For $1.5 \leqslant \varkappa \leqslant 1.85$ we apply Theorem 3 with $r = 3$ and $n = 4$. It is easily checked, that

$$P_5(z, 3, \varkappa) > 0 \quad \text{for } z \geqslant 1.6\varkappa - 1.41,\ 1.5 \leqslant \varkappa < 1.85$$

and this completes the proof.

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ABT. FÜR MATH. III
UNIVERSITÄT ULM
Oberer Eselsberg
D-7900 Ulm

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Equidistribution of Frobenius classes and the volumes of tubes

by

B. Z. Moroz (Bonn)

1. Let G be a compact Lie group that fits in an exact sequence

$$1 \to \mathcal{T} \to G \xrightarrow{j} H \to 1, \tag{1}$$

where $\mathcal{T}$ is an n-dimensional real torus and H is a finite group. Given a countable index set $\mathcal{P}$ and a set of conjugacy classes $\{\sigma_p |\ p \in \mathcal{P}\}$ in G, we are interested in the following equidistribution problem. Let

$$|\cdot|\colon \mathcal{P} \to \boldsymbol{R}_+$$

be a map satisfying the asymptotic formula (8) below and let $\mathcal{A} \subseteq G$. For each x in $\boldsymbol{R}_+$, let

$$\mathcal{N}(\mathcal{A}, x) = \operatorname{card}\{p|\ p \in \mathcal{P},\ \sigma_p \cap \mathcal{A} \neq \emptyset,\ |p| < x\}.$$

One studies the asymptotics of $\mathcal{N}(\mathcal{A}, x)$ as $x \to \infty$. Without loss of generality we can assume that $\mathcal{A}$ is invariant under conjugation, i.e.

$$\tau^{-1}\mathcal{A}\tau = \mathcal{A} \quad \text{for } \tau \in G, \tag{2}$$

so that

$$\mathcal{N}(\mathcal{A}, x) = \operatorname{card}\{p|\ p \in \mathcal{P},\ \sigma_p \subseteq \mathcal{A},\ |p| \leqslant x\}. \tag{3}$$

The manifold G inherits the natural Riemannian structure from $\mathcal{T}$. Let μ be the Haar measure on G normalized by the condition $\mu(G) = 1$, and suppose that $\mathcal{A}$ satisfies the following condition:

$$\mu(\mathcal{U}_\delta(\partial\mathcal{A})) = O(C(\mathcal{A})\delta^\alpha) \quad \text{with } \alpha > 0, \tag{4}$$

where $\partial\mathcal{A}$ denotes the boundary of $\mathcal{A}$ and where $\mathcal{U}_\delta(\mathcal{A})$ denotes the δ-neighbourhood of $\mathcal{A}$, i.e. the subset

$$\{x|\ x \in G,\ \varrho(x, \mathcal{A}) < \delta\}; \tag{5}$$

here $\delta > 0$ and ϱ denotes the Riemannian metric on G. Consider now the set $\hat{G}$ of all the simple characters of G; let ψ be an irreducible representation of G and let

(6) $$\psi|_{\mathcal{T}} = \operatorname{diag}(\lambda_1, \ldots, \lambda_l), \quad \chi = \operatorname{tr}\psi, \quad \lambda_j \in \hat{\mathcal{T}}, \quad 1 \leqslant j \leqslant l.$$

In view of the isomorphism $\hat{\mathcal{T}} \cong \boldsymbol{Z}^n$, one can choose a basis

$$\{\mu_j|\ 1 \leqslant j \leqslant n\}$$

of $\hat{\mathcal{T}}$. Let

(7) $$\lambda_i = \prod_{j=1}^{n} \mu_j^{m_{ij}}, \quad m_{ji} \in \boldsymbol{Z}, \quad 1 \leqslant i \leqslant l,$$

we write then

$$w(\lambda_i) = \prod_{j=1}^{n} (1+|m_{ij}|), \quad w(\chi) = \max_{1 \leqslant i \leqslant l} w(\lambda_i).$$

THEOREM 1. *If $\mathcal{A}$ satisfies* (4) *and*

(8) $$\sum_{\substack{p \in \mathcal{P} \\ |p| < x}} \chi(\sigma_p) = g(\chi)B(x) + O\big(b(x, w(\chi))\big), \quad \chi \in \hat{G},$$

where $g(\chi) = 1$ if χ is the character of the identical representation and $g(\chi) = 0$ for any other character and where

(9) $$\sum_{m=1}^{\infty} b(x, m)m^{-\nu} = b_1(x, \nu) < \infty$$

for some ν in $\boldsymbol{R}_+$, then (assuming (2) *and* (3))

(10) $$\mathcal{N}(\mathcal{A}, x) = \mu(\mathcal{A})B(x)\left(1 + O\left(\frac{C(\mathcal{A})}{\mu(\mathcal{A})}\left(\frac{b_1(x, \nu)}{B(x)}\right)^{\alpha/(\alpha+\nu n)}\right)\right).$$

Proof. Since, by definition, $\varrho(g_1, g_2) = \infty$ when $j(g_1) \neq j(g_2)$, we have

$$\mathcal{U}_\delta(\{1\}) \subseteq \mathcal{T},$$

therefore there is a C^∞-function φ_δ: $G \to [0, 1]$ satisfying the following conditions:

$$\int_G \varphi_\delta(g)d\mu(g) = 1, \quad \varphi_\delta \text{ is } H\text{-invariant}, \quad \varphi_\delta(g) = 0 \text{ for } g \notin \mathcal{U}_\delta(\{1\}).$$

Let f_+ and f_- be the characteristic functions of $\mathcal{U}_\delta(\mathcal{A})$ and $\mathcal{A} \backslash \mathcal{U}_\delta(G \backslash \mathcal{A})$ respectively, and let

$$g_\pm(\beta) = \int_G f_\pm(\gamma)\varphi_\delta(\gamma^{-1}\beta)d\mu(\gamma).$$

Then $g_\pm \in C^\infty(G)$ and $g_\pm$ is H-invariant (since $f_\pm$ and φ_δ are). Moreover,

$$g_\pm(\beta) = \int_{\mathcal{U}_\delta(\{1\})} f_\pm(\beta\gamma^{-1})\varphi_\delta(\gamma)d\mu(\gamma),$$

so that

$$g_\pm(\beta) \geqslant 0 \text{ for } \beta \in G, \quad g_+(\beta) = 1 \text{ for } \beta \in \mathcal{A}, \quad g_-(\beta) = 0 \text{ for } \beta \notin \mathcal{A}.$$

Thus

(11) $$\sum_{|p| < x} g_-(\sigma_p) \leqslant \mathcal{N}(\mathcal{A}, x) \leqslant \sum_{|p| < x} g_+(\sigma_p).$$

We write

(12) $$g_\pm = \sum_{\chi \in \hat{G}} c_\pm(\chi)\chi$$

and substitute (8) in (12) to obtain

(13) $$\sum_{|p| < x} g_\pm(\sigma_p) = c_\pm(1)B(x) + O\Big(\sum_{\chi \neq 1} |c_\pm(\chi)| b\big(x, w(\chi)\big)\Big).$$

It follows from (12) that

$$c_\pm(1) = \int_G g_\pm(\beta)d\mu(\beta),$$

or recalling the definition of $g_\pm$, $f_\pm$, and φ_δ,

$$c_\pm(1) = \int_G f_\pm(g)d\mu(g) = \mu(\mathcal{A}) \pm \mu(\mathcal{U}_\delta(\partial\mathcal{A})).$$

Therefore it follows from (4) and (13) that

(14) $$\sum_{|p| < x} g_\pm(\sigma_p) = \mu(\mathcal{A})B(x) + O\big(B(x)\delta^\alpha C(\mathcal{A})\big) + O\Big(\sum_{\chi \neq 1} |c_\pm(\chi)| b\big(x, w(\chi)\big)\Big).$$

To estimate $c_\pm(\chi)$ let us suppose that χ satisfies (7) and (6) and write

$$G = \bigcup_{\gamma \in H} \mathcal{T}h_\gamma, \quad j(h_\gamma) = \gamma.$$

Then (12) gives:

(15) $$c_\pm(\chi) = \int_{\mathcal{T}} d\mu(\alpha) \sum_{\gamma \in H} g_\pm(\alpha h_\gamma)\overline{\chi(\alpha h_\gamma)}.$$

In view of (6),

$$\chi(dh_\gamma) = \sum_{i=1}^{l} \lambda_i(\alpha)\psi_{ii}(h_\gamma).$$

Therefore

(16) $$c_\pm(\chi) = \sum_{\gamma \in H} \sum_{i=1}^{l} \overline{\psi_{ii}(h_\gamma)} \int_{\mathcal{T}} d\mu(\alpha) g_\pm(\alpha h_\gamma)\overline{\lambda_i(\alpha)}.$$

It follows from (7) and the definition of $g_\pm$ that (cf., e.g., [2], § 3)

(17) $$\int_{\mathcal{T}} d\mu(\alpha) g_\pm(\alpha h_\nu)\overline{\lambda_i(\alpha)} = O(\delta^{-\nu n} w(\lambda_i)^{-\nu})$$

for each ν in $\mathbf{Z} \cap \mathbf{R}_+$. A classical argument (cf., e.g., [8], § 8.1) shows that, in fact,

$$w(\chi) = O(w(\lambda_i)), \quad 1 \leqslant i \leqslant l,$$

for a simple character χ and that

(18) $$\operatorname{card}\{\chi|\ \chi \in \hat{G},\ w(\chi) = m\} = O(1), \quad m \in \mathbf{Z},\ m \geqslant 1.$$

In view of (9), (14), (17) and (18), we conclude that

(19) $$\sum_{|p|<x} g_\pm(\sigma_p) = \mu(\mathcal{A})B(x) + O(B(x)\delta^\alpha C(\mathcal{A})) + O(\delta^{-\nu n} b_1(x, \nu)).$$

Taking $\delta = (b_1(x, \nu)/B(x))^{1/(\alpha+\nu n)}$ one deduces (10) from (11) and (19). This completes the proof of Theorem 1.

COROLLARY 1. *Assume that $\partial\mathcal{A}$ is contained in an analytic subset of dimension $n-1$. Then relations* (8) *and* (9) *imply* (10) *with* $\alpha = 1$.

Proof. By a geometric lemma discussed in the Appendix to this paper, a compact analytic set $\mathcal{B}$ of codimension α satisfies an estimate

$$\mu(\mathcal{U}_\delta(\mathcal{B})) = O(C(\mathcal{B})\delta^\alpha).$$

2. To describe an arithmetical application of Theorem 1 let k be a finite extension of $\mathbf{Q}$, the field of rational numbers, and let $W(k)$ denote the (absolute) Weil group of k defined as a projective limit of the relative Weil groups $W(K|k)$, where K varies over all the finite Galois extensions of k (cf. [10], [11]). Let us recall that

$$W(K|k) \cong \mathbf{R}_+^* \times W_1(K|k)$$

with compact $W_1(K|k)$ and that $W(K|k)$ is defined as a group extension

$$1 \to C_K \to W(K|k) \to G(K|k) \to 1,$$

where C_K denotes the idèle-class group of K and where $G(K|k)$ is the Galois group of K over k. Let $S(k)$ be the set of all the prime divisors of k, and let I_p and σ_p be the inertia subgroup and the Frobenius class in $W(k)$ for $p \in S(k)$. Consider a finite dimensional continuous representation

$$\psi\colon\ W(k) \to \mathrm{GL}(V)$$

acting in a complex vector space V; let

$$V_p = \{e|\ e \in V,\ \psi(g)e = e \text{ for } g \in I_p\}$$

be the subspace of I_p-invariant vectors and let χ denote the character of ψ. We define $\chi(\sigma_p)$ to be equal to the trace of the operator $\psi(\tau_p)$ on V_p for $\tau_p \in \sigma_p$ and notice that this definition does not depend on the choice of τ_p in σ_p. One can show that the set

$$S_0(\psi) := \{p|\ p \in S(k),\ V_p \neq V\}$$

is finite and that ψ factors through $W(K|k)$ for a finite extension $K|k$. We say that ψ is *normalized* if ψ factors through $W_1(K|k)$ for a finite Galois extension $K|k$.

THEOREM 2. *Let $\mathcal{M}$ be a finite set of normalized (finite dimensional continuous) representations of $W(k)$, let*

$$\check{\mathcal{M}} = \{\chi|\ \chi = \operatorname{tr}\psi \text{ for some } \psi \text{ in } \mathcal{M}\},$$

and choose g_0 in $W(k)$ and ε in the interval $0 < \varepsilon < 1$. There is a positive constant $a(\mathcal{M}; g_0, \varepsilon)$ such that

(20) $$\operatorname{card}\{p|\ p \in S(k),\ |\chi(\sigma_p) - \chi(g_0)| < \varepsilon,\ N_{k/\mathbf{Q}}p < x\}$$
$$= a(\mathcal{M}; g_0, \varepsilon)\int_2^x \frac{du}{\log u} + O(x\exp(-c_1\sqrt{\log x})), \quad c_1 > 0,$$

and

(21) $$a(\mathcal{M}; g_0, \varepsilon) > c_3\varepsilon^{c_2},$$

where c_j, $1 \leqslant j \leqslant 3$, and the implied by the O-symbol constant depend at most on $\mathcal{M}$ (but not on g_0, ε, x).

Proof. Let $K|k$ be a finite Galois extension such that each ψ in $\mathcal{M}$ factors through $W_1(K|k)$ and let $[K:k] = n+1$. Consider the (closed) subgroup

$$G_0 = \bigcap_{\psi \in \mathcal{M}} \operatorname{Ker}\psi$$

of $W(k)$ and let $G = W(k)/G_0$. It follows from the definitions that G fits into the exact sequence (1): indeed the restriction $\psi|_{C_K}$ of the representation ψ to C_K is equivalent to a direct sum of (normalized) grossencharacters of K, therefore there is a finite set $\mathcal{N}$ of the grossencharacters of K for which

$$G_0 \supseteq \bigcap_{\lambda \in \mathcal{N}} \operatorname{Ker}\lambda;$$

since clearly

$$C_K/(\bigcap_{\lambda \in \mathcal{N}} \operatorname{Ker}\lambda) \cong \mathcal{T} \times H_0,$$

where $\mathcal{T}$ is a real torus of dimension not exceeding $[K:\mathbf{Q}] - 1$ and H_0 is a finite Abelian group, it follows that $G/\mathcal{T}$ is a finite group. We let

$$S_0(\mathcal{M}) = \bigcup_{\psi \in \mathcal{M}} S_0(\psi)$$

and denote by $\bar{\sigma}_p$ the image of the Frobenius class under the natural homomorphism

$$\varphi\colon\ W(k)\to G.$$

For $p\in S(k)\backslash S_0(\mathcal{M})$ the set $\bar{\sigma}_p$ is a conjugacy class in G. Moreover, it can be deduced from the Hecke's Primzahlsatz, [1] (cf. also [5], Theorem 4) that, for each χ in $\hat{G}$, we have:

$$\sum_{|p|<x}\chi(\sigma_p) = g(\chi)\int_2^x\frac{du}{\log u}+O\left(x\exp\left(-c_4\frac{\log x}{\log w(\chi)+\sqrt{\log x}}\right)\right) \tag{22}$$

with $c_4>0$, where $|p|:=N_{k/\mathbf{Q}}p$. Let

$$\mathcal{B} = \{g|\ g\in W(k),\ |\chi(g)-\chi(g_0)|<\varepsilon\ \text{for}\ \chi\in\hat{\mathcal{M}}\}$$

and let

$$\mathcal{A} = \varphi(\mathcal{B}).$$

The set $\partial\mathcal{A}$ may be regarded as a semialgebraic set, therefore it satisfies (4) with $C(\mathcal{A})$ and α independent of ε and g_0 (cf. [12], Corollary 4.5). Estimate (20) follows now from Theorem 1, in view of (22). To deduce the inequality (21) we appeal to [3], Proposition 5 (cf. also [4], p. 461 and [6], Theorem 2, p. 99).

Remark 1. Theorem 2 may be regarded as a generalization of both Chebotarev's density theorem and the prime number theorem for grossencharacters due to E. Hecke. It confirms our conjecture stated in [3], p. 23, and in [6], p. 139–140.

Appendix. We reproduce here an argument kindly communicated to the author by Professor J.-P. Serre in his letter of April 24th, 1986 (cf. also [9], p. 45).

Lemma. *Let $\mathfrak{h}$ be a compact subset of the analytic set*

$$\mathcal{C} = \{x|\ x\in\mathbf{R}^n, f(x)=0\},$$

where $f\colon \mathbf{R}^n\to\mathbf{R}$ is an analytic function, and let d denote the (real) dimension of $\mathcal{C}$. Then

$$\int_{\mathcal{U}_\delta(\mathfrak{h})} dx < C(\mathfrak{h})\delta^{n-d}\quad \textit{for}\ 0<\delta<1. \tag{23}$$

Sketch of the proof. It follows from the Hironaka's theorem on resolution of singularities that

$$\mathfrak{h}\subseteq\bigcup_{j=1}^{l(\mathfrak{h})}B_j,\qquad B_j = g_j(I^d),$$

where $I:=[0,1]$ and g_j is a continuous map with the Lipschitz property, i.e.

$$|g_j(x+y)-g_j(x)|<C_j|y|,\qquad C_j>0.$$

Therefore

$$\int_{\mathcal{U}_\delta(\mathfrak{h})} dx \leqslant \sum_{j=1}^{l(\mathfrak{h})}\int_{\mathcal{U}_\delta(B_j)} dx. \tag{24}$$

Let

$$I(\nu,N) = \left[\frac{\nu}{N},\frac{\nu+1}{N}\right],\qquad 0\leqslant\nu\leqslant N-1,$$

and let

$$B_{j,\vec{\nu}} = g_j(I(\nu_1,N)\times\ldots\times I(\nu_d,N)),\qquad \vec{\nu}:=(\nu_1,\ldots,\nu_d).$$

Then

$$\int_{\mathcal{U}_\delta(B_j,\vec{\nu})} dx = O\left(\left(\delta+\frac{1}{N}\right)^n\right)$$

with an O-constant depending at most on C_j, $1\leqslant j\leqslant l(\mathfrak{h})$, and therefore

$$\int_{\mathcal{U}_\delta(B_j)} dx \leqslant \sum_{\vec{\nu}}\int_{\mathcal{U}_\delta(B_j,\vec{\nu})} dx = O\left(N^d\left(\delta+\frac{1}{N}\right)^n\right).$$

Choosing N to be equal to $[1/\delta]$ one obtains an estimate

$$\int_{\mathcal{U}_\delta(B_j)} dx = O(\delta^{n-d}). \tag{25}$$

Relation (23) is a consequence of (24) and (25).

Remark 2. As it has been pointed out in [9] one should try to prove this lemma by elementary methods making no use of the theory of resolution of singularities.

Remark 3. One can deduce the inequality (21) by a direct computation (cf. [7], n°5), but we shall not do it here.

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MAX-PLANCK-INSTITUT FÜR MATHEMATIK
UNIVERSITÄT BONN
Gottfried-Claren-Straße 26
5300 Bonn 3
West Germany

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ACTA ARITHMETICA
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Über die Restklasse modulo 2^{e+2} des Wertes $2^e n\zeta(1-2^e n, \mathfrak{K})$ der Zetafunktion einer Idealklasse aus dem reell-quadratischen Zahlkörper $Q(\sqrt{D})$ mit $D \equiv 3 \bmod 4$

von

HEINRICH LANG (Münster)

1. Einleitung. Es sei D eine quadratfreie natürliche Zahl mit der Eigenschaft $D \equiv 3 \bmod 4$ und $K = Q(\sqrt{D})$ der zugehörige reell-quadratische Zahlkörper. Ferner sei $2m \geqslant 2$ eine gerade natürliche Zahl. Ausgehend von den durch K. Barner [1] und C. L. Siegel [10] hergeleiteten Darstellungen für den Wert $\zeta(1-2m, \mathfrak{K})$ der Zetafunktion einer Idealklasse $\mathfrak{K}$ von K läßt sich zeigen, daß $2m\zeta(1-2m, \mathfrak{K})$ in dem Ring

$$Z_2 = \left\{\frac{p}{q} \in Q \mid p, q \in Z \text{ und } 2 \nmid q\right\}$$

der für 2 ganzen rationalen Zahlen liegt. Spaltet man von $2m$ die höchste Potenz von 2 ab und schreibt

$$2m = 2^e n \quad \text{mit } 2 \nmid n,$$

so kann man die Restklasse von $2m\zeta(1-2m, \mathfrak{K})$ mod 2^{e+2} explizit durch die Bernoullischen Zahlen B_μ und die Komponenten T, U der Grundeinheit

$$\varepsilon = T + U\sqrt{D} > 1$$

von K beschreiben. Dazu werde für $\nu \in \{0, \ldots, 4m-1\}$ das Polynom

$$(1.1) \quad F_\nu(x, y) = \frac{1}{\nu!}\frac{\partial^\nu}{\partial x^\nu}\sum_{\varrho=0}^{2m-1}\binom{2m-1}{\varrho}\frac{(-1)^{\varrho+1}}{2\varrho+1}D^{2m-1-\varrho}x^{2\varrho+1}y^{4m-2-2\varrho}$$
$$= \sum_{\varrho=0}^{2m-1}\binom{2m-1}{\varrho}\binom{2\varrho+1}{\nu}\frac{(-1)^{\varrho+1}}{2\varrho+1}D^{2m-1-\varrho}x^{2\varrho+1-\nu}y^{4m-2-2\varrho}$$

aus $Z_2[x, y]$ eingeführt. Für $\nu \geqslant 1$ hat man dafür offensichtlich:

$$F_\nu(x, y) = \frac{1}{\nu!}\frac{\partial^{\nu-1}}{\partial x^{\nu-1}}(x^2 - Dy^2)^{2m-1}.$$