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A generalization of an Euler formula in partition theory

by

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In this note we generalize the well-known Euler's formula

$$np(n) = \sum_{r=1}^{n} p(n-r)\sigma(r),$$

where $p(n)$ is the number of unrestricted partitions of n and $\sigma(n)$ is the sum of all the divisors of n.

Let $S_k(n)$ denote the sum of the kth powers of all the summands in all the partitions of n. Let $\sigma_k(n) = \sum_{d|n} d^k$, where k is any integer.

THEOREM 1. *We have*

$$S_k(n) = \sum_{r=1}^{n} p(n-r)\sigma_k(r).$$

Proof. If we add up the kth powers m^k, of all the summands m, in the $p(n)$ partitions of n we get

$$S_k(n) = \sum_{m=1}^{n} \sum_{\varrho=1}^{[\frac{n}{m}]} m^k p(n-\varrho m).$$

Let $\varrho m = r$, then

$$S_k(n) = \sum_{r=1}^{n} p(n-r) \sum_{m|r} m^k = \sum_{r=1}^{n} p(n-r)\sigma_k(r).$$

Similarly

$$S_{-k}(n) = \sum_{r=1}^{n} p(n-r)\sigma_{-k}(r) = \sum_{r=1}^{n} \frac{p(n-r)\sigma_k(r)}{r^k}.$$

Putting $k = 1, 0, -1$ in the expression for $S_k(n)$ above, we have the following particular cases:

$$S_1(n) = np(n) = \sum_{r=1}^{n} p(n-r)\sigma(r),$$

$$S_0(n) = \sum_{r=1}^{n} p(n-r)d(r),$$

$$S_{-1}(n) = \sum_{r=1}^{n} \frac{p(n-r)\sigma(r)}{r}.$$

The formula for $S_0(n)$ has appeared on page 218 in [1].

Now let $S_k^*(n)$ denote the sum of all kth powers of all the summands in all the partitions of n into primes. Let $\sigma_k^*(n) = \sum_{p_i|n} p_i^k$, where the p_i are primes and k is any integer.

THEOREM 2. $S_k^*(n) = \sum_{r=1}^{n} q(n-r)\sigma_k^*(r)$, *where* $q(n)$ *is the number of partitions of* n *into primes.*

The proof is similar to that of Theorem 1. Putting $k = 1$, we have

$$S_1^*(n) = nq(n) = \sum_{r=1}^{n} q(n-r)\sigma_1^*(r),$$

where $\sigma_1^*(n)$ denotes the sum of all the prime divisors of n.

Finally we note that in Theorem 2, the primes may be replaced by any subset of the natural numbers.

Reference

[1] D. H. Lehmer, *Calculating moments of partitions*, Proceedings of the Second Manitoba Conference on Numerical Mathematics, University Manitoba, Winnipeg, Man. (1972), pp. 217–220.

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Some remarks on Fermat's conjecture

by

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In a recent note [7] (Theorem 1 cf. also [5]) it was shown: if p is a fixed odd prime, then there exist at most finitely many triples of integers x, y, z which satisfy

$$(1) \qquad x^p + y^p = z^p, \quad (x, y, z) = 1, \quad y > x > 0,$$

and $y - x = k$, where k is a fixed natural number.

Refinements of the effective methods of Baker now allow us to improve the above result. Namely, we can prove:

THEOREM 1. *All solutions in positive integers* x, z, *and odd primes* p, *of the equation*

$$(2) \qquad x^p + (x+k)^p = z^p, \quad (x, k) = 1$$

are bounded by effectively computable constants depending only on the positive integer k.

The new feature is that we now can bound the prime p in terms of k; indeed, as we shall see, in terms of the prime factors of k. We shall give explicit bounds for p and establish some improvements of the above theorem.

1. Bounding the exponent. The following lemma is convenient for bounding p in (2).

LEMMA A. *Let* a, b, q *be integers and let* p *be an odd prime. If* $b > a > 0$, $p > |q|$ *then there is an effectively computable absolute constant* $C > 0$ *such that*

(i) $$|1 - p^q(a/b)^p| > b^{-C(\log p)^3},$$

(ii) *for each prime* $l \neq p$

$$|1 - p^q(a/b)^p|_l > b^{-Cl(\log p)^3}.$$

The first result is implied by Theorem 2 of van der Poorten and Loxton [9] (or by Theorem 2 of Baker [4]) on noting that for $u \geq \frac{1}{2}$ one has $|\log u| \leq 2|1-u|$. The second, l-adic, result is a special case of Theorem 2 of