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Corrigendum to the paper "On the arithmetical theory of continued fractions, II" Acta Arith. 7 (1962), pp. 287–298

by

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Miss M. Lozach has pointed out an error in the proof of Theorem 2. Contrary to what is stated in the last three lines on p. 290 the fundamental solution of the polynomial Pell equation $X^2 - f(x)Y^2 = \pm 1$ may be furnished not by the shortest period of the continued fraction expansion of $\sqrt{f(x)}$, but by the shortest pseudoperiod. To be precise, if k is the least non-negative integer such that T_k/U_k is a reduct of the expansion of $\sqrt{f(x)}$ and for some constant c we have $T_k^2 - f(x)U_k^2 = c$ then the fundamental solution of the polynomial Pell equation is given by $T_k/\sqrt{|c|}$, $U_k/\sqrt{|c|}$, even though $k+1$ is not the length K of the shortest period of the said expansion. M. Lozach has proved (see [1]) that in such a case $K = 2(k+1)$. This however has no influence on the validity of Theorem 2. Indeed we have then

$$T_{K-1} + U_{K-1}\sqrt{f(x)} = \left(\frac{T_k + U_k\sqrt{f(x)}}{\sqrt{c}}\right)^2$$

hence

$$2T_{K-1} = \frac{2T_k^2 + 2U_k^2 f(x)}{c} = \frac{4T_K^2 - 2c}{c} = \pm\left(\frac{2T_k}{\sqrt{|c|}}\right)^2 - 2$$

and if $2T_{K-1}(n)$ is not a rational integer, $2T_k/\sqrt{|c|}$ also is not. The argument given in the paper applies.

Reference

[1] M. Lozach, *Appendice à "Équation de Pell et points d'ordre fini" par Y. Hellegouarch et M. Lozach*, Prépublication de l'Université de Caen.