

Table des matières du tome V fascicule 2

	Pages
L. Moser, On the minimal overlap problem of Erdős	117
A. Mąkowski, Sur quelques problèmes concernant les sommes de quatre cubes	121
G. Lomadse, Über die Darstellung der Zahlen durch einige quaternäre quadratische Formen	125
P. Erdős, Remarks on number theory II. Some problems on the σ function	171
W. Staś, Über eine Anwendung der Methode von Turán, auf die Theorie des Restgliedes im Primidealsatz	179
K. Mahler, An arithmetic property of groups of linear transformations . .	197
E. S. Barnes, The construction of perfect and extreme forms II . . .	205
L. Rédei und P. Turán, Zur Theorie der algebraischen Gleichungen über endlichen Körpern	223
C. Pommerenke, Über die Gleichverteilung von Gitterpunkten auf m -dimensionalen Ellipsoiden	227
Remarque concernant le travail de A. Schinzel et W. Sierpiński: „Sur certaines hypothèses concernant les nombres premiers”	259

La revue est consacrée à toutes les branches de l'arithmétique et de la théorie des nombres, ainsi qu'aux fonctions ayant de l'importance dans ces domaines.

Prière d'adresser les textes dactylographiés à l'un des rédacteurs de la revue ou bien à la Rédaction de

ACTA ARITHMETICA
Warszawa 10 (Pologne), ul. Śniadeckich 8.

La même adresse est valable pour toute correspondance concernant l'échange de Acta Arithmetica.

Les volumes IV et suivants de ACTA ARITHMETICA sont à obtenir chez
Ars Polona, Warszawa 5 (Pologne), Krakowskie Przedmieście 7.

Prix d'un volume 7,50 \$.

Les volumes I-III (rééditions) sont à obtenir chez
Johnson Reprint Corp., 111 Fifth Ave., New York, N. Y.

PRINTED IN POLAND

WROCŁAWSKA Drukarnia Naukowa

On the minimal overlap problem of Erdős

by

L. MOSER (Edmonton, Canada)

Let \mathcal{E} be a separation of the integers $1, 2, \dots, 2n$ into two disjoint classes $\{a_i\}$ and $\{b_j\}$ with n elements in each class, say

$$(1) \quad \begin{aligned} \{a_i\} : a_1 < a_2 < \dots < a_n, \quad \sum_{i=1}^n a_i = A, \\ \{b_j\} : b_1 < b_2 < \dots < b_n, \quad \sum_{j=1}^n b_j = B. \end{aligned}$$

Let M_k denote the number of solutions of $a_i - b_j = k$, i. e.

$$(2) \quad M_k = \sum_{a_i - b_j = k} 1 \quad (-2n \leq k \leq 2n)$$

and let

$$(3) \quad M = M(n) = \min_{\mathcal{E}} \max_k M_k.$$

Erdős [1] conjectured that for n even, $M = \frac{1}{2}n$ but later (written communication) disproved this and showed by a probability method that for large n , $M \leq \frac{4}{9}n$. Independently, Selfridge, Motzkin and Ralston [3] used the electronic computer SWAC to find an \mathcal{E} with $n = 15$ and $\max_k M_k = 6$. A slight modification of their example shows that for infinitely many n ,

$$(4) \quad M \leq 0,4n.$$

On the other hand, Erdős [2] proved that

$$(5) \quad M \geq 0,25n$$

and Scherk (written communication) improved this to

$$(6) \quad M > \left(1 - \frac{1}{\sqrt{2}}\right)n > 0,2929n.$$

The object of this note is to prove

$$(7) \quad M > \frac{\sqrt{2}}{4} (n-1) > 0,3525(n-1).$$

We can combine our method with that of Scherk to obtain

$$(8) \quad M > \sqrt{4-\sqrt{15}}(n-1) > 0,3570(n-1).$$

However, the proof of (8) is somewhat involved and the improvement over (7) is small so we will not prove (8) here.

We proceed to the proof of (7). First consider

$$(9) \quad P = \sum_k kM_k = \sum_{i,j} (a_i - b_j) = n(A - B).$$

Next note that

$$(10) \quad Q = \sum_k k^2 M_k = \sum_{i,j} (a_i - b_j)^2 = n(1^2 + 2^2 + \dots + (2n)^2) - 2AB.$$

Finally compute

$$(11) \quad R = \sum_k (k - P/n^2)^2 M_k = \sum_{i,j} (a_i - b_j - P/n^2)^2 = Q - (A - B)^2.$$

Since $(A - B)^2 + 2AB = \frac{1}{2}(A + B)^2 + \frac{1}{2}(A - B)^2$ and $A + B = 1 + 2 + \dots + 2n$ we easily find that

$$(12) \quad R = \frac{1}{6}n^2(4n^2 - 1) - \frac{1}{2}(A - B)^2 < \frac{2}{3}n^4.$$

We now obtain a lower bound for R which involves M . Note that in the sum $R = \sum_{i,j} (a_i - b_j - P/n^2)^2$, the distinct elements $a_i - b_j - P/n^2$ differ by integers. Further, the total number of terms in this sum is n^2 , and no term can be repeated more than M times. Hence R is decreased by replacing it by $2M$ terms of 0^2 each, $2M$ terms of 1^2 each, \dots , $2M$ terms of r^2 each, where r is determined by

$$(13) \quad 2Mr \leq n^2 < 2M(r+1).$$

With this value of r we therefore have

$$(14) \quad R \geq 2M(0^2 + 1^2 + \dots + r^2) = \frac{1}{3}Mr(r+1)(2r+1);$$

using the right hand side of (13) gives

$$(15) \quad R > \frac{M}{3} \left(\frac{n^2}{2M} - 1 \right) \left(\frac{n^2}{2M} \right) \left(\frac{n^2}{M} - 1 \right).$$

Combining (12) and (15) yields the required result (7).

References

- [1] P. Erdős, *Some results on number theory*, Riveon Lematematika 9 (1955), p. 48.
- [2] — *Problems and results in additive number theory*, Colloque sur la théorie des nombres, 1955, p. 135-137.
- [3] T. S. Motzkin, K. E. Ralston and J. L. Selfridge, *Minimal overlap under translation*, Abstract, Bull. Amer. Math. Soc. 62 (1956), p. 558.

Reçu par la Rédaction le 14. 6. 1958