wobei $A > 4$ so groß gewählt sei, daß (15) für $T > 16A$ anwendbar ist. Dann gilt $N^{1/2} \ll qT \ll N$. Für $\frac{1}{4}N < y \leqslant N$ hat man

$$\frac{1}{\varphi(q)}\frac{y}{T} \gg \frac{hq}{\varphi(q)}.$$

Die linke Seite von (15) wird damit

$$\gg N\left(\frac{hq}{\varphi(q)}\right)^2 \varphi(q)\delta(q),$$

und wir erhalten aus (15)

$$h \ll \prod_{p|(q,N)}\left(1-\frac{1}{p}\right)\log^2 N.$$

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The linear sieve, revisited

by

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1. Introduction. Two of the most famous unsolved number theory problems are the twin-prime and Goldbach conjectures. These problems, and others, lend themselves to an application of the linear sieve. Chen's theorem, [3], currently the best result known about the Goldbach conjecture, has as its basis a linear sieve, for example.

The linear sieve functions (defined in Section 3) have been derived in at least two different ways: by means of a complicated combinatorial identity, [9], and by an equally complicated set construction, [7]. It has been said that these functions are the result of iterating the Selberg upper bound function an infinite number of times using the Buchstab identity. However, this iteration process is internal, and not readily apparent, in the articles cited.

The purpose of this article is to show that the linear sieve functions are indeed the results of an external iteration of the Selberg upper bound function.

2. Notation, assumption, and the Selberg sieve. We follow the notation of Halberstam–Richert ([4] and [5]).

Let $\mathfrak{A}$ be a finite sequence of integers, and let $\mathfrak{A}_d$ denote the subsequence of $\mathfrak{A}$ all of whose elements are divisible by d. We use $|\mathfrak{A}|$ and $|\mathfrak{A}_d|$ to denote the number of elements of $\mathfrak{A}$ and $\mathfrak{A}_d$, respectively.

Let $\mathscr{P}$ be a set of primes and define (the empty product being 1)

$$P(z) = \prod_{\substack{p\in\mathscr{P}\\ p<z}} p.$$

Define the sifting function $\mathscr{S}(\mathfrak{A};\mathscr{P},z)$ for any z to be $\mathscr{S}(\mathfrak{A};\mathscr{P},z) = |\{a\in\mathfrak{A}\colon (a, P(z)) = 1\}|$; in other words, $\mathscr{S}(\mathfrak{A};\mathscr{P},z)$ is the number of elements of $\mathfrak{A}$ remaining after we have removed all those with prime factors less than z that belong to $\mathscr{P}$.

We need some additional notation in order to study $\mathscr{S}(\mathfrak{A};\mathscr{P},z)$. Let $\omega(d)$ be a multiplicative function with $\omega(1) = 1$, $\omega(p) = 0$ for $p\notin\mathscr{P}$, and $\omega(p)$

$\geqslant 0$ for $p \in \mathscr{P}$. Thus

$$\omega(d) = \prod_{p|d} \omega(p), \quad \text{for} \quad \mu(d) \neq 0.$$

We let X be a convenient approximation to $|\mathfrak{A}|$, and define, for all d,

$$R_d = |\mathfrak{A}_d| - \frac{\omega(d)}{d} X, \quad \text{for} \quad \mu(d) \neq 0.$$

In practice $\omega(d)$ is chosen to make R_d small on average.

In this paper we are studying linear sieves, those in which $\omega(p)$ is equal to 1, on average, in the sense that

$$(\Omega_2) \qquad \sum_{w \leqslant p < y} \frac{\omega(p) \log p}{p} - \log \frac{y}{w} \leqslant A_2$$

holds, if $2 \leqslant w < y$, for some constant A_2. We further assume

$$(\Omega_1) \qquad 0 \leqslant \frac{\omega(p)}{p} \leqslant 1 - \frac{1}{A_1},$$

for some suitable constant $A_1 > 1$.

In order to state our sieve results we need one more important function, namely

$$W(z) = \prod_{p<z} \left(1 - \frac{\omega(p)}{p}\right) = \sum_{d|P(z)} \frac{\mu(d)\,\omega(d)}{d}.$$

With these notations and assumptions, we are able to state the *Selberg sieve estimate*, namely

$$(1) \qquad \mathscr{S}(\mathfrak{A}; \mathscr{P}, z) \leqslant XW(z) \cdot \frac{1}{\sigma(\tau)} \left\{1 + O\left(\frac{(\log\log \xi)^3}{\log \xi}\right)\right\} + \sum_{\substack{d < \xi^2 \\ d|P(z)}} 3^{\nu(d)} |R_d|,$$

where $\tau = \log \xi^2/\log z$, $\nu(d)$ is the number of distinct prime divisors of d, and $\sigma(u)$ is the continuous solution of the system

$$(2) \qquad \sigma(u) = u/2e^{\gamma}, \quad \text{for} \quad 0 \leqslant u \leqslant 2,$$

$$(3) \qquad \left(\frac{\sigma(u)}{u}\right)' = -\frac{1}{u^2}\,\sigma(u-2), \quad \text{for} \quad u > 2.$$

The function $1/\sigma(u)$ is referred to as the *Selberg upper bound function.*

3. The linear sieve and statement of results. The Selberg sieve estimate is an upper bound on $\mathscr{S}(\mathfrak{A}; \mathscr{P}, z)$. By means of the Buchstab identity,

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) = \mathscr{S}(\mathfrak{A}; \mathscr{P}, w) - \sum_{\substack{w \leqslant p < z \\ p \in \mathscr{P}}} \mathscr{S}(\mathfrak{A}_p; \mathscr{P}, p),$$

that holds for all $w \leqslant z$, and the fundamental lemma

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) = XW(z)\left\{1 + O\left(\frac{1}{\log \xi}\right)\right\} + \Theta \sum_{\substack{d < \xi^2 \\ d|P(z)}} 3^{\nu(d)} |R_d|,$$

that holds for $z \leqslant z_1 = \exp\{\log \xi/\log\log \xi\}$ and some $|\Theta| \leqslant 1$, we obtain lower bound sieve estimates from upper, and vice versa. The first few iterations of this process, starting with the Selberg upper bound, were investigated by Porter, [10].

In this paper we show that repeated iteration in the manner of Porter arrives at the following sieve inequalities, where $F(u)$ and $f(u)$ are the so-called *linear sieve functions.*

THEOREM 1. *Let (Ω_1) and (Ω_2) hold. For $\tau = \log \xi^2/\log z > 0$, and any $\varepsilon > 0$,*

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) \leqslant XW(z) F(\tau) \{1 + \varepsilon + o_{\varepsilon}(1)\} + \sum_{\substack{d < \xi^2 \\ d|P(z)}} 3^{\nu(d)} |R_d|,$$

and

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) \geqslant XW(z) \{f(\tau) - \varepsilon + o_{\varepsilon}(1)\} - \sum_{\substack{d < \xi^2 \\ d|P(z)}} 3^{\nu(d)} |R_d|,$$

where the little-o terms tend to zero as ξ (or z) goes to ∞, and the relationship between $F(u)$ and $f(u)$ is

$$F(u) = 2e^{\gamma}/u, \quad \text{for} \quad u \leqslant 2,$$

$$f(u) = 0, \quad \text{for} \quad u \leqslant 2,$$

$$(uF(u))' = f(u-1), \quad \text{for} \quad u > 2,$$

and

$$(uf(u))' = F(u-1), \quad \text{for} \quad u > 2,$$

where both $F(u)$ and $f(u)$ are continuous.

4. The iteration. Let $F_0(u)$ denote $1/\sigma(u)$, and define a series of upper and lower bound functions (indicated by F_r and f_r, respectively) in the following way,

$$(4) \qquad f_0(u) = \begin{cases} 0, & \text{for} \quad u \leqslant 1, \\ \max\{0,\ 1 - (1/u) \int_u^{\infty} (F_0(t-1) - 1)\,dt\}, & \text{for} \quad u > 1, \end{cases}$$

and for $r \geqslant 1$ let

$$(5) \qquad F_r^*(u) = 1 + (1/u) \int_u^{\infty} (1 - f_{r-1}(t-1))\,dt,$$

(6) $$f_r^*(u) = 1-(1/u)\int_u^\infty (F_r(t-1)-1)\,dt,$$

(7) $$f_r(u) = \max\{0, f_r^*(u)\} = \begin{cases} 0, & \text{for } u \leqslant \beta_r, \\ f_r^*(u), & \text{for } u > \beta_r, \end{cases}$$

and

(8) $$F_r(u) = \begin{cases} 1/\sigma(u), & \text{for } u \leqslant \alpha_r, \\ F_r^*(u), & \text{for } u > \alpha_r, \end{cases}$$

where α_r is the largest point of intersection of $1/\sigma(u)$ and $F_r^*(u)$.

The sequences $\{f_r(u)\}$, $\{F_r(u)\}$, $\{\alpha_r\}$, and $\{\beta_r\}$ all decrease, and converge, to $f(u)$, $F(u)$, α, and β, respectively. The relationships in the limit are

$$F(u) = \begin{cases} 1/\sigma(u), & \text{for } u \leqslant \alpha, \\ 1+(1/u)\int_u^\infty (1-f(t-1))\,dt, & \text{for } u > \alpha, \end{cases}$$

and

$$f(u) = \begin{cases} 0, & \text{for } u \leqslant \beta, \\ 1-(1/u)\int_u^\infty (F(t-1)-1)\,dt, & \text{for } u > \beta, \end{cases}$$

or, in differential form

(9) $$F(u) = 1/\sigma(u), \quad \text{for } u \leqslant \alpha,$$

(10) $$f(u) = 0, \quad \text{for } u \leqslant \beta,$$

(11) $$(uF(u))' = f(u-1), \quad \text{for } u > \alpha,$$

and

(12) $$(uf(u))' = F(u-1), \quad \text{for } u > \beta,$$

where $F(u)$, $f(u)$ are both continuous. Furthermore, by [1] and the iteration process, $F(u)$ and $f(u)$ have the following properties:

$$F(u) = 1+O(e^{-u/2}) \quad \text{as } u \to \infty,$$

$$f(u) = 1+O(e^{-u/2}) \quad \text{as } u \to \infty,$$

and

$$0 \leqslant f(u) < 1 < F(u) \quad \text{for all } u > 0.$$

By using methods similar to Porter [10] or Halberstam–Richert [5], and keeping track of the big-O terms, we obtain the following sieve inequalities.

THEOREM 2. *Let* (Ω_1), (Ω_2) *hold. For* $\tau = \log\xi^2/\log z > 0$, *there are constants* A *and* B, *dependent on* A_1, A_2, *and* $1/\sigma(u)$, *such that*

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) \leqslant XW(z)F_r(\tau)\left\{1+B\left(A^{2r}\frac{(\log\log\xi)^{3+2r}}{\log\xi}\right)\right\} + \sum_{\substack{d<\xi^2 \\ d|P(z)}} 3^{\nu(d)}|R_d|,$$

and

$$\mathscr{S}(\mathfrak{A}; \mathscr{P}, z) \geqslant XW(z)\left\{f_r(\tau)-B\left(A^{2r+1}\frac{(\log\log\xi)^{4+2r}}{\log\xi}\right)\right\} - \sum_{\substack{d<\xi^2 \\ d|P(z)}} 3^{\nu(d)}|R_d|.$$

Clearly, Theorem 2 is a more precise theorem than Theorem 1. The remainder of this paper will be devoted to proving that the iteration process forces $\alpha = \beta = 2$, so Theorem 1 will follow as a corollary of Theorem 2.

5. Analysis of the sum and difference functions. Given the limit functions of the iteration, define the sum and difference functions as follows:

$$\Phi(u) = F(u)+f(u),$$

and

$$\Psi(u) = F(u)-f(u).$$

By using Laplace transforms, or adjoint operators like Iwaniec [7], one derives the equations

(13) $$2 = \int_\delta^{\delta+1} \Phi(u-1)h(u)\,du + \delta\Phi(\delta)h(\delta),$$

and

(14) $$\delta(\delta-1)\Psi(\delta) = \int_{\delta-1}^{\delta} u\Psi(u)\,du,$$

valid for any $\delta \geqslant \max\{\alpha, \beta\}$, where

$$h(u) = \int_0^\infty \exp\left\{-\int_0^x \frac{1-e^{-t}}{t}\,dt - xu\right\}dx.$$

We analyze these equations to find functional relationships between α and β.

In this analysis we make the assumption that $\alpha > 0$, since $\alpha = 0$ implies that $F(u)$ and $f(u)$ are the linear sieve functions, as is shown by Iwaniec, [7]. Furthermore, since both $1/\sigma(u)$ and $F(u)$ have the form c/u for $u \leqslant 2$, we further assume that $\alpha \geqslant 2$.

We analyze equations (13) and (14) in each of the four possible configurations for α and β: $\alpha < \beta-1$, $\beta-1 \leqslant \alpha < \beta$, $\beta \leqslant \alpha < \beta+1$, and $\beta+1 \leqslant \alpha$. We do this by making suitable choices for δ in each of the four cases, and using the appropriate formulas for $F(u)$ and $f(u)$. These formulas are: for $\alpha \leqslant \beta$,

(15) $$f(u) = 0, \quad \text{for} \quad u \leqslant \beta,$$

(16) $$F(u) = \begin{cases} 1/\sigma(u), & \text{for} \quad u \leqslant \alpha, \\ \alpha/u\sigma(\alpha), & \text{for} \quad \alpha \leqslant u \leqslant \beta+1, \end{cases}$$

and for $\beta \leqslant \alpha$,

(17) $$f(u) = \begin{cases} 0, & \text{for} \quad u \leqslant \alpha, \\ (1/u)\int\limits_{\beta}^{u}\big(1/\sigma(t-1)\big)\,dt, & \text{for} \quad \beta \leqslant u \leqslant \alpha+1, \end{cases}$$

(18) $$F(u) = 1/\sigma(u), \quad \text{for } u \leqslant \alpha.$$

By analyzing equation (14) we arrive at the relationship $\beta = \beta_1(\alpha)$, where $\beta_1(x)$ is defined as follows:

(19) $$\beta_1(x) = \begin{cases} 2, \quad \text{for} \quad x \leqslant 2, \\ \text{the } y \text{ that satisfies} \\ x(x-1)/\sigma(x) = \int\limits_{y}^{x+1} \big(l(t)/\sigma(t-1)\big)\,dt, \quad \text{for} \quad 2 < x, \end{cases}$$

where

$$l(t) = \begin{cases} t-1, & \text{for} \quad x-1 \leqslant t, \\ x-2, & \text{for} \quad t \leqslant x-1, \end{cases}$$

and by analyzing equation (13) we get $\beta = \beta_2(\alpha)$, where $\beta_2(x)$ is defined as the y that satisfies

(20) $$2 - xh(x)/\sigma(x) = \int\limits_{y}^{x+1} k(t-1)\,g(t)\,dt,$$

where

$$k(t-1) = \begin{cases} 1/\sigma(t-1), & \text{for} \quad t \leqslant x+1, \\ x/(t-1)\sigma(x), & \text{for} \quad t \geqslant x+1, \end{cases}$$

and

$$g(t) = \begin{cases} h(t), & \text{for} \quad x-1 \leqslant t, \\ h(x-1), & \text{for} \quad t \leqslant x+1. \end{cases}$$

We will not carry out either of these derivations, as they are technical and not illuminating. In each of the eight cases, the derivation consists primarily of integration by parts, interchanging orders of integration, rearranging, and using easily derived properties of $h(u)$, where applicable.

We now have two curves, $y = \beta_i(x)$, $i = 1, 2$, that intersect at the point (α, β). It is easy to show that $2 = \beta_i(2)$, $i = 1, 2$, so what we need to show is that no other intersection can arise through the iteration process.

6. To show $\alpha < 8$. In this section we show that α_1 (and hence α) is less than 8. This will restrict where we look for applicable intersections of $\beta_i(x)$, $i = 1, 2$.

We let $m(u) = 1 - \sigma(u)$, and note that $m''(u) \geqslant 0$, $m'''(u) \leqslant 0$, for all $u > 0$, by (2) and (3). We also obtain the equation

(21) $$\int\limits_{u}^{\infty} m(t)\,dt = -um(u) + \int\limits_{u-2}^{u} m(t)\,dt,$$

and these imply

(22) $$m(t) \leqslant m(u)\,e^{-(1-2/u)(t-u)},$$

for all $t \geqslant u \geqslant 2$, which we use in the form

(23) $$\left(\frac{1}{\sigma(t)} - 1\right) \leqslant \left(\frac{1}{\sigma(u)} - 1\right) e^{-2/3(t-2)},$$

for $t \geqslant u \geqslant 6$. By using this inequality as we iterate from $1/\sigma(u)$ to $F_1(u)$, we obtain

(24) $$F_1^*(u) - 1 \leqslant \frac{9}{4u(u-1)}\left(\frac{1}{\sigma(u-2)} - 1\right),$$

for $u \geqslant 8$.

In order to show that $\alpha_1 \leqslant 8$ we need to show $F_1^*(u) < 1/\sigma(u)$, for $u \geqslant 8$. By (24) it suffices to show that

(25) $$\frac{9}{4u(u-1)} \cdot \frac{1}{\sigma(6)}\, m(u-2) < m(u)$$

holds, for $u \geqslant 8$, since $\sigma(u)$ is an increasing function.

We first find an upper bound on $1/\sigma(6)$. By the concavity of $m(u)$ we have

(26) $$\int\limits_{u-2}^{u} m(t)\,dt \leqslant m(u-2) + m(u),$$

and this, coupled with (21), yields

(27) $$m(u) \leqslant \frac{1}{u-1}\, m(u-2),$$

for all $u > 2$. Therefore, $m(6) \leqslant m(4)/5 \leqslant m(2)/15 \leqslant 1/15$, so

(28) $$1/\sigma(6) \leqslant 15/14.$$

To get an upper bound on $m(u-2)$ we use (21) and (22) to obtain

(29) $$\int_{u-2}^{u} m(t)\,dt \leqslant (u+2)\,m(u).$$

Since $m'''(t) \leqslant 0$ for all $t > 0$, we see that

$$\int_{u-2}^{u} m(t)\,dt \geqslant \int_{-2}^{0} \left(m(u)+m'(u)\,x+\frac{m''(u)}{2}\,x^2\right)dx,$$

for all $u > 2$, by looking at the parabola that fits under $m(t)$ and agrees at u. This, along with (3) and (21), implies

(30) $$(u+2)\,m(u) \geqslant m(u)\,[2-2/u]+m(u-2)\left[\frac{2}{u}-\frac{8}{6u(u-2)}\right]+m(u-4)\,\frac{8}{6u(u-2)},$$

for $u > 4$. By (27) with u replaced by $u-2$ we obtain

(31) $$m(u) \geqslant m(u-2)\left[\frac{1}{u(u+2/u)}\right]\left[2+\frac{8(u-4)}{6(u-2)}\right],$$

for $u > 4$.

By using (25), (28), and (31), we see that $F_1^*(u) < 1/\sigma(u)$ for $u \geqslant 8$ if

$$\frac{9\cdot 15}{4\cdot 14(u-1)} < \frac{26}{9}\,\frac{1}{u+1/4}$$

holds for $u \geqslant 8$. Simple algebra shows this to hold, so we now know $\alpha < 8$.

7. The third function. In the last section we showed that the α generated by the iteration process is less than 8. We also know that $\alpha = 2$, $\beta = 2$ satisfy $\beta_i(x)$, $i = 1, 2$. In this section we derive a third function, $\beta_3(x)$, that shows that the intersections of the curves $y = \beta_i(x)$, $i = 1, 2$, are separated, thus enabling us to trust computer calculations.

We use equations (19) and (20) to calculate $\beta_i'(x)$ for $i = 1, 2$. Straightforward, though tedious, calculations show

(32) $$U_i(x, y)\,\beta_i'(x) = \begin{cases} \dfrac{\sigma(x-2)}{\sigma^2(x)}, & \text{for } \beta_i(x) \geqslant x-1,\\[2ex] \dfrac{\sigma(x-2)}{\sigma^2(x)}-\dfrac{1}{x-1}\displaystyle\int_{y}^{x-1}\dfrac{1}{\sigma(t-1)}\,dt, & \text{for } \beta_i(x) < x-1,\end{cases}$$

where

$$U_1(x, y) = -\frac{l(y)}{\sigma(y-1)\,j(x)},$$

and

$$U_2(x, y) = \begin{cases} \dfrac{(y-1)\,h(y)}{xh(y-1)\,\sigma(x)}, & \text{for } y > x+1,\\[2ex] \dfrac{g(y)}{\sigma(y-1)\,h(x)}, & \text{for } x+1 \geqslant y.\end{cases}$$

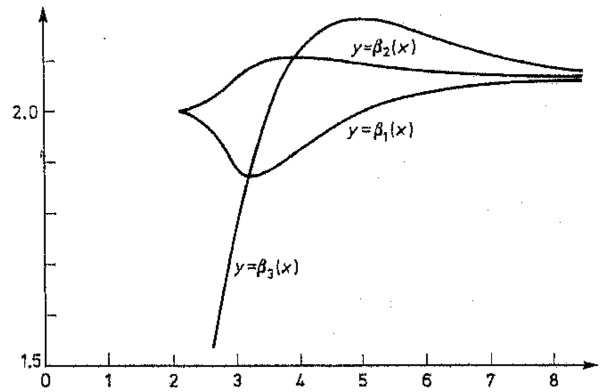
We note that $U_1(x, y) < 0$ and $U_2(x, y) > 0$ for all $x, y > 0$.

We now define the function $\beta_3(x)$ for $x > 2$ to be the y that makes the integral equation

(33) $$\frac{\sigma(x-2)}{\sigma^2(x)} = \frac{1}{x-1}\int_{y}^{x-1}\frac{1}{\sigma(t-1)}\,dt$$

hold. The importance of the curve $y = \beta_3(x)$ is that the derivatives of $\beta_1(x)$ and $\beta_2(x)$ are of opposite signs when both are either above or below $\beta_3(x)$. In other words, if $\beta_3(x)$ is at all well-behaved, the intersections of $\beta_1(x)$ and $\beta_2(x)$ will be separated.

8. Conclusion. We now calculate $y = \beta_i(x)$, $i = 1, 2, 3$, for $2 \leqslant x < 8$ (see figure below). From these graphs and the discussion at the end of the last section, it is clear that the point (2, 2) is the only intersection of $\beta_1(x)$ and $\beta_2(x)$ in this region.



Therefore, we have proved what we set out to; namely, that the external iteration method described here does indeed produce the linear sieve functions previously derived via the combinatorial internal iteration methods of [7] and [9].

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