

In [103] Hua improved a result of Buchstab on the rate of convergence of the solution of a certain differential difference equation with retarded argument and prescribed initial behavior. The elegant account contains a very useful lemma (Lemma 5): Suppose $F(u)$ is a positive function satisfying the functional inequality

$$F(u) \leq \frac{1}{u} \int_{u-1}^u F(t) dt, \quad u \geq u_0.$$

Then

$$F(u) \leq \exp \left\{ -u \left(\log u + \log \log u - 1 + \frac{\log \log u}{\log u} + O \left(\frac{1}{\log u} \right) \right) \right\}.$$

Differential difference equations of a similar kind have featured in many arithmetical investigations of late, and Hua's lemma has proved very helpful. (See also Hua's survey [B4], pp. 70–71.)

Hua and Wang wrote several papers ([131], [138], [147], [148]) on replacing statistical Monte Carlo methods in numerical analysis by effective deterministic procedures based on number theoretic ideas; their work, and other contributions chiefly from the Chinese & Russian schools, are described in their book [B9]. Their methods rest on the construction of special integral bases of algebraic number fields, the construction of simultaneous rational approximations to these by means of a given set of independent units or by linear recurrences, and on discrepancy measures. Numerical information for dimensions 2 to 18 is given in an appendix.

Publications of Loo-keng Hua

I. Articles

1. *Study on Sturm Theorem* (in Chinese), Science 14 (1929), 545–548.
2. *On incorrectness of Shu Jia Ju's paper* (in Chinese), Science 15 (1930), 307–309.
3. *Study on function $T^{-1}\{H(x)\}$* (in Chinese), Science 15 (1931), 871–888, 1055–1062.
4. *A Theorem of integral calculus*, Science 15 (1931), 1716–1719.
5. *Generalization and trial of additive angles formula in trigonometry* (in Chinese), Science 15 (1931), 1930–1945.
6. *A new function*, Science 15 (1931), 1051–1058.
7. *On pseudo-periodic functions*, Trans. Sci. Soc. China 8 (1934), 15–18; also Tôhoku Math J. 40 (1934), 27–33.
8. *On the representation of integers by circulant*, Trans. Sci. Soc. China 8 (1934), 19–21; also Tôhoku Math. J. 39 (1934), 316–321.
9. *On a theorem of Hermite*, Trans. Sci. Soc. China 8 (1934), 157–158.
10. *A note on Minkowski's theorem of homogeneous linear forms*, Trans. Sci. Soc. China 8 (1934), 160–161.
11. *On the hypergeometric functions of higher order*, Tôhoku Math. J. 39 (1934), 253–263.
12. *Note on diophantine equation of two circulants*, Tôhoku Math. J. 40 (1934), 34–35.
13. *Note on Pell's equation*, Tôhoku Math. J. 40 (1934), 36.
14. *Waring's problem for cubes*, Bull. Calcutta Math. Soc. 26 (1934), 139–140.
15. *On a certain kind of operations connected with linear algebra*, Tôhoku Math. J. 41 (1935), 222–246.
16. *A proof of Hadamard's theorem*, Tôhoku Math. J. 41 (1935), 247–248.
17. *The representation of integers as sums of the cubic function $(x^3+5x)/6$* , Tôhoku Math. J. 41 (1935), 356–360.
18. *On the representation of integers by the sums of seven cubic functions*, Tôhoku Math. J. 41 (1935), 361–366.
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20. *On an easier Waring-Kamke problem*, Sci. Rep. Tsing Hua Univ. A3 (1935), 247–260.
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22. *On Waring's problem with polynomial summands*, Amer. J. Math. 58 (1936), 553–562; also Chinese Math. Soc. 1 (1936), 23–61.
23. *Note on boundedly convergent power series*, Sci. Rep. Tsing Hua Univ. A3 (1936), 345–351.
24. *A problem on the additive theory of number, of several variables*, Math. Zeitschr. 41 (1936), 708–712.
25. *On Waring's problem*, Tôhoku Math. J. 42 (1936), 210–225.
26. *An easier Waring-Kamke problem*, J. London Math. Soc. 11 (1936), 4–5.
27. (with S. S. Shu) *On Fourier transforms in L^p in the complex domain*, J. Math. Phys. 15 (1936), 249–263.

28. *A problem in the additive theory of numbers of several variables*, J. London Math. Soc. 12 (1937), 257–261.
29. *A generalization of an easier Waring–Kamke problem*, J. London Math. Soc. 12 (1937), 262–264.
30. *On a generalized Waring problem*, Proc. London Math. Soc. (2) 43 (1937), 161–182.
31. *On the representation of integers as the sums of the k -th powers of primes*, Dokl. Akad. Nauk SSSR (N. S.) 17 (1937), 167–168.
32. *Some results in the additive prime-number theory*, Quart. J. Math. Oxford Ser. 9 (1938), 68–80.
33. *Some results in the additive prime number theory*, Dokl. Akad. Nauk SSSR (N. S.) 18 (1938), 3.
34. *Some results in the additive theory of numbers*, Dokl. Akad. Nauk SSSR (N. S.) 18 (1938), 4.
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36. *On Waring's problem*, Quart. J. Math. Oxford Ser. 9 (1938), 199–202.
37. *On Tarry's problem*, Quart. J. Math. Oxford Ser. 9 (1938), 315–320.
38. *On an exponential sum*, J. London Math. Soc. 13 (1938), 54–61; also J. Chinese Math. Soc. 2 (1940), 301–312 (with further progress.)
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40. *A generalization of Lendesdorff's Theorem*, Proc. Indian Acad. Sci. 7 (1938), 390–392.
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47. *On Waring's problem with cubic polynomial summands*, J. Indian Math. Soc. 4 (1940), 127–135; also Sci. Rep. Tsing Hua Univ. A4 (1940), 55–83.
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100. *On exponential sums over an algebraic number field*, Canadian J. Math. 3 (1951), 44–51.
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118. *Some definite integrals*, Acta Math. Sinica 6 (1956), 302–312.
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II. Books and monographs

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3. *Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains* (in Chinese), Academic Press, Peking 1958; Revised edition, 1965; Russian translation, Izd. inostran. lit., Moskva 1959; English translation, AMS, Providence, Rhode Island 1963.
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5. (with Wang Yuan) *Numerical Integration and its Applications (revised Numerical Calculation of Integral)* (in Chinese), Academic Press, Peking 1963.
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8. *Starting with Unit Circle* (in Chinese), Academic Press, Peking 1977.
9. (with Wang Yuan) *Application of Number Theory to Numerical Analysis* (in Chinese), Academic Press, Peking 1978.
10. (with Wu Zhi Qian and Lin Wei) *Systems of Partial Differential Equations of the Second Order of Two Unknown Functions and Two Independent Variables with Constant Coefficients* (in Chinese), Academic Press, Peking 1978.

III. Popular books (all in Chinese)

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7. *Starting with Sun Zi's "Magic Calculation" (Chinese remainder theorem)*, People's Education Press, Peking 1964.
8. *Popular Talk on Overall Planning Method and its Supplement (revised)*; Chinese Industry Press, Peking 1965.
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