

$$\text{for } D = \det(k_{ij}) \text{ we have } D \neq 0.$$  

Define

$$\langle \mathcal{A} \rangle = \max_{1 \leq v \leq n} |x_v|; \quad r(\mathcal{A}) = (\sum_{v=1}^{n-1} x_v^2)^{1/2}.$$  

Then there exists a bound $d = d(A) > 0$ such that for every $\mathcal{A} \in \mathbb{R}^{n+1}_+$

$$\frac{1}{d} \langle \mathcal{A} \rangle \leq r(\mathcal{A}) \leq d \langle \mathcal{A} \rangle.$$  

In particular $d(1) = 1$, where 1 denotes the identity.

Let

$$\Phi(A, w) = \{x \in \mathbb{R}^{n+1}_+ | x_v \leq (1 + \varepsilon) r(\mathcal{A})^{w} \} \cup \{x \in \mathbb{R}^{n+1}_+ | r(\mathcal{A}) \leq 1\};$$

$$\Psi(A) = \{x \in \mathbb{R}^{n+1}_+ | x_v \leq c \langle \mathcal{A} \rangle \}.$$  

We will show more generally that the hypotheses of Theorem 1 (a) respectively 1 (b) imply that there exist infinitely many integer points $\mathcal{G}$ with

$$\mathcal{G} \in \Phi(A, w) \quad \text{and} \quad |L(\mathcal{G})| \leq (1 + \delta) d^{2n} \{G\}^{-n}$$

or

$$\mathcal{G} \in \Psi(A) \quad \text{and} \quad |L(\mathcal{G})| \leq \delta \{G\}^{-n}.$$  

Similarly it will be proved that

$$(9) \quad \text{Theorem 2 holds with } \Psi \text{ replaced by } \Psi(A).$$

3. The case $n = 2$ of (8) is due to W. M. Schmidt [6], who was basically interested in diophantine approximation by positive integers. He also has given an example [6], remark C, showing that Theorems 1 (b) and 2 no longer hold if the numbers $1, \alpha_1, \ldots, \alpha_n$ are allowed to be $Q$-linearly dependent. For $n = 2$, results similar to Theorems 1 (a) and 2 could be proved by his method [6].

4. For $n \geq 3$ the following result of W. M. Schmidt gives a "lower bound" for the sets $\Phi(A, w)$ and $\Psi(A)$ in (7) and (8), making clear that these cannot be replaced by the set $\Theta$ of points with positive coordinates, $\Theta = \{x \in \mathbb{R}^{n+1}_+ | x_v > 0, v = 1, \ldots, n\}$, since [6], remark F:

$$c(n, \varepsilon) > 0 \text{ there are } x_1, \ldots, x_n \text{ and } c(n, \varepsilon) > 0 \text{ such that } 1, \alpha_1, \ldots, \alpha_n \text{ are } Q\text{-linearly independent and } |L(\mathcal{G})| \geq c(n, \varepsilon) \{G\}^{-2+n}.$$  

5. C. A. Rogers [5] has studied the "dual" problem to the one we deal with; G. Harman [4] has investigated analogous metrical problems (see also [7]).

6. I am grateful to Professor W. M. Schmidt for encouragement.

II. Proofs of (7) and (8).

(i) Since the number $\delta$ in (8) does not depend on $A$, we may replace $c$ in the definition of $\Psi(A)$ by any positive number, say 3. Hence

$$(10) \quad \Psi(A) = \{x \in \mathbb{R}^{n+1}_+ | x_v \leq 3 \langle \mathcal{A} \rangle \}.$$  

Let $w$ and $v$ be the numbers defined in (9) and (1) and let $c = c(A, n)$ $= (1 + \delta) d^{2n}$. We shall give inductive proofs for (7) and (8). Therefore in what follows, we always suppose that one of the following assumptions holds.

Assumption A. There are at most finitely many integer points $\mathcal{G}$ verifying

$$(7) \quad \mathcal{G} \in \Phi(A, w) \text{ and } |L(\mathcal{G})| \leq (1 + \delta) d^{2n} \{G\}^{-n}.$$  

Assumption B. The numbers $1, \alpha_1, \ldots, \alpha_n$ are $Q$-linearly independent and there are at most finitely many integer points $\mathcal{G}$ verifying

$$(8) \quad \mathcal{G} \in \Psi(A) \text{ and } |L(\mathcal{G})| \leq \delta \{G\}^{-n}.$$  

Then, the intersection of $\Phi(A, w)$ with any linear subspace of $\mathbb{R}^{n+1}_+$ being infinite, in both cases the numbers

$$(11) \quad 1, \alpha_1, \ldots, \alpha_n \text{ are } Q\text{-linearly independent}.$$  

Let $\mathcal{G}_1, \mathcal{G}_2, \ldots$ be a sequence of minimal points, i.e., of integer points with $\langle \mathcal{G}_j \rangle < \langle \mathcal{G}_j \rangle < \ldots$ such that for no $j = 1, 2, \ldots$ there is an integer point $\mathcal{G} \neq \mathcal{G}$ with the properties $|L(\mathcal{G})| \leq |L(\mathcal{G}_j)|$ and $\langle \mathcal{G} \rangle < \langle \mathcal{G}_j \rangle$.

For $j = 1, 2, \ldots$ put

$$\xi_j(\mathcal{G}) = \xi_j; \quad x_j(\mathcal{G}) = x_j, \quad r(\mathcal{G}) = r_j.$$
Since any minimal point \( \mathcal{g}_j \) can be replaced by \( -\mathcal{g}_j \), we may suppose \( x_{n_j} \geq 0 \), \( j = 1, 2, \ldots \). By Dirichlet's theorem we have with (6)
\[
|L_j| \leq d^n N_j^{1/w}, \quad j = 1, 2, \ldots
\]
Henceforth let \( m \) be a sufficiently large subscript. It then follows from (12) that \( \mathcal{g}_m \neq \Phi(A, w) \) if \( A \) holds and that \( \mathcal{g} \neq \Psi(A) \) if \( B \) holds. Therefore
\[
1 < r_m < x_{nm}^{1/w} \leq N_m^{1/w}, \quad \text{if } A \text{ holds}
\]
and in any case
\[
r_m < \frac{1}{2} x_{nm} \leq \frac{1}{2} N_m
\]
by (10). Hence
\[
\langle \mathcal{g}_m \rangle = x_{nm} = N_m.
\]
(ii) If \( A \) holds, then
\[
N_m \geq N_m^{1/(w + 1)} r_m.
\]
Proof. Let \( [x] \) denote the integral part of the real number \( x \). For the integer point
\[
\mathcal{h} = \mathcal{h}(m) = \left[ r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} \right] \mathcal{g}_m
\]
we have by (13)
\[
r(\mathcal{h}) \geq \left( r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} - 1 \right) r_m = (1 + o(1)) r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)}, \quad m \to \infty,
\]
and
\[
|x_n(\mathcal{h})| = \langle \mathcal{h} \rangle \leq x_{nm} r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} \leq r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)}.
\]
Hence
\[
\mathcal{h} \in \Phi(A, w).
\]
It follows from (12) that
\[
|L(\mathcal{h})| \leq d^n r_m^{1/(w + 1)} \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} N_m^{1/(w + 1)} = d^n \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} r_m^{-\eta/(w + 1)} N_m^{2/(w - 1)}.
\]
By the definition (0) of \( w \) we have \( n(n + 1) = (1 + nw)/(w - 1) \). Now assume indirectly that
\[
N_m^{1/(w + 1)} > \left( \frac{N_m}{r_m} \right)^{1/(w - 1)},
\]
and hence that
\[
N_m^{1/(w + 1)} - \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} = \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} - \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} - \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} - \left( \frac{N_m}{r_m} \right)^{1/(w - 1)} = \left( \frac{N_m}{r_m} \right)^{1/(w - 1)}.
\]
By (19) and (17) we then have
\[
|L(\mathcal{h})| \leq d^n \left( \frac{N_m}{r_m} \right)^{-\eta/(w + 1)} r_m^{-\eta/(w + 1)} \leq d^n \langle \mathcal{h} \rangle^{-\eta}.
\]
Combined with (18) this contradicts \( A \), which proves (16). In what follows, the constant implied by \( \langle \cdot \rangle \) does not depend on \( m \).
(iii) If \( B \) holds, then
\[
|L_m| \leq N_m^{-\eta/(w + 1)}.
\]
Proof ([6], Lemma 1). Since \( n - 1 < v \) we can pick \( \sigma \) so large that
\[
D \sigma^{v - 1} < \delta/\sigma^\eta d',
\]
where \( D \) is defined in (5). Let \( N > 1 \). According to Minkowski, the set
\[
\{ \mathcal{h} \in \mathbb{R}^{n+1} | |x_n| \leq \frac{1}{2} N^{1/(w + 1 - \eta)}, \quad n = 1, \ldots, n - 1; \quad |x_n| \leq N; \}
\]
contains an integer point \( \mathcal{g} \neq \mathcal{h} \). Since \( n > v \) we have \( |\mathcal{g}| \leq d \langle \mathcal{g} \rangle \leq N^{1/(w + 1 - \eta)} d/\sigma \) and hence
\[
|L(\mathcal{g})| \leq D \sigma^{v - 1} N^{-\eta/(w + 1 - \eta)} < \delta (N^{1/(w + 1 - \eta)} d/\sigma)^{v - \eta} \leq \delta \langle \mathcal{h} \rangle^{-\eta}.
\]
Now, if \( N \) is large then \( |L(\mathcal{g})| \) is small and \( \langle \mathcal{h} \rangle \) is large by (11). Therefore, since \( B \) is supposed to hold, \( \mathcal{g} \neq \Psi(A) \) for large \( N \). It follows that \( r(\mathcal{g}) < |x_n(\mathcal{g})|/3 < 3N/3 \) and \( \langle \mathcal{g} \rangle \leq N \). Hence, given \( N \) large enough, there exists an integer point \( \mathcal{g} \neq \mathcal{h} \) such that \( \langle \mathcal{g} \rangle \leq N \) and \( |L(\mathcal{g})| \leq N^{-\eta/(w + 1 - \eta)} \). Combined with the definition of minimal points, this implies (20).
(iv) We have
\[
|L_{m+1}| \leq 2 \frac{N_m+1}{N_m} |L_m| + |L_{m+1}|.
\]
Proof. Consider the integer point
\[
\mathcal{h} = \mathcal{h}(m) = \mathcal{g}_{m+1} - \left[ \frac{x_n+1}{x_n} \right] + \mu \mathcal{g}_m,
\]
where \( \mu \in (0, 1) \) is chosen such that
\[
|x_n(\mathcal{h})| \leq x_{nm}/2 \leq N_m/2.
\]
By (15) \( x_{m+1}/x_m = N_{m+1}/N_m \). Hence
\[
(23) \quad |L(\mathcal{X'})| \leq |L_{m+1}| + \left(\frac{N_{m+1}}{N_m} + 1\right)|L_m|
\]
and
\[
(24) \quad r(\mathcal{X'}) \leq r_{m+1} + \left(\frac{N_{m+1}}{N_m} + 1\right)r_m.
\]
We next show that
\[
(25) \quad |L(\mathcal{X'})| \leq c \{\mathcal{X'}\}^{-\nu}, \quad \text{if A holds},
\]
\[
(26) \quad \leq \delta \{\mathcal{X'}\}^{-\nu}, \quad \text{if B holds}.
\]
Indeed, if A holds, it follows from (24), (13), (16) and (0) that for \( \nu = 1, \ldots, n-1 \)
\[
(27) \quad |x_\nu(\mathcal{X'})| \leq r(\mathcal{X'}) < N^{1/\nu}_m + N^{m+1}_m + N_{m+1}^{1/\nu}.
\]
Using (23), (12) and (16) we find
\[
(28) \quad |L(\mathcal{X'})| \leq d^\nu \left(\frac{N^{1/\nu}_m + N^{m+1}_m + N_{m+1}^{1/\nu}}{N_m} + \frac{N^{m+1}_m + N_{m+1}^{1/\nu}}{N_m}\right) \leq (1 + o(1)) d^\nu N^{-\nu}_m, \quad m \to \infty.
\]
Now, by (22) and (27) we have either \( \{\mathcal{X'}\} \leq N_m d/2 \) — which implies (25) in view of the first estimate in (28) — or \( \mathcal{X'} \notin \mathcal{Y}(A, w) \) if A holds and \( \mathcal{X'} \notin \mathcal{F}(A) \) if B holds, from which we get (25) using the second estimate in (28).

If B holds, we have by (23), (20) and (1)
\[
|L(\mathcal{X'})| \leq N^{1/\nu}_m + N^{m+1}_m + N_{m+1}^{1/\nu}.
\]
Since \( \{\mathcal{X'}\} \leq N_{m+1} \), this proves (26).

Now \( \mathcal{X'}(m) \neq \emptyset \) for all \( m \). If \( m \) is large, \( |L(\mathcal{X'})| \) is small by (23), hence \( \{\mathcal{X'}\} \) is large by (11). In view of (25), (26) this implies \( \mathcal{X'} \notin \mathcal{F}(A, w) \) if A holds and \( \mathcal{X'} \notin \mathcal{Y}(A) \) if B holds, such that in both cases
\[
r(\mathcal{X'}) < |x_0(\mathcal{X'})|/3 \leq N_m/6
\]
by (22). Hence \( \langle \mathcal{X'} \rangle < N_m \). It follows from the definition of minimal points that
\[
|L_{m+1}| < |L(\mathcal{X'})|.
\]
This, combined with (23), proves (21).
(v) We have
\[
(29) \quad N_{m+1} \geq N_m + N_{m-1}.
\]
Proof. Consider the integer points \( \mathcal{M}_{v} = \mathcal{M}(m), v = 1, 2, 3 \);
\[
\mathcal{M}_1 = \mathcal{G}_{m+1} - \mathcal{G}_m; \quad \mathcal{M}_2 = \mathcal{G}_m - \mathcal{G}_{m-1}; \quad \mathcal{M}_3 = \mathcal{G}_{m-1} - \mathcal{G}_{m-2}.
\]
Assume indirectly that \( N_{m+1} < N_m + N_{m-1} \). Then by (15) and (14)
\[
|x_\nu(\mathcal{M}_v)| < N_m; \quad |r(\mathcal{M}_v)| < (N_{m+1} + N_m)/3 < N_m, \quad v = 1, 2, 3.
\]
Hence \( \langle \mathcal{M}_v \rangle < N_m, v = 1, 2, 3 \). Since \( \mathcal{M}_v \neq \emptyset \) this implies \( |L(\mathcal{M}_v)| \geq |L_{m+1}|, v = 1, 2, 3 \). Noting that \( 0 < |L_{m+1}| < |L_{m+1}| \) for \( m = m-1, m, m+1 \), we get
\[
L_{m+1} L_m < 0; \quad L_m L_{m-1} < 0; \quad L_{m+1} L_{m-1} < 0.
\]
This contradiction proves (29).

(vi) Define the sequences \( a_0, a_1, \ldots, b_0, b_1, \ldots \) by
\[
a_0 = a_1 = 1; \quad a_k = a_{k-1} + a_{k-2} \quad \text{for } k \geq 2;
\]
\[
b_0 = 1; \quad b_1 = 2; \quad b_k = 2b_{k-1} + b_{k-2} \quad \text{for } k \geq 2.
\]
Then \( a_0, a_1, \ldots \) is the Fibonacci sequence and for \( k = 1, 2, \ldots \) we have according to [3], §10.13,
\[
(30) \quad a_k = \frac{1}{\sqrt{5}} \left( \frac{1 + \sqrt{5}}{2} \right)^k - \left( \frac{1 - \sqrt{5}}{2} \right)^k,
\]
\[
(31) \quad b_k = \frac{2}{\sqrt{2}} \left( \frac{1 + \sqrt{2}}{2} \right)^k - \left( \frac{1 - \sqrt{2}}{2} \right)^k.
\]
(vii) For any positive integer \( k \) we have
\[
(32) \quad (a) \quad N_{m+k} \geq a_{k+1} N_{m-1}.
\]
\[
(33) \quad (b) \quad |L_{m-1}| \leq b_k \frac{N_{m+k}}{N_m} |L_{m+k-1}| + b_{k-1} \frac{N_{m+k-1}}{N_m} |L_{m+k}|.
\]
\[
(34) \quad (c) \quad N_{m+k} \leq b_k^{(n-1)} \left( 2d^\nu N^{-\nu}_m |L^{-\nu}_m| \right)^{1/(n-1)}.
\]
Proof. Formula (32) is an immediate consequence of (29). To prove (33) we note that for \( k = 1 \) (33) holds by (21). Assume now (33) to hold for \( k \). Then, from (21) with \( m \) replaced by \( m+k \) we find
\[
|L_{m-1}| \leq b_k \frac{N_{m+k}}{N_m} \left( \frac{2}{N_{m+k}} |L_{m+k-1}| + |L_{m+k}| \right) + b_{k-1} \frac{N_{m+k-1}}{N_m} |L_{m+k}|
\]
\[
\leq \left( 2b_k + b_{k-1} \right) \frac{N_{m+k+1}}{N_m} + b_k \frac{N_{m+k}}{N_m} |L_{m+k+1}|,
\]
which proves (33), since \( 2b_k + b_{k-1} = b_{k+1} \).

From (33) we get (34) using (12).
(viii) The case \( n \geq 3 \). By (32) and (30) respectively (34) and (31) there are positive bounds \( d_1, d_2 \), which do not depend on \( k \) and such that
\[
d_k \left( \frac{1 + \sqrt{5}}{2} \right)^k \leq d_k \left( 1 + \sqrt{2} \right)^{1/(n-1)} \leq d_k \left( 1 + \sqrt{2} \right)^{1/2}, \quad n \geq 3.
\]
Since \( 1 + \sqrt{5}/2 > (1 + \sqrt{2})^{1/2} \), this cannot hold for large \( k \). This contradiction proves (7) and (8) for \( n \geq 3 \).
(Observing that if \( B \) holds, (20) and (33) yield a better estimate for \( N_{m+1}^{1/4} \) than (34), one could prove (8) even for \( n = 2 \) in the same way.)

(ix) The case \( n = 2 \). Following the arguments in [2], Lemma 3, and using (12), we see that

(35) the minimal points \( \mathcal{G}_{m-1}, \mathcal{G}_m, \mathcal{G}_{m+1} \) are linearly independent for infinitely many \( m \).

For such an \( m \) we have

\[
\begin{vmatrix}
\xi_{m-1,1} & \xi_{m-1,2} & \xi_{m-1,3} \\
\xi_{m,1} & \xi_{m,2} & \xi_{m,3} \\
\xi_{m+1,1} & \xi_{m+1,2} & \xi_{m+1,3}
\end{vmatrix}
\]

\[
= \begin{vmatrix}
\xi_{m-1,1} & \xi_{m-1,2} & \xi_{m-1,3} \\
\xi_{m,1} & \xi_{m,2} & \xi_{m,3} \\
\xi_{m+1,1} & \xi_{m+1,2} & \xi_{m+1,3}
\end{vmatrix}
\]

Let \( D_n, v = m-1, m, m+1, \) denote the algebraic complement of \( L_n \) in this last matrix. Then we have by (13)

\[
|D_n| \leq 2N_m^{1/4}N_{m+1}^{1/2}, \quad \text{if } A \text{ holds,}
\]

\[
\leq 2N_mN_{m+1}, \quad \text{if } B \text{ holds,}
\]

and by (21), (12) and (20)

\[
|L_n| \leq |L_{m-1}| \leq \frac{3N_m^{1/4}N_{m+1}^{1/2}}{N_m} \leq (N_mN_{m+1})^{-1}, \quad \text{if } A \text{ holds,}
\]

\[
\leq N_m^{-1}N_{m+1}^{-1}N_m^{-1/4}, \quad \text{if } B \text{ holds,}
\]

such that in both cases

\[
L_nD_n = o(1), \quad m \to \infty, \quad v = m - 1, m, m+1.
\]

This contradicts (35), (36) and hence proves (7) and (8) for \( n = 2 \).

III. On the proof of (9).

(i) We can prove (9) by similar arguments. Assume again that the numbers 1, \( \sigma_1, \ldots, \sigma_q \) are \( \mathcal{Q} \)-linearly independent and that there are at most finitely many integer points \( \mathcal{Y} \) verifying (4) with \( \mathcal{Y} \) replaced by \( \mathcal{Y} \). Define the sequence \( \mathcal{G}_1, \mathcal{G}_2, \ldots \) of minimal points as in II, (i). Then (15) holds and by the arguments in [6], Lemma 1 (see II, (iii)) we have

(37) \[
|L_m| \leq N_m^{1/2}N_m^{1-n}.
\]

(ii) Let

\[
\omega_0 = \omega_0(n, x) = \frac{n^{n^1}/(n-1)(n^1-1)}{N_m^{1-n}}
\]

and let \( \omega > \omega_0 \). According to Khinchin's transference principle ([1], Chap. V, Theorem IV), the assumption (2) implies

\[
|L_m| > N_m^{1-n}.
\]

It follows from (37) that

(38) \[
N_m \geq N_m^{1/2}N_m^{1-n}N_m^{1-n}.
\]

(iii) We have

\[
1 - \frac{u_0}{u_0 + 1 - n} - \frac{u_0}{u_0 + 1 - n}(-u_0) = -u_0.
\]

Proof. Equation (39) is equivalent to

\[
(1 + u_0)(u_0 + 1 - n)(n + \omega_0) - u_0(n + \omega_0) - u_0 = 0
\]

and hence to

\[
u_0^2 - u_0\left(n - 1 + \frac{1}{n + \omega_0}\right) = n + 1 - 0.
\]

Noting that

\[
n - 1 + \frac{1}{n + \omega_0} = \frac{n^2 - 2nx + x + 1}{nx},
\]

we find that (39) is equivalent to the equation

\[
u_0^2 - u_0(n - 1 + \omega_0 - 1) - nx + n - 1 = 0,
\]

which holds in view of the definition (3) of \( u_0 \).

(iv) We have

\[
|L_{m-1}| \leq 2N_m^{1/4}N_m^{1/2} + |L_{m+1}|.
\]

Proof. Define the integer point \( \mathcal{X} = \mathcal{X}(m) \) as in II, (iv). Then (23) holds and using (15), (23), (37) and (38) we get

\[
|L(\mathcal{X})| \leq N_m^{1/2}N_m^{1-n} + N_m^{1/2}N_m^{1-n} - N_m^{1/2}N_m^{1-n}.
\]

Since \( u < u_0 \), by (39) we can choose \( \omega > \omega_0 \) such that with (40) and

\[
\mathcal{X} < \mathcal{Y} < N_{m+1}
\]

Hence, in view of the indirect assumption, \( \mathcal{X}(m) \notin \mathcal{Y}(A) \) for large \( m \). By the arguments in II, (iv) this implies (21).

(v) The rest of the proof is the same as for (7) and (8).
Über die Anzahl quadratvoller Zahlen in kurzen Intervallen und ein verwandtes Gitterpunktproblem
Corrigendum zu Acta Arithmetica L (1988), 195–201 (1)

von

PETER GEORG SCHMIDT (Marburg)

6. Vorbemerkungen. Wie ich in §7 darlegen werde, ist der auf B. R. Srinivasan zurückgehende Hilfssatz 2 nicht gesichert. Darauf beruhen aber die Sätze 1 und 2 und (1.1)–(1.2). Statt dessen werde ich in §8 das etwas schwächere Resultat $\zeta_{2,3,6}(x) \ll (x \log x)^{1/7}$ erzielen, also (1.1) für $1/7 < \theta < 1/2$ beweisen. Herr H. Menzer (Jena) hat mir in einem Brief vom 27.3.88 mitgeteilt, daß er $\zeta_{2,3,6}(x) \ll x^{8/48} \log^{2} x$ zeigen kann.


W. G. Nowak [13] äußerte, ohne jedoch konkrete Fehlerhinweise zu geben, Zweifel an der Korrektheit der Srinivasanschen Theorie. Ich teile seine Auffassung:

Srinivasan übersicht, daß die in [15] auf Seite 333 auftretenden Größen $\xi_{2}$ im allgemeinen von $x_{2}$ abhängen: $\xi_{2} = \xi_{2}(x_{2})$. Statt $(\partial f/x_{2})^{2} f(x_{2}, x_{2}) \gg \rho_{2}$ brauchte er $(\partial f/x_{2})^{2} f(x_{2}(x_{2}), x_{2}) \gg \rho_{2}$. Gravierender dürfte diese Ungenauigkeit in den Beweisen der Lemmata 4, 5 und 6 in [16] sein (siehe etwa [16], Seite 181, Zeilen 10–17). Sie wird sich meines Erachtens, wenn überhaupt, nur mit großem Aufwand beheben lassen. Auf den Lemmata 4, 5 und 6 in [16] basieren aber die dortigen Theoreme 1 und 2 und darauf die in [8] entwickelte Theorie der Exponentenpaare. Daher ist auch [9], Theorem 5 und folglich Hilfssatz 2 nicht gesichert.

8. Corrigendum.

Satz 4. Für $x \to \infty$ gilt $\zeta_{2,3,6}(x) \ll (\log x)^{2/7}$.

Satz 4 folgt unmittelbar aus Hilfssatz 3 und dem Korollar zu

(1) In dieser Arbeit finden sich die §§ 1–5, Sätze 1–3, Hilfssätze 1–4, Formeln (1.1)–(5.3) und Literaturhinweise [1]–[11].