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Bounds for solutions of additive equations in an algebraic number field II

by

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1. Introduction. We use the conventions and notation introduced in [4] throughout this paper. Let $\alpha_1, \ldots, \alpha_s$ be a set of integers in K. Consider the additive form

$$A(a, \lambda) = \alpha_1 a_1 \lambda_1^k + \ldots + \alpha_s a_s \lambda_s^k, \tag{1}$$

where $a = (a_1, \ldots, a_s)$ and $\lambda = (\lambda_1, \ldots, \lambda_s)$ are vectors. A set of numbers a, λ is called a *nontrivial solution of the equation*

$$A(a, \lambda) = 0 \tag{2}$$

if each a_i is 1 or -1, and the λ_i $(1 \leqslant i \leqslant s)$ are totally nonnegative integers of K, not all zero. Write

$$\|\lambda\| = \max(\|\lambda_1\|, \ldots, \|\lambda_s\|) \quad \text{and} \quad |A| = \max(1, \|\alpha_1\|, \ldots, \|\alpha_s\|).$$

In this paper, we shall prove the following theorem by the combination of the methods of Schmidt [1] and Siegel [3].

THEOREM. *Suppose $s \geqslant c_1(k, n, \varepsilon)$. Then the equation* (2) *has a nontrivial solution with*

$$\|\lambda\| \ll |A|^{\varepsilon}. \tag{3}$$

This gives a generalization of a theorem due to Schmidt [1]. He first established the estimation (3) for the case of rational field.

If a, λ is a nontrivial solution of (2) with (3) for the case of $k = 2$, then $a, \Lambda = (\Lambda_1, \ldots, \Lambda_s)$, with $\Lambda_i = \lambda_i^2$ $(1 \leqslant i \leqslant s)$, is a nontrivial solution of the linear equation

$$\alpha_1 a_1 \Lambda_1 + \ldots + \alpha_s a_s \Lambda_s = 0,$$

having

$$\|\Lambda\| \ll |A|^2.$$

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Therefore we may suppose $k>1$ throughout this paper.

If A is not identically zero, put

$$A' = \frac{c_2}{\sigma} A,$$

where σ is a nonzero element in the integral ideal $(\alpha_1, \ldots, \alpha_s)$ with the least norm in absolute value and $c_2 = c_2(K)$ is a rational integer such that $\sigma | c_2 \alpha_i$ $(1 \leqslant i \leqslant s)$. (See, e.g., Lemma 3 in [4].) We may suppose without loss of generality that

$$|N(\sigma)|^{1/n} \ll \|\sigma\| \ll |N(\sigma)|^{1/n},$$

because we can choose a unit η of K such that $\eta\sigma$ satisfies the above relation, and use $\eta\sigma$ instead of σ. (See, e.g., Lemma 1 in [4].) If $A \equiv 0$, then we put $A' = A$.

2. Reductions. One can prove by Siegel's method that if $s \geqslant c_3(k, n)$, then (2) has a nontrivial solution with

(4) $$\|\lambda\| \ll |A|^{c_4(k,n)}.$$

Let X be the set of x such that if $s \geqslant c_5(k, n, x)$, then (2) has a nontrivial solution with

$$\|\lambda\| \ll |A|^{x}.$$

(4) shows that X is not empty. Let x be the greatest lower bound of X. The conclusion of the theorem is $x=0$. We will suppose that $x>0$ and we will reach a contradiction.

We may choose y such that

(5) $$0<y<x \quad \text{and} \quad x+kx^2-kxy-k^2x^2y<y.$$

(See Schmidt [1], p. 222.) Take z so small that

(6) $$y+12xz<x, \quad z<y/10, \quad z<1/10.$$

Then pick x' with

(7) $$\max\big(y+12xz,\ x-xz/(2n)\big)<x'<x.$$

We proceed to prove that $x' \in X$. It will suffice to prove the assertion when $|A|$ is large, say $|A| \geqslant c_6(k, K, x')$. (See Wang Yuan [4].) And we may suppose clearly that $\alpha_i \neq 0$ $(1 \leqslant i \leqslant s)$. Finally, pick x'' with

(8) $$\max\big(y+12xz,\ x-xz/(2n)\big)<x''<x'$$

and choose ε_1 such that

(9) $$(1+\varepsilon_1)x''+4\varepsilon_1/k<x'.$$

Since

$$1 \leqslant |N(\alpha_j)| = |\alpha_j^{(1)} \ldots \alpha_j^{(n)}| \leqslant |\alpha_j^{(i)}|\,|A|^{n-1}$$

we have

$$\min_{i,j} |\alpha_j^{(i)}| \geqslant |A|^{-n+1}.$$

Divide the interval $[-n+1, 1]$ into a finite number of intervals $\{I\}$ of length $\leqslant \varepsilon_1$. One of these intervals I_1 will be such that there are not less than s_1 numbers among α_i's satisfying

$$|\alpha_j^{(1)}| = |A|^{e_1}, \quad e_1 \in I_1,$$

where $s_1 \geqslant s \Big/ \left(\left[\frac{n}{\varepsilon_1}\right]+1\right)$. We may suppose without loss of generality that $\alpha_1, \ldots, \alpha_{s_1}$ satisfy the above relation. Similarly, there exists $I_2 \in \{I\}$ such that there are at least s_2 numbers in $\alpha_1, \ldots, \alpha_{s_1}$ satisfying

$$|\alpha_j^{(2)}| = |A|^{e_2}, \quad e_2 \in I_2,$$

where

$$s_2 \geqslant s_1 \Big/ \left(\left[\frac{n}{\varepsilon_1}\right]+1\right) \geqslant s \Big/ \left(\left[\frac{n}{\varepsilon_1}\right]+1\right)^2.$$

We may suppose that $\alpha_1, \ldots, \alpha_{s_2}$ have the above property. Continuing this process, we obtain t numbers among α_i's which we may suppose to be $\alpha_1, \ldots, \alpha_t$ such that

(10) $$|\alpha_j^{(i)}/\alpha_l^{(i)}| \leqslant |A|^{\varepsilon_1}, \quad 1 \leqslant j, l \leqslant t,\ 1 \leqslant i \leqslant n,$$

where $t \geqslant s \Big/ \left(\left[\frac{n}{\varepsilon_1}\right]+1\right)^n$.

Suppose that if $\alpha_1, \ldots, \alpha_t$ satisfy (10) and $t \geqslant c_7(k, n, x')$, the equation

$$\alpha_1 a_1' \lambda_k'^k + \ldots + \alpha_t a_t' \lambda_t'^k = 0$$

has a nontrivial solution satisfying

$$\max_{i,j} |\lambda_j'^{(i)}| \ll |A|^{x'}.$$

Take $c_5(k, n, x') = \left(\left[\frac{n}{\varepsilon_1}\right]+1\right)^n c_7(k, n, x')$ and set $a_i = a_i'$, $\lambda_i = \lambda_i'$ $(1 \leqslant i \leqslant t)$ $a_i = 1$, $\lambda_i = 0$ $(t < i \leqslant s)$. We have a nontrivial solution of (2) with

$$\|\lambda\| \ll |A|^{x'}.$$

Hence we may suppose that the coefficients of (2) satisfy

(11) $$|\alpha_j^{(i)}/\alpha_l^{(i)}| \leqslant |A|^{\varepsilon_1}, \quad 1 \leqslant j, l \leqslant s, \quad 1 \leqslant i \leqslant n.$$

Let σ_i $(1 \leqslant i \leqslant s)$ be a set of totally nonnegative units such that

(12) $$|N(\alpha_j)|^{1/n} \ll |\alpha_j^{(i)}\sigma_j^{(i)k}| \ll |N(\alpha_j)|^{1/n}, \quad 1 \leqslant j \leqslant s,\ 1 \leqslant i \leqslant n.$$

(See, e.g., Lemma 1 in [4].) Let $a^n = |A|^{n\varepsilon_1} \max |N(\alpha_i)|$ and let P_i be the largest rational integer such that

$$|N(\alpha_i)|\, P_i^{kn} \leqslant a^n, \quad 1 \leqslant i \leqslant s.$$

Since $|A| \geqslant c_6$ and $a^n/|N(\alpha_i)| \geqslant |A|^{n\varepsilon_1}$, we may suppose

$$P_i \geqslant 2^{-1/(kn)} \left(\frac{a^n}{|N(\alpha_i)|}\right)^{1/(kn)}, \quad 1 \leqslant i \leqslant s.$$

Hence

$$|N(\alpha_i)|\, P_i^{kn} \geqslant \tfrac{1}{2} a^n.$$

Set

(13) $$\alpha_i' = \alpha_i \sigma_i^k P_i^k \quad \text{and} \quad \lambda_i = \sigma_1^{-1} \sigma_i P_i \lambda_i', \quad 1 \leqslant i \leqslant s.$$

Then

$$A_1(a, \lambda') = \alpha_1' a_1 \lambda_1'^k + \ldots + \alpha_s' a_s \lambda_s'^k = \sigma_1^k A(a, \lambda),$$

where $\lambda' = (\lambda_1', \ldots, \lambda_s')$. By (11) and (12), we have

$$a \ll |\alpha_j^{(i)'}| \ll a, \quad 1 \leqslant j \leqslant s,\ 1 \leqslant i \leqslant n$$

and

(14) $$|\sigma_1^{(i)-1}\sigma_j^{(i)}| = \left|\frac{\sigma_j^{(i)k}\alpha_j^{(i)}\alpha_j^{(i)-1}}{\sigma_1^{(i)k}\alpha_1^{(i)}\alpha_1^{(i)-1}}\right|^{1/k}$$
$$\ll |N(\alpha_j)|^{1/(kn)} |N(\alpha_1)|^{-1/(kn)} |A|^{\varepsilon_1/k} \ll |A|^{2\varepsilon_1/k},$$
$$1 \leqslant j \leqslant s,\ 1 \leqslant i \leqslant n.$$

Since

$$|N(\alpha_i)| = |A|^{n\varepsilon_1} |N(\alpha_i)| \max_j |N(\alpha_j)| / |A|^{n\varepsilon_1} \max_l |N(\alpha_l)|$$
$$= a^n |N(\alpha_i)|\, |A|^{-n\varepsilon_1} / \max |N(\alpha_l)| \geqslant a^n |A|^{-2n\varepsilon_1},$$

we have

(15) $$P_i^{kn} \leqslant \frac{a^n}{|N(\alpha_i)|} \leqslant |A|^{2n\varepsilon_1}.$$

Suppose that the equation $A_1(a, \lambda') = 0$ has a nontrivial solution such that

$$\|\lambda'\| \ll |A_1|^{x''} \ll a^{x''} \ll |A|^{(1+\varepsilon_1)x''}.$$

Then it follows by (9), (13), (14) and (15) that (2) has a nontrivial solution satisfying

$$\|\lambda\| \ll |A|^{(1+\varepsilon_1)x'' + 4\varepsilon_1/k} \ll |A|^{x'}.$$

Thus it will suffice to prove that if $s \geqslant c_8$ and if α_i $(1 \leqslant i \leqslant s)$ satisfy

(16) $$c_9 a < |\alpha_j^{(i)}| < c_{10} a, \quad 1 \leqslant j \leqslant s,\ 1 \leqslant i \leqslant n,$$

where $c_9 = c_9(k, K)$, $c_{10} = c_{10}(k, K)$ and $a > c_6(k, K, x')$, then (2) has a nontrivial solution with

(17) $$\|\lambda\| \ll a^{x''}.$$

Of course c_8 depends on k, n, x'', but since k, n, x, y, z, x', x'' are fixed, we will not indicate the dependency of c_8 (and of subsequent constants) on these parameters.

We shall first prove that this assertion can be derived by the following Proposition 1, and then give the proof of the proposition.

PROPOSITION 1. *Suppose that α_i $(1 \leqslant i \leqslant s)$ satisfy* (16). *If $s \geqslant c_{11}$, then either* (2) *has a nontrivial solution with* (17) *or there is a nonzero integer χ such that*

(18) $$A(a, \lambda) = \chi, \quad \chi \in P(a^{6z}), \quad \lambda_i \in P(a^y), \quad 1 \leqslant i \leqslant s,$$

where each a_i is 1 *or* -1 *and λ_j $(1 \leqslant j \leqslant s)$ are totally nonnegative integers of K, not all zero.*

We may suppose that $c_5(k, n, 2x)$ and c_{11} are integers. Denote $v = c_5$, $u = c_{11}$ and $s = uv$. Replace the indices $1 \leqslant l \leqslant s$ by double indices $1 \leqslant i \leqslant v$, $1 \leqslant j \leqslant u$. Then (1) becomes

$$A(a, \lambda) = \sum_{i=1}^{v} A_i(a_i, \lambda_i),$$

where $a_i = (a_{i1}, \ldots, a_{iu})$, $\lambda_i = (\lambda_{i1}, \ldots, \lambda_{iu})$ and

$$A_i(a_i, \lambda_i) = \sum_{j=1}^{u} \alpha_{ij} a_{ij} \lambda_{ij}^k, \quad 1 \leqslant i \leqslant v.$$

If there is an equation, say $A_i(a_i, \lambda_i) = 0$, which has a nontrivial solution having

$$\|\lambda_i\| \ll a^{x''},$$

then we have directly a nontrivial solution of (2) with (17). Otherwise it follows by Proposition 1 that there are nonzero integers $\chi_1, \ldots, \chi_v$ satisfying

$$A_i(u_i, \lambda_i) = \chi_i, \quad \chi_i \in P(a^{6z}), \quad \lambda_{ij} \in P(a^y), \quad 1 \leqslant i \leqslant v,\ 1 \leqslant j \leqslant u,$$

where each a_{ij} is 1 or -1. Since the equation

$$B(b, \mu) = \chi_1 b_1 \mu_1^k + \ldots + \chi_v b_v \mu_v^k = 0$$

has a nontrivial solution with

$$\|\mu\| \ll \max \|\chi_i\|^{2x} \ll a^{12xz},$$

the equation (2) has a nontrivial solution $b_i a_{ij}$, $\mu_i \lambda_{ij}$ $(1 \leqslant i \leqslant v,\ 1 \leqslant j \leqslant u)$ satisfying

$$\max_{i,j} \|\mu_i \lambda_{ij}\| \leqslant \|\mu\| \max_i \|\lambda_i\| \ll a^{y+12xz} \ll a^{x''}$$

by (8). Hence it remains only to prove Proposition 1.

3. The circle method. Let

$$t = a^{z/n} \quad \text{and} \quad h = a^{1+ky-z/n}. \tag{19}$$

Let $\Gamma(t)$ be the set consisting of $\gamma = x_1 \varrho_1 + \ldots + x_n \varrho_n$ satisfying $(x_1, \ldots, x_n) \in G_n$, x_i $(1 \leqslant i \leqslant n)$ rational numbers, $\gamma \to \mathfrak{a}$ and $N(\mathfrak{a}) \leqslant t^n$. For any $\gamma \in \Gamma(t)$, subject to $\gamma \to \mathfrak{a}$, we define the basic domain B_γ by

$$\{(x_1, \ldots, x_n)\colon (x_1, \ldots, x_n) \in G_n,\ \xi = x_1 \varrho_1 + \ldots + x_n \varrho_n$$
$$\text{such that } h\|\xi - \gamma_0\| < 1 \text{ for some } \gamma_0 \equiv \gamma \,(\mathrm{mod}\ \delta^{-1})\}.$$

We may prove that if $\gamma_1 \neq \gamma_2$, then $B_{\gamma_1} \cap B_{\gamma_2} = \emptyset$. In fact, suppose there is a $\xi \in B_{\gamma_1} \cap B_{\gamma_2}$, i.e., $h\|\xi - \gamma_{0i}\| < 1$, where $\gamma_{0i} \equiv \gamma_i \,(\mathrm{mod}\ \delta^{-1})$, $i = 1, 2$. For simplicity, we set $\gamma_{0i} = \gamma_i$ $(i = 1, 2)$. Denote

$$\max(h|\xi^{(i)} - \gamma_j^{(i)}|,\ t^{-1}) = \sigma_j^{(i)}, \quad 1 \leqslant j \leqslant 2,\ 1 \leqslant i \leqslant n.$$

Then

$$\prod_{i=1}^{n} \sigma_j^{(i)} < 1, \quad \max_i \sigma_j^{(i)-1} \leqslant t, \quad j = 1, 2,$$

and thus

$$|\gamma_1^{(i)} - \gamma_2^{(i)}| \leqslant |\xi^{(i)} - \gamma_1^{(i)}| + |\xi^{(i)} - \gamma_2^{(i)}| \leqslant h^{-1}(\sigma_1^{(i)} + \sigma_2^{(i)})$$
$$= h^{-1} \sigma_1^{(i)} \sigma_2^{(i)} (\sigma_1^{(i)-1} + \sigma_2^{(i)-1}) \leqslant 2h^{-1} \sigma_1^{(i)} \sigma_2^{(i)} t.$$

Suppose $\gamma_i \to \mathfrak{a}_i$ $(i = 1, 2)$. We have

$$N(\mathfrak{a}_1 \mathfrak{a}_2) |N(\gamma_1 - \gamma_2)| < (2h^{-1} t^3)^n < D^{-1},$$

since $a \geqslant c_6$. On the other hand, $\mathfrak{a}_1 \mathfrak{a}_2 (\gamma_1 - \gamma_2) \delta$ is an integral ideal, and thus

$$N(\mathfrak{a}_1 \mathfrak{a}_2) |N(\gamma_1 - \gamma_2)| \geqslant N(\delta^{-1}) = D^{-1}.$$

This gives a contradiction, and therefore the assertion follows.

We define the supplementary domain E by

$$E = G_n - \bigcup_{\gamma \in \Gamma(t)} B_\gamma. \tag{20}$$

We use the notations

$$\xi = x_1 \varrho_1 + \ldots + x_n \varrho_n, \quad dx = dx_1 \ldots dx_n, \quad B = a^y, \quad H = a^{6z},$$

$$S_i(\xi) = \sum_{\lambda \in P(B)} E(a_i \xi \alpha_i \lambda^k), \quad 1 \leqslant i \leqslant s, \quad S(\xi) = \prod_{i=1}^{s} S_i(\xi), \tag{21}$$

and

$$F(\xi) = \sum_{\chi \in P(H)} S(\xi) E(-\xi\chi),$$

where a_i $(1 \leqslant i \leqslant s)$ are defined by

$$a_{2p-1} = \frac{|\alpha_{2p-1}^{(p)}|}{\alpha_{2p-1}^{(p)}}, \quad a_{2p} = -\frac{|\alpha_{2p}^{(p)}|}{\alpha_{2p}^{(p)}} \quad (1 \leqslant p \leqslant r_1),$$
$$a_j = 1 \quad (2r_1 + 1 \leqslant j \leqslant s). \tag{22}$$

Let Z denote the number of solutions of the equation

$$a_1 \alpha_1 \lambda_1^k + \ldots + a_s \alpha_s \lambda_s^k = \chi$$

in totally nonnegative integers $\lambda_1, \ldots, \lambda_s$, χ satisfying

$$\chi \in P(H), \quad \lambda_i \in P(B), \quad 1 \leqslant i \leqslant s.$$

Then

$$Z = \sum_{\gamma \in \Gamma(t)} \int_{B_\gamma} F(\xi)\, d\xi + \int_E F(\xi)\, d\xi. \tag{23}$$

We shall show that under the assumption made in Proposition 1, either (2) has a nontrivial solution with (17) or Z is > 1.

4. Supplementary domain. In this section $a_i = \pm 1$ $(1 \leqslant i \leqslant s)$ which are not restricted by (22).

LEMMA 1 (Schmidt). *Suppose that*

$$T \geqslant c_{12}(k, K, \varepsilon_2), \quad C \geqslant T^{1 - 1/G + \varepsilon_2} \quad \textit{and} \quad \Big|\sum_{\lambda \in P(T)} E(\xi \lambda^k)\Big| \geqslant C,$$

where $G = 2^{k-1}$. Then there exist a totally nonnegative integer α and an integer β such that

$$\|\alpha\xi - \beta\| \ll \left(\frac{T^n}{C}\right)^G T^{\varepsilon_2 - k}$$

and

$$0 < \|\alpha\| \ll \left(\frac{T^n}{C}\right)^G T^{\varepsilon_2}.$$

See, e.g., Wang Yuan [3].

LEMMA 2. *Suppose that $s \geqslant c_{13}$ and $\xi \in E$. Then either*

$$|F(\xi)| < H^n B^{n(s-k)} a^{-2n} \tag{24}$$

or there is a nontrivial solution of (2) *with* (17).

Proof. Take ε_2 such that

$$0 < \varepsilon_2 < c_{14} < 1/(2G), \tag{25}$$

where c_{14} is a constant to be determined later. Set

(26) $$m = c_5(k, n, x+\varepsilon_2) \quad \text{and} \quad h = m^2.$$

Choose c_{13} $(> 8kn)$ sufficiently large such that if $s \geqslant c_{13}$, then

$$\frac{n(k+2/y)}{s-h+1} < \varepsilon_2,$$

and by (21), we have

$$(B^k a^2)^{n/(s-h+1)} = B^{n(k+2/y)/(s-h+1)} < B^{\varepsilon_2}.$$

If (24) fails to hold, then

$$H^n |S_1(\xi)\ldots S_s(\xi)| \geqslant F(\xi) \geqslant H^n B^{n(s-k)} a^{-2n}.$$

We may suppose without loss of generality that

$$|S_1(\xi)| \geqslant \ldots \geqslant |S_s(\xi)|.$$

Hence we have

$$|S_h(\xi)|^{s-h+1} B^{n(h-1)} \geqslant B^{n(s-k)} a^{-2n},$$

and thus by (25),

$$|S_i(\xi)| \geqslant B^{n(s-h-k+1)/(s-h+1)} a^{-2n/(s-h+1)}$$
$$\geqslant B^n (B^k a^2)^{-n/(s-h+1)} \geqslant B^{n-\varepsilon_2} > B^{n-1/G+\varepsilon_2}, \quad 1 \leqslant i \leqslant h.$$

Take $C = B^{n-\varepsilon_2}$ in Lemma 1. Then it follows that there are totally nonnegative integers σ_i $(1 \leqslant i \leqslant s)$ and integers β_j $(1 \leqslant j \leqslant s)$ such that

(27) $$\|\xi\alpha_i \sigma_i - \beta_i\| \ll B^{-k+2G\varepsilon_2},$$
$$0 < \|\sigma_i\| \ll B^{2G\varepsilon_2}, \quad 1 \leqslant i \leqslant h.$$

Denote $\tau_i = \beta_i \sigma_i^{k-1}$ $(1 \leqslant i \leqslant h)$. Then

$$\|\xi\alpha_i \sigma_i^k - \tau_i\| \leqslant \|\sigma_i\|^{k-1} \|\xi\alpha_i \sigma_i - \beta_i\| \ll B^{-k+2kG\varepsilon_2}, \quad 1 \leqslant i \leqslant h.$$

Therefore by (16), we have

$$\|\alpha_i \sigma_i^k \tau_j - \alpha_j \sigma_j^k \tau_i\| \leqslant \|(\xi\alpha_j \sigma_j^k - \tau_j)\alpha_i \sigma_i^k\| + \|(\xi\alpha_i \sigma_i^k - \tau_i)\alpha_j \sigma_j^k\|$$
$$\ll aB^{-k+4kG\varepsilon_2}, \quad 1 \leqslant i, j \leqslant h,$$

i.e., the vectors $f_i = (\alpha_i \sigma_i^k, \tau_i)$ $(1 \leqslant i \leqslant h)$ satisfy

(28) $$\|\det(f_i, f_j)\| \ll aB^{-k+4kG\varepsilon_2}, \quad 1 \leqslant i, j \leqslant h.$$

Let β be a nonzero element in the integral idea $(\alpha_1 \sigma_1^k, \tau_1)$ with the least norm in absolute value. Then $\beta | c_2 \alpha_1 \sigma_1^k$ and $\beta | c_2 \tau_1$. (See § 1.) Set

$$c_2 \alpha_1 \sigma_1^k = \beta\sigma, \quad c_2 \tau_1 = \beta\tau \quad \text{and} \quad f = (\sigma, \tau).$$

Then

$$c_2 f_1 = \beta f.$$

We may choose β such that

(29) $$|N(\sigma)|^{1/n} \ll \|\sigma\| \ll |N(\sigma)|^{1/n}.$$

(See, e.g., Lemma 1 in [4].) We have also two integers σ' and τ' such that

$$\beta = \alpha_1 \sigma_1^k \tau' - \tau_1 \sigma',$$

therefore

$$c_2 \beta = \beta\sigma\tau' - \beta\tau\sigma',$$

i.e.,

(30) $$c_2 = \sigma\tau' - \tau\sigma'.$$

Set $g = (\sigma', \tau')$. Then by (30),

$$f_i = c_2^{-1} \varphi_i f + c_2^{-1} \psi_i g,$$

where

$$\varphi_i = \begin{vmatrix} \alpha_i \sigma_i^k & \sigma' \\ \tau_i & \tau' \end{vmatrix} \quad \text{and} \quad \psi_i = \begin{vmatrix} \sigma & \alpha_i \sigma_i^k \\ \tau & \tau_i \end{vmatrix}, \quad 1 \leqslant i \leqslant h$$

are integers. By (27), we have

$$1 \leqslant N(\sigma_j) \leqslant |\sigma_j^{(i)}| B^{2(n-1)G\varepsilon_2},$$

i.e.,

(31) $$|\sigma_j^{(i)}| \geqslant B^{-2(n-1)G\varepsilon_2}, \quad 1 \leqslant j \leqslant h,\ 1 \leqslant i \leqslant n.$$

Since $c_2 \alpha_1 \sigma_1^k = \beta\sigma$, we have by (16), (27), (29) and (31) that

(32) $$a\|\sigma\|^{-1} B^{2kG\varepsilon_2} \gg |\beta^{(i)}| \gg a\|\sigma\|^{-1} B^{-2k(n-1)G\varepsilon_2}, \quad 1 \leqslant i \leqslant n.$$

Therefore it follows by (28) that

(33) $$\|\psi_i\| = \|\det(f_i, f)\| \leqslant c_2 \max_i |\beta^{(i)}|^{-1} \|\det(f_i, f_1)\|$$
$$\ll \|\sigma\| B^{-k+6knG\varepsilon_2} = M, \quad 1 \leqslant i \leqslant h.$$

1. Suppose $M \geqslant 1$. Replace the indices $1 \leqslant l \leqslant h$ by double indices $1 \leqslant i, j \leqslant m$. Define

$$A_i(a_i, \lambda_i) = \sum_{j=1}^{m} \psi_{ij} a_{ij} \lambda_{ij}^k, \quad 1 \leqslant i \leqslant m,$$

where $a_i = (a_{i1}, \ldots, a_{im})$ and $\lambda_i = (\lambda_{i1}, \ldots, \lambda_{im})$ $(1 \leqslant i \leqslant m)$. It follows by (26), (33) and the definition of the set X that the equation

$$A_i(a_i, \lambda_i) = 0$$

has a nontrivial solution having

$$\|\boldsymbol{\lambda}_i\| \ll M^{x+\varepsilon_2}, \quad 1 \leqslant i \leqslant m. \tag{34}$$

Let

$$\boldsymbol{g}_i = \sum_{j=1}^{m} a_{ij} \lambda_{ij}^k \boldsymbol{f}_{ij} = c_2^{-1} \Big(\sum_{j=1}^{m} a_{ij} \lambda_{ij}^k \varphi_{ij} \Big) \boldsymbol{f}, \quad 1 \leqslant i \leqslant m.$$

The first coordinate of $\boldsymbol{g}_i$ is

$$\beta_i = c_2^{-1} \sigma \sum_{j=1}^{m} a_{ij} \lambda_{ij}^k \varphi_{ij}, \quad 1 \leqslant i \leqslant m.$$

Therefore $c_2 \beta_i/\sigma$ $(1 \leqslant i \leqslant m)$ are integers. If $\beta_1, \ldots, \beta_m$ are not all zero, then let χ be a nonzero element in the integral ideal $(\beta_1, \ldots, \beta_m)$ with least norm in absolute value and satisfying

$$|N(\chi)|^{1/n} \ll \|\chi\| \ll |N(\chi)|^{1/n}.$$

Then $\sigma | c_2 \chi$ and by (29),

$$\|\sigma\| \ll \|\chi\|.$$

Consider the form

$$B(\boldsymbol{b}, \boldsymbol{\mu}) = \sum_{i=1}^{m} \beta_i b_i \mu_i^k,$$

where $\boldsymbol{b} = (b_1, \ldots, b_m)$ and $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_m)$. Since

$$\beta_i = \sum_{j=1}^{m} a_{ij} \lambda_{ij}^k \alpha_{ij} \sigma_{ij}^k, \quad 1 \leqslant i \leqslant m,$$

we have by (16), (27) and (34),

$$\|\beta_i\| \ll aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k}, \quad 1 \leqslant i \leqslant m,$$

and therefore the equation $B(\boldsymbol{b}, \boldsymbol{\mu}) = 0$ has a nontrivial solution satisfying

$$\|\boldsymbol{\mu}\| \ll \max\left(1, \frac{aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k}}{\|\sigma\|}\right)^{x+\varepsilon_2}.$$

Consequently (2) has a nontrivial solution

$$b_i a_{ij}, \quad \mu_i \sigma_{ij} \lambda_{ij} \quad (1 \leqslant i, j \leqslant m),$$

$$a_l = 1, \quad \lambda_l = 0 \quad (h < l \leqslant s)$$

satisfying

$$\|\boldsymbol{\lambda}\| \ll B^{2G\varepsilon_2} M^{x+\varepsilon_2} \max\left(1, \frac{aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k}}{\|\sigma\|}\right)^{x+\varepsilon_2}$$

$$\ll B^{2G\varepsilon_2} \max\left(M, \frac{aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k+1}}{\|\sigma\|}\right)^{x+\varepsilon_2}$$

$$= B^{2G\varepsilon_2} I^{x+\varepsilon_2}, \quad \text{say}.$$

Since $c_2 \alpha_1 \sigma_1^k = \beta\sigma$, by (27), (29) and (33), we have

$$\|\sigma\| \ll aB^{2kG\varepsilon_2} (\max_i |\beta^{(i)}|)^{-1} \ll aB^{2kG\varepsilon_2},$$

$$M \ll aB^{-k+8knG\varepsilon_2},$$

and therefore

$$\begin{aligned} I &\ll aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k+1} \|\sigma\|^{-1} \\ &= aB^{2kG\varepsilon_2} M^{(x+\varepsilon_2)k+1} M^{-1} B^{-k+6knG\varepsilon_2} \\ &\ll aB^{-k+8knG\varepsilon_2} (aB^{-k+8knG\varepsilon_2})^{(x+\varepsilon_2)k}. \end{aligned}$$

Since $a > c_6$, we have

$$\|\boldsymbol{\lambda}\| \ll a^{x+kx^2-kxy-k^2x^2y+c_{15}(k,n)\varepsilon_2}.$$

Hence if c_{14} is sufficiently small, then by (5) and (8), we have the desired (17).

2. Suppose $M < 1$. We revert to the indices $1 \leqslant i \leqslant h$. By (33), we have $\psi_i = 0$ $(1 \leqslant i \leqslant h)$, i.e., $c_2 \boldsymbol{f}_i$ $(1 \leqslant i \leqslant h)$ are integral multiples of the integral vector $\boldsymbol{f}$. Let

$$B(\boldsymbol{b}, \boldsymbol{\mu}) = \sum_{i=1}^{h} \alpha_i \sigma_i^k b_i \mu_i^k,$$

where $\boldsymbol{b} = (b_1, \ldots, b_h)$ and $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_h)$. Let χ be a nonzero element in the integral ideal $(\alpha_1 \sigma_1^k, \ldots, \alpha_h \sigma_h^k)$ with least norm in absolute value and satisfying $|N(\chi)|^{1/n} \gg \|\chi\| \gg |N(\chi)|^{1/n}$. Then $\sigma | c_2 \chi$, and thus $\|\sigma\| \ll \|\chi\|$. Hence the equation $B(\boldsymbol{b}, \boldsymbol{\mu}) = 0$ has a nontrivial solution satisfying

$$\|\boldsymbol{\mu}\| \ll \max_i (\|\alpha_i \sigma_i^k\| \|\sigma\|^{-1})^{x+\varepsilon_2} \ll (aB^{2kG\varepsilon_2} \|\sigma\|^{-1})^{x+\varepsilon_2}.$$

It derives a nontrivial solution of (2):

$$a_i = b_i, \ \lambda_i = \sigma_i \mu_i \quad (1 \leqslant i \leqslant h),$$

$$a_j = 1, \quad \lambda_j = 0 \quad (h < j \leqslant s).$$

If c_{14} is sufficiently small, then

$$\begin{aligned} \|\boldsymbol{\lambda}\| &\ll B^{2G\varepsilon_2} (aB^{2kG\varepsilon_2} \|\sigma\|^{-1})^{x+\varepsilon_2} \\ &= a^{x+\varepsilon_2+2Gy\varepsilon_2+2kGy(x+\varepsilon_2)\varepsilon_2} \|\sigma\|^{-x-\varepsilon_2} \\ &\leqslant a^{x+xz/(2n)} \|\sigma\|^{-x}. \end{aligned}$$

If $\|\sigma\| \geqslant a^{z/n}$, then by (8), we have

$$\|\lambda\| \leqslant a^{x-xz/(2n)} \leqslant a^{x''},$$

i.e., (17) is true. Now we suppose that $\|\sigma\| < a^{z/n}$. Then by (27) and (31), we have

$$\|\xi-\sigma^{-1}\tau\| = \|\xi-\alpha_1^{-1}\sigma_1^{-1}\beta_1\| \leqslant \max_i \frac{1}{|\alpha_1^{(i)}\sigma_1^{(i)}|}\|\alpha_1\sigma_1\xi-\beta_1\|$$

$$\ll a^{-1}B^{2(n-1)G\varepsilon_2-k+2G\varepsilon_2} < a^{-1-ky+z/n} = h^{-1}$$

if c_{14} is sufficiently small. Let $\sigma^{-1}\tau\delta = \mathfrak{b}/\mathfrak{a}$, $(\mathfrak{a}, \mathfrak{b}) = 1$. Then $\mathfrak{a}|\sigma$, and thus

$$N(\mathfrak{a}) \leqslant |N(\sigma)| < a^z = t^n.$$

This means that $\xi \in B_\gamma$, where $\gamma \equiv \sigma^{-1}\tau \pmod{\delta^{-1}}$. The lemma is proved.

5. Basic domain. We use the notations

(35) $$\xi-\gamma = \zeta, \quad \eta = y_1\omega_1+\ldots+y_n\omega_n, \quad dy = dy_1\ldots dy_n,$$

$$G_i(\gamma) = N(\mathfrak{a})^{-1}\sum_{\mu \pmod{\mathfrak{a}}} E(a_i\alpha_i\mu^k\gamma) \quad (1 \leqslant i \leqslant s),$$

$$G(\gamma) = \prod_{i=1}^{s} G_i(\gamma),$$

$$I_i(\zeta, B) = \int_{P(B)} E(a_i\alpha_i\eta^k\zeta)\,dy \quad (1 \leqslant i \leqslant s)$$

and

$$I(\zeta, B) = \prod_{i=1}^{s} I_i(\zeta, B),$$

where a_i $(1 \leqslant i \leqslant s)$ are defined by (22).

LEMMA 3. *Suppose that $\xi \in B_\gamma$. Then*

$$S_i(\xi) = G_i(\gamma)I_i(\zeta, B)+O(a^{2z/n}B^{n-1}).$$

See, e.g., Lemma 12 in Wang Yuan [4].

LEMMA 4.

$$I_i(\zeta, B) \ll \prod_{i=1}^{s} \min(B, a^{-1/k}|\zeta^{(i)}|^{-1/k}), \quad 1 \leqslant i \leqslant s.$$

See Siegel [3], p. 335.

LEMMA 5.

$$\int_{B_\gamma} S(\xi)E(-\chi\xi)\,dx = G(\gamma)E(-\chi\gamma)\int_{E_n} I(\zeta, B)\,dx+O(B^{(s-k)n}a^{-n-7z}).$$

Proof. Let

$$\zeta^{(p)} = u_p \quad (1 \leqslant p \leqslant r_1), \qquad \zeta^{(q)} = u_q e^{i\varphi_q} \quad (r_1+1 \leqslant q \leqslant r_1+r_2).$$

The Jacobian of $x_1, \ldots, x_n$ with respect to u_p, u_q, φ_q is

$$D^{1/2}2^{r_2}\prod_q u_q.$$

Suppose $\xi \in B_\gamma$. Then by Lemmas 3 and 4, we have

$$S(\xi) = G(\gamma)I(\zeta, B)+O(B^{ns-1}a^{2z/n}).$$

Since by (19),

$$\int_{B_\gamma} dx \ll h^{-n} = a^{-n-kny+z},$$

and by (6),

$$(1+2/n)z-y < -7z,$$

we have

(36) $$\int_{B_\gamma} S(\xi)E(-\chi\xi)\,dx$$

$$= G(\gamma)E(-\chi\gamma)\int_{B_\gamma} I(\zeta, B)E(-\chi\zeta)\,dx+O(B^{n(s-k)}a^{-n-7z}).$$

In the integral on the right-hand side of (36) we replace $E(-\chi\zeta)$ by 1. The error is

$$B^{ns}\int_{B_\gamma} \|\chi\zeta\|\,dx \ll B^{ns}Hh^{-n-1} \ll B^{n(s-k)}a^{-n-7z}$$

by (19) and (21). Hence

(37) $$\int_{B_\gamma} S(\xi)E(-\chi\xi)\,dx = G(\gamma)E(-\chi\gamma)\int_{B_\gamma} I(\zeta, B)\,dx+O(B^{n(s-k)}a^{-n-7z}).$$

If $(x_1, \ldots, x_n)$ is a point of E_n-B_γ, then the inequality $h|\zeta^{(i)}| \geqslant 1$ is true for at least one index i. Since $s \geqslant c_{13}$, it follows by Lemma 4 that

$$\int_{E_n-B_\gamma} I(\zeta, B)\,dx \ll \int_{E_n-B_\gamma} \Big(\prod_{i=1}^{n} \min(B, a^{-1/k}|\zeta^{(i)}|^{-1/k})\Big)^s dx$$

$$\ll \Big(\int_{h^{-1}}^{\infty} a^{-s/k}u^{-s/k}\,du\Big)\Big(\int_0^{\infty} \min(B^s, a^{-s/k}v^{-s/k})\,dv\Big)^{r_1-1}$$

$$\times\Big(\int_{-\pi}^{\pi}\int_0^{\infty} \min(B^{2s}, a^{-2s/k}w^{-2s/k})\,w\,dw\,d\varphi\Big)^{r_2}$$

$$+\Big(\int_0^{\infty} \min(B^s, a^{-s/k}u^{-s/k})\,du\Big)^{r_1}\Big(\int_{-\pi}^{\pi}\int_{h^{-1}}^{\infty} a^{-2s/k}v^{-2s/k+1}\,dv\,d\varphi\Big)$$

$$\times\Big(\int_{-\pi}^{\pi}\int_{0}^{\infty}\min(B^{2s}, a^{-2s/k}w^{-2s/k})\,w\,dw\,d\varphi\Big)^{r_2-1}$$

$$\ll a^{-s/k}h^{s/k-1}a^{-(r_1-1)}B^{(s-k)(r_1-1)}a^{-2r_2}B^{2(s-k)r_2}$$

$$+a^{-r_1}B^{(s-k)r_1}a^{-2s/k}h^{2s/k-2}a^{-2(r_2-1)}B^{2(s-k)(r_2-1)}$$

$$\ll B^{(s-k)(n-1)}a^{-s/k-n+1+(1+ky-z/n)(\frac{s}{k}-1)}$$

$$+B^{(s-k)(n-2)}a^{-2s/k-n+2+(1+ky-z/n)(\frac{2s}{k}-2)}$$

$$\ll B^{n(s-k)}a^{-n}(a^{-\frac{zs}{kn}+\frac{z}{n}}+a^{-\frac{2zs}{kn}+\frac{2z}{n}})$$

$$\ll B^{n(s-k)}a^{-n-7z}.$$

The lemma follows by substitution into (37).

6. Singular integral. Let

$$\eta' = y_1'\omega_1+\ldots+y_n'\omega_n, \qquad \zeta' = x_1'\varrho_1+\ldots+x_n'\varrho_n,$$
$$dy' = dy_1'\ldots dy_n', \qquad dx' = dx_1'\ldots dx_n',$$
$$\eta = B\eta' \quad \text{and} \quad \zeta = a^{-1-ky}\zeta'.$$

The Jacobians of $y_1, \ldots, y_n$ and $x_1, \ldots, x_n$ with respect to $y_1', \ldots, y_n'$ and $x_1', \ldots, x_n'$ are B^n and $(a^{-1-ky})^n$ respectively. Define $\gamma_i = \alpha_i/a$ $(1 \leqslant i \leqslant s)$. Then

$$\alpha_i\eta^k\zeta = \gamma_i\eta'^k\zeta', \qquad 1 \leqslant i \leqslant s,$$

and by (16), we have

(38) $$c_9 < |\gamma_j^{(i)}| < c_{10}, \qquad 1 \leqslant j \leqslant s,\ 1 \leqslant i \leqslant n.$$

Let us write η' and ζ' as η and ζ again and let

$$I_i(\zeta) = \int_P E(a_i\gamma_i\eta^k\zeta)\,dy \quad (1 \leqslant i \leqslant s) \quad \text{and} \quad I(\zeta) = \prod_{i=1}^{s} I_i(\zeta),$$

where $P = P(1)$. Then

$$I_i(\zeta, B) = B^n I_i(\zeta), \qquad 1 \leqslant i \leqslant s,$$

and thus

(39) $$\int_{E_n} I(\zeta, B)\,dx = B^{n(s-k)}a^{-n}\int_{E_n} I(\zeta)\,dx.$$

LEMMA 6.

$$\int_{E_n} I(\zeta)\,dx = D^{(1-s)/2}k^{-ns}|N(\gamma_1\ldots\gamma_s)|^{-1/k}\prod_p F_p\prod_q H_q,$$

where

$$F_p = \int_{U_p}\prod_{i=1}^{s} w_i^{1/k-1}\,dw_1\ldots dw_{s-1}$$

in which U_p denotes the domain

$$0 \leqslant w_i \leqslant |\gamma_i^{(p)}|, \quad 1 \leqslant i \leqslant s, \qquad w_{2p} = w_{2p-1} \pm w_1 \pm \ldots \pm w_s,$$

here the sign before w_i is the sign of $a_i\gamma_i^{(p)}$ (cf. (22)), and where

$$H_q = \int_{V_q}\prod_{i=1}^{s} w_i^{1/k-1}\,dw_1\ldots dw_{s-1}\,d\varphi_1\ldots d\varphi_{s-1}$$

in which V_q denotes the domain

$$0 \leqslant w_i \leqslant |\gamma_i^{(q)}|^2 \quad (1 \leqslant i \leqslant s), \qquad -\pi \leqslant \varphi_j \leqslant \pi \quad (1 \leqslant j \leqslant s-1),$$
$$w_s = |w_1^{1/2}e^{i\varphi_1}+\ldots+w_{s-1}^{1/2}e^{i\varphi_{s-1}}|^2.$$

The proof is similar to that of Lemma 16 in Wang Yuan [4].

7. The proof of the theorem. We have

$$\sum_{\gamma\in\Gamma(t)} 1 \ll \sum_{N(\mathfrak{a})\leqslant t^n} N(\mathfrak{a}) \ll \sum_{d\leqslant t^n} d^2 \ll t^{3n} = a^{3z}.$$

If (24) holds, then by (23), (39) and Lemmas 2, 5 and 6, we have

$$Z = \sum_{\gamma\in\Gamma(t)}\int_{B_\gamma} F(\xi)\,dx + O(H^n B^{n(s-k)}a^{-n-4z})$$
$$= J_0\,\mathfrak{S}(t, H)B^{n(s-k)}a^{-n}+O(H^n B^{n(s-k)}a^{-n-4z}),$$

where

$$J_0 = D^{(1-s)/2}k^{-ns}|N(\gamma_1\ldots\gamma_s)|^{-1/k}\prod_p F_p\prod_q H_q$$

and

$$\mathfrak{S} = \mathfrak{S}(t, H) = \sum_{\chi\in P(H)}\sum_{\gamma\in\Gamma(t)} G(\gamma)E(-\chi\gamma).$$

Let $\sum^*$ denote a sum, where γ runs over a reduced residue system of $(\mathfrak{a}\delta)^{-1}$, mod δ^{-1}. Then

$$\mathfrak{S} = \sum_{N(\mathfrak{a})=1}\sum_{\gamma}{}^{*} G(\gamma)\sum_{\chi\in P(H)} E(-\chi\gamma)+\sum_{1<N(\mathfrak{a})\leqslant t^n}\sum_{\gamma}{}^{*} G(\gamma)\sum_{\chi\in P(H)} E(-\chi\gamma)$$
$$= \mathfrak{S}_1+\mathfrak{S}_2, \qquad \text{say}.$$

We have

$$\mathfrak{S}_1 = \sum_{\chi\in P(H)} 1 \gg H^n.$$

(See, e.g., [4].) If $N(\mathfrak{a}) > 1$, then

$$\sum_{\chi(\mathrm{mod}\,\mathfrak{a})} E(-\chi\gamma) = 0.$$

Therefore if the domain $\chi \in P(H)$ has to be split up into a union of complete residue set (mod $\mathfrak{a}$), plus a few other, remaining elements, say R elements, then

$$R \ll H^{n-1} N(\mathfrak{a})^{1/n},$$

and thus

$$\mathfrak{S}_2 \ll \sum_{N(\mathfrak{a}) \leqslant t^n} \sum_{\gamma}{}^{*} H^{n-1} N(\mathfrak{a})^{1/n} \ll H^{n-1} \sum_{N(\mathfrak{a}) \leqslant t^n} N(\mathfrak{a})^{1+1/n}$$

$$\ll H^{n-1} \sum_{d \leqslant t^n} d^3 \ll H^{n-1} t^{4n} \ll H^{n-2z}.$$

Hence

$$\mathfrak{S} \gg H^n.$$

It follows by (38) that $J_0 > c_{16}$, and therefore

$$Z > c_{17} H^n B^{n(s-k)} a^{-n} > 1$$

if $a > c_6(k, K, x')$. The theorem is proved.

Remarks. 1. The inequality (3) can be replaced by

$$\max_i |N(\lambda_i)| \ll \max\left(1, |N(\alpha_1)|, \ldots, |N(\alpha_s)|\right)^{\varepsilon}.$$

(See [4].)

2. Consider the equation

$$A(\lambda) = \sum_{i=1}^{s} \alpha_i \lambda_i^k = 0, \tag{40}$$

where $\alpha_1, \ldots, \alpha_s$ are given integers in K. If k is an odd number, then

$$a_i \lambda_i^k = (a_i \lambda_i)^k \quad (1 \leqslant i \leqslant s).$$

If $r_1 = 0$, i.e., K is totally complex, then the singular integral J_0 is always positive. Therefore in these two cases it follows by the theorem that if $s \geqslant c_{18}(k, n, \varepsilon)$, the equation (40) has a solution in integers $\lambda_1, \ldots, \lambda_s$, not all zero, satisfying

$$\|\lambda\| \ll |A|^{\varepsilon}.$$

3. We can further consider the problem of the estimation of bounds for solutions of certain diophantine inequalities in an algebraic number field (cf. [2]).

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