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On sets characterizing additive arithmetical functions, II

by

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To the memory of Professor Paul Turán

As in [1], \( f \) denotes an additive arithmetical function, \( A \) and \( B \) are subsequences of the natural numbers, consisting of the elements \( a_1 < a_2 < a_3 < \ldots \) and \( b_1 < b_2 < b_3 < \ldots \), respectively. \( A \) is called a \( U \)-set, if

\[
\left( (a_k) = 0, \ h = 1, 2, \ldots, \right) \ \text{imply} \ f = 0.
\]

In [1] we proved the following assertions:

I. Let \( A \) be a \( U \)-set. Then

\[
\liminf \frac{a_{k+1}}{a_k} < 1,
\]

moreover, if we put \( \frac{a_{k+1}}{a_k} = e_k \), then

(1) \[
\liminf (e_1 \cdots e_k) = 0 \quad (\text{Theorem 2/I}).
\]

In fact, if \( A \) does not satisfy (1), then we can construct an additive \( f \), which is "arbitrarily strongly" unbounded, though \( f(a_k) = 0 \) for all \( k \) (Theorem 4).

II. Let \( a_k \) be an arbitrary sequence of positive numbers satisfying

\[
\liminf (a_1 \cdots a_k) = 0 \quad \text{and} \quad a_k \geq 2^{-k}.
\]

Then there exists an \( A \), for which

\[
\frac{a_{k+1}}{a_k} > a_k
\]

holds, and \( A \) is a \( U \)-set, moreover, if

(2) \[
\sum_{k=1}^{\infty} f(a_k) \text{ is convergent},
\]

then \( f = 0 \) (Theorem 2/II).