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Giovanni Ricci (1904–1973)

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Giovanni Ricci was born in Florence on August 17, 1904, and there he spent his youth.

He read mathematics in Pisa at the Scuola Normale Superiore, where he graduated on December 15, 1925, on presenting a thesis in differential geometry. His thesis was later published [1]* in the Annals of the School under the title: "Le transformazioni di Christoffel e di Darboux per le superficie rotonde, coniche e cilindriche. Alcune generazioni, per rotolamento del cono e del cilindro di rotazione".

He spent the subsequent two years at the University of Rome, as assistant professor, whence, in 1928, he went back to the Scuola Normale Superiore, where he was professor for about eight years. The time he spent at the Scuola Normale Superiore was very fruitful in scientific research and gave birth to some papers which stand out among his publications.

During those years, his research was directed at investigating various topics in differential geometry, theory of functions, theory of series and number theory. In the last subject, which indeed was his main field of research, the topics he studied most thoroughly were properties of integer valued polynomials, Tauberian theorems, Hilbert's seventh problem and additive arithmetics of prime numbers (focusing attention on Goldbach's conjecture) and of power free numbers.

We cannot give here a complete description of his work; we want to recall at least two highly significant results of his, on Hilbert's seventh problem and on the Goldbach conjecture. As regards the former topic, namely investigation whether, given algebraic numbers α and β, α^β is an algebraic or a transcendental number, let us recall that in 1934 Gelfond proved that α^β is transcendental for every couple α, β ($\alpha \neq 0, 1$; β irrational).

Ricci took up the subject again, trying to prove that α^β is transcendental whenever α and β are Liouville numbers. In fact, he could prove this for particular subclasses of Liouville numbers, speedily approximable by means

* Numbers in brackets refer to the first part of the bibliography. Publications of Giovanni Ricci.

of rational numbers, thus giving a valuable contribution to a field investigated by many important researchers (as e.g., J. F. Koksma, K. Mahler, C. L. Siegel).

In order to sketch the results he obtained, let us recall the following one ([26], [27]):

$\xi^{\alpha\eta}$ is a transcendental number if $\xi \neq 0, 1$ (ξ and η being algebraic numbers) and there exists some unlimited sequence $\{q_r\}$ of integer numbers such that, given proper values p_r (p_r, q_r being integer numbers for every r), we have

$$\left|\frac{p_r}{q_r} - \alpha\right| < \frac{1}{q_r^w}$$

where $w = \log^{r+\varepsilon} q_r$, $\varepsilon > 0$.

As to the Goldbach problem, Ricci's research aimed at reducing Schnirelmann's constant ([29], [31]).

As is well known, Schnirelmann began the solution of Goldbach's problem by showing the existence of a number S such that any sufficiently large positive integer can be obtained as a sum of S prime numbers at most. Schnirelmann's research, however, provided little information on the value of S, giving only an upper bound excessively large and removed very far from the value of S.

Further research somewhat improved the bound on S and in 1936 Ricci noticeably sharpened any bound given before by showing that $S \leqslant 67$. Ricci obtained this result by using the sieve of Viggo Brun and the theory of sequence density; as we known the result was to be substantially improved later but at the time it was greatly appreciated because simultaneously there appeared an analogous result by Heilbronn, Landau and Scherk with the weaker limitation $S \leqslant 71$.

The problem centres at the density of sequences; the aim is to show that for a suitable S the sequence of prime numbers (whose density, as we know, is 0), if added to itself $S-1$ times, generates a sequence of density 1.

In order to prove his result, Ricci mainly used Viggo Brun's method, which relies on combinatorial considerations on the basis of the following idea: if many terms are to be added and every term is corrupted by some indeterminacy, it may be expedient to ignore the terms which are very small or which eliminate each other because of opposite sign, since by suppressing them we also remove their indeterminacy. In spite of the apparent simplicity of this method, by using it patiently and cleverly, Ricci was able to prove the quoted result.

In the meantime his stay in Pisa was terminating and at the end of 1936, Ricci moved to the University of Milano, where he was professor of Mathematical Analysis for over 36 years until he died on September 9, 1973.

In Milano he was much move occupied than in Pisa with teaching and

administrative work: consequently his research progressed at a slower pace. He still worked in number theory, mainly studying the distribution of prime numbers, but he concentrated more on connecting known results and commenting on them than on finding new ones.

In the same period, however, he also studied the theory of analytical functions. We shall at least recall the results he obtained about the position of singularities on the boundary of the convergence circle of some analytical element ([34], [39]); the idea had already been investigated by such authors as Vivanti, Pringsheim, Hadamard and Fabry.

Ricci's scientific activity brought him many academic awards: he was first a corresponding member and later full member of the Istituto Lombardo, the Academy of Science and Humanities, founded in Milano by Napoleon, then a corresponding member of the Accademia di Torino, the foundation of which was connected with Lagrange, and finally of the Accademia Nazionale dei Lincei, the most distinguished Italian accademy, which had included even Galilei among its members.

However, as we had already hinted, during his stay in Milano, which lasted for more than half his life, he was largely committed to teaching and administration.

First of all let us recall his patient work at the library of the Institute of Mathematics of the University of Milano, which he had built from scratch (the University of Milano had only been founded in 1924) giving it a modern and efficient structure.

This achievement was due to his love of books and was accomplished by through his continuous, passionate work for over 30 years and his generous contributions. After his death, to honour his activity, the library was given his name.

His organizing activities included the directorship of the Mathematics Institute from 1959 to 1970 and presidency of the Unione Matematica Italiana from 1964 to 1967.

Since he was a man of strong personality, who left his stamp on many of his disciples, let us complete the story of his life by describing the kind of man he was, in research, in teaching and in everyday life.

Three main features stand out in the memories of the people who lived close to him: his wonderful teaching, his mental balance and his deep aesthetic sense.

He was a born teacher, partly owing to his mastery of the language, since he came from Toscana, the region where the best Italian is spoken; but, most of all, he was an excellent teacher because of his extreme clarity of mind and his profound intellectual honesty, which forced him to present topics in such a way that their logical connections became absolutely obvious; those who listened to him often thought that what he had explained was a chain of trivial statements (as he used to say). Of great help was also his wide mathematical knowledge, deriving both from the school he had followed and from his continuous interest in research.

His level-headedness and moderation always proved very helpful in dealing with colleagues and pupils, as was clearly manifested during the student movements which shook the Italian universities in 1968 and left their trace on many of them, including of course the University of Milano. He showed in full his tolerant and understanding attitude, and his skill in coping with situations very different from those which had been familiar to him in the past, without losing sight of the final aim of serious study.

On several occasions he was indeed able to show his ability to understand other people's point of view and to come to an agreement, as far as the situation permitted, in such a way as to minimize the risk of giving scope to senseless ideas and to exploit those which contained some good.

His profound aesthetic sense was quite an important aspect of his personality, and it seemed to be related to his fondness for library organization, and his love of books, which he always regarded as objects of high aesthetic value.

He said to me once: "Opening a book neatly printed on good paper and properly bound is one of the pleasures allowed to man".

To his everyday work, his aesthetic sense brought clarity and a sober and balanced language, both in research and in teaching. Of course it also had other, more evident, manifestations, for instance his love of music, which he often combined with the pleasures of mathematical studies through a subtle transposition of activities which he felt to be very close to each other.

His colleagues in Milano still remember a passage from one of his lectures, where he compared the brilliant and harmonious statements of classical analysis to the "Largo" in Hendel's Xerses, and the tortuous developments of discrete analysis to Ravel's "Bolero".

Ricci's character reveals itself to us in its complex and rich humanity and his work in education and his inspiring activity in university life appear as important as his scientific contribution.

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Publications of Giovanni Ricci

First part

1. *Le trasformazioni di Christoffel e di Darboux per le superficie rotonde, coniche e cilindriche. Alcune generazioni per rotolamento del cono e del cilindro di rotazione.* (Tesi di laurea, 15 dicembre 1925), Ann. R. Scuola Norm. Sup. Pisa, Cl. Sc. Fis., Mat., 15 (1927), 64 pp.
2. *Determinazione del massimo limite e del minimo limite della somma di funzioni periodiche continue, per la variabile indefinitamente crescente*, Boll. U. M. I. 9 (1930), pp. 211–217.
3. *Sulle funzioni simmetriche di interi costituenti uno o piú sistemi ridotti di resti, secondo un modulo composto.*, Ann. Univ. Toscane, nuova serie, 13 (1930), pp. 177–198.
4. *Sulle somme di potenze simili degl'interi naturali e sui coefficienti del fattoriale*, ibid. pp. 199–216.
5. *Un perfezionamento dei teoremi di Sylvester, N. Nielsen, Saalschutz, Lipschitz sui numeri di Bernoulli*, Giornale di Matem. di Battaglini 69 (1931), pp. 1–4.
6. *Sui coefficienti binomiali e polinomiali. Una dimostrazione del teorema di Staudt–Clausen sui numeri di Bernoulli*, ibid., pp. 9–13.
7. *Sulle funzioni simmetriche delle radici del'unità secondo un modulo composto*, Ann. di Mat. 9 (1930), pp. 181–193.
8. *Due proprietà caratteristiche delle funzioni a rapporto incrementale limitato*, Bool. U. M. I. 10 (1931), pp. 131–134.
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17. *Sulle serie di potenze che rappresentano funzioni razionali a coefficienti razionali e con i poli appartenenti ad una progressione geometrica*, Comment. Math. Helv. 6 (1933–34), pp. 223–234.
18. *Sulla deformazione delle doppie infinità di sfere per flessione della superficie dei centri*, Atti R. Istit. Veneto 93 (1933–1934), parte II, pp. 1535–1556.
19. *Una proprietà caratteristica delle congruenze di sfere di Ribaucor illimitatamente deformabili*, Verhandlungen des Internationaler Mathematiker-Kongress, Zürich 1932, Bd. II, pp. 154–156. Riassunto. Giornale di Matem. di Battaglini 72 (1934), pp. 220–234.