

with

$$G_{l+2} = \text{diag}(n(g_{\mathfrak{S}})^{l/2+1})$$

for a prime ideal  $\mathfrak{p}$  not dividing the discriminant  $\mathfrak{d}_K$  of  $K/k$ .

Equation (52) also holds for the prime divisors of  $\mathfrak{d}_K$ . We leave the proof to the reader, if he is interested.

The  $U_{l+2}(\varepsilon)$  are often equal to the identity, namely either if the totally positive units of  $k$  are squares of other units, or if they are norms of units of every maximal order of  $K$ .

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(837)

## Corrigendum to the paper "Elementary methods in the theory of $L$ -functions, VII. Upper bound for $L(1, \chi)$ "

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by

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In the course of the proof of Lemma 1 the inequality

$$(1) \quad c(n) = 1 - \sum_{j=1}^s \frac{\beta_j}{\beta_j + 1} \geq \prod_{j=1}^s \frac{1}{\beta_j + 1} = \frac{1}{d(b)}$$

on p. 401, line (–4) is obviously incorrect and also Lemma 1 is not valid in the given form (it is only valid for the special case  $\theta(d) \neq 0$  which corresponds to the case  $D = p$  prime). This fault may be corrected in the following way.

We shall show that on adding to the right side of (2.4)

$$(2) \quad \frac{1}{2} \sum_{q \leq Q} \sum_{\substack{q|n \\ n \leq x}} d\left(\frac{n}{q}\right) + \frac{1}{2} \sum_{q \leq Q} \sum_{\substack{q' \leq Q \\ q' \neq q}} \sum_{\substack{q' | n \\ n \leq x}} d\left(\frac{n}{qq'}\right)$$

Lemma 1 is already valid.

After Lemma 1 we added the sum

$$(3) \quad \frac{1}{2} \sum_{q \leq Q} \sum_{\substack{q' \leq Q \\ qq' \leq x}} \frac{x}{qq'} \log \frac{x}{qq'}$$

to the right of (2.4) (p. 403, line 3) to make more handy the expression and now the same sum occurs on the right of (2.4) (apart from an insignificant error term  $o(x \log x)$ ), thus the Corollary and all the later parts of the proof remain unchanged.

In case of  $r \geq 1$  (cases II-IV) the proof is valid (only  $B$  in (2.8) is modified by the new terms) but in case of  $r = 0$  (case I) we have to prove:

$$(4) \quad c(n) = 1 - \sum_{i=1}^s \frac{\beta_i}{\beta_i+1} + \frac{1}{2} \sum_{i=1}^s \sum_{j=1, j \neq i}^s \frac{\beta_i \beta_j}{(\beta_i+1)(\beta_j+1)} + \frac{1}{2} \sum_{i=1}^s \frac{\beta_i-1}{\beta_i+1} \\ \geq \prod_{i=1}^s \frac{1}{\beta_i+1} = \frac{1}{d(b)}.$$

We shall distinguish the following cases:

I/1. If  $s = 1$ , then

$$(5) \quad c(n) = 1 - \frac{\beta_1}{\beta_1+1} = \frac{1}{\beta_1+1}.$$

I/2. If  $s = 2$ , then

$$(6) \quad c(n) \geq 1 - \frac{\beta_1}{\beta_1+1} - \frac{\beta_2}{\beta_2+1} + \frac{\beta_1 \beta_2}{(\beta_1+1)(\beta_2+1)} = \frac{1}{(\beta_1+1)(\beta_2+1)}.$$

I/3. If  $s = 3$ , then

$$(7) \quad c(n) = \frac{1}{2} \sum_{i=1}^s \frac{\beta_i}{\beta_i+1} \left( \sum_{\substack{j=1 \\ j \neq i}}^s \frac{\beta_j}{\beta_j+1} - 1 \right) + 1 - \frac{1}{2} \sum_{i=1}^s \frac{1}{\beta_i+1} \\ \geq 1 - \frac{1}{2} \cdot 3 \cdot \frac{1}{2} = \frac{1}{4} > \prod_{i=1}^3 \frac{1}{\beta_i+1}.$$

I/4. If  $s = 4$ , and there is an  $i$  ( $1 \leq i \leq 4$ ) with  $\beta_i \neq 1$ , then as in (7)

$$(8) \quad c(n) \geq 1 - \frac{1}{2} \left( 3 \cdot \frac{1}{2} + \frac{1}{3} \right) = \frac{1}{12} > \prod_{i=1}^4 \frac{1}{\beta_i+1}.$$

I/5. If  $s = 4$  and  $\beta_i = 1$  for  $1 \leq i \leq 4$ , then from (4)

$$(9) \quad c(n) = 1 - 4 \cdot \frac{1}{2} + \frac{1}{2} \cdot 4 \cdot 3 \cdot \frac{1}{4} = \frac{1}{2} > \frac{1}{16} = \prod_{i=1}^4 \frac{1}{\beta_i+1}.$$

I/6. If  $s \geq 5$  then by the modification of (7)

$$(10) \quad c(n) = \frac{1}{2} \sum_{i=1}^s \frac{\beta_i}{\beta_i+1} \left( \sum_{\substack{j=1 \\ j \neq i}}^s \frac{\beta_j}{\beta_j+1} - 1 - \frac{1}{\beta_i} \right) + 1 \geq 1 > \prod_{i=1}^s \frac{1}{\beta_i+1}.$$

Finally some misprints in the paper:

1. On p. 403 line 11

$$\text{for } > 2 \sum_{p \in P} \frac{x}{p} - \sum_{q \in Q} \frac{x}{q} = \dots \quad \text{read } > [x] - 2 \sum_{p \in P} \frac{x}{p} - \sum_{q \in Q} \frac{x}{q} = \dots;$$

2. On p. 404, line 6 in (2.16)

$$\text{for } \sum_s \sum_{s' \leq x/s} \frac{1}{ss'} \log \frac{x}{ss'} \quad \text{read } \sum_s \sum_{s' \leq x/s} \frac{1}{ss'} \log \frac{x}{ss'};$$

3. On p. 404, line 8 for  $s \neq s'$  read  $S \neq S'$ ;

4. On p. 404, line 13 for  $s = s'$  read  $S = S'$ ;

5. On p. 405, line (-9)

$$\text{for } \sum_{s \in S} \frac{1}{s} \quad \text{read } \sum_{s \in S'} \frac{1}{s} \log s.$$

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