Soit η₀ une unité fondamentale positive de \( K \).

\[
a_i^{(k)} = \eta_0^{a_i} \frac{\zeta_i}{\zeta_j}, \quad a_j^{(k)} = \eta_0^{a_j} \frac{\zeta_h}{\zeta_j}
\]

où \( i, j, h \) sont des fonctions de \( k \) dans \([1, N]\) et \( u_1, u_2 \) des entiers dépendants de \( k \).

Les relations (IV.11) permettent de borner les entiers \( u_2 \) et \( u_1 \). Alors \( a_i^{(k)} \) et \( a_j^{(k)} \) ne prennent qu'un nombre fini de valeurs.

Références


Reçu le 19.1.1979 et dans la forme modifiée le 7.6.1979 (1130)
The integers in set (ii) above are of the form \(a\mathcal{P}\). We will show that for each \(b_i\) that is a member of the sequence, there corresponds a unique \(a\mathcal{P}\) that cannot be a member. It will follow that there are at most \(n-1\) numbers \(a_i\) in the sequence, which is a contradiction.

Assume that \(b_i\) is a member of the sequence. Let \(m\) be the smallest integer such that

\[
p(p^m b_i - 1) \geq n.
\]

Then

\[
\frac{p(p^m b_i - 1)}{b_i, p(p^m b_i - 1)} = p(p^m b_i - 1) \geq n.
\]

Thus \(p(p^m b_i - 1)\) cannot be a member of the sequence. Furthermore, no two \(a\mathcal{P}\)'s of this form can be equal, for if \(p(p^m b_i - 1) = p(p^m b_j - 1)\), \(b_i \neq b_j\).

(7)

\[
p^{m}b_i = p^{m}b_j.
\]

So \(p|b_i\) or \(p|b_j\), which contradicts the definition of the \(b_i\).

We next show that for all but at most one \(b_i\), \(p^{m}b_i - 1\) is less than \(n\) and thus an element of \(C\).

If \(p^{m}b_i - 1 \geq n\) and \(p(p^{m-1}b_i - 1) < n\) then

(8)

\[
n < p^{m}b_i < n + p
\]
or

(9)

\[
\frac{n}{p} < p^{m-1}b_i < \frac{n + 1}{p}.
\]

There can be only one integer that satisfies this inequality. By the reasoning used after equation (7) it can be satisfied by only one \(b_i\).

We have now paired all but at most one of the elements of \(B\) with elements of \(C\). If an element of \(C\) is still unaccounted for it must equal \(n-1\). For if \(n-1\) had already been excluded we would have

\[
(n-1)p = p(b_i p^m - 1) \quad \text{for some } i \text{ and } m.
\]

So \(n = b_i p^m\), and \(p\) divides \(n\), but if this is so there can be no \(b_i\) satisfying (9).

We finally show that if \(b_n\) is the single remaining element of \(B\), then

\[
\frac{(n-1)p}{b_n(n-1)p} \geq n.
\]

Let \((b_n, n-1) = q\), \(b_n = b_n q\), and \(n-1 = n_0 q\). Putting these values in (8) we get,

\[
n_0 q + 1 < p^n b_0 q \leq n_0 q + p, \quad n_0 + 1/q < p^n b_0 \leq n_0 + p/q.
\]

But \(p^n b_0\) is an integer, so \(p/q \geq 1\). Equality is impossible since this would imply \(p/b_n\). Therefore

\[
\frac{(n-1)p}{b_n(n-1)p} = \frac{(n-1)p}{q} > n-1.
\]

Thus \(b_n\) and \(p(n-1)\) cannot both be members of the sequence.

For every \(b_i\) that is a member of the sequence there corresponds a number \(a\mathcal{P}\) that is not a member. Thus the members of the sequence must come from the \(k\) multiples of \(p\) less than \(n\), and a single member of each of the \(n-1-k\) pairs of incompatible elements of \(B\) and \(C\). This allows at most \(n-1\) integers in the sequence, which is a contradiction. This completes the proof.

References


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