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W R O C Ł A W S K A   D R U K A R N I A   N A U K O W A

Johannes G. van der Corput (1890–1975)
A biographical note

by

N. G. de Bruijn

The dutch mathematician J. G. van der Corput was born at Rotter-
ı on September 4th, 1890. From 1908 to 1914 he studied at Leyden
ersity, where analysis and analytical number theory were taught
J. C. Kluyver (1860–1932), who was the first one to promote the study
igid modern analysis and analytical number theory in the Netherlands.
ı der Corput had to serve as an officer in the army from 1914 to 1917.
m 1917 to 1920 he taught mathematics in secondary shools at Leeuwar-
and Utrecht. In this period he got his doctorate from Leyden uni-
ity. Part of the year 1920 he spent with E. Landau at Göttingen.
m 1920 to 1922 Van der Corput was A. Denjoy's assistant at Utrecht
ersity, and from 1922 to 1923 professor at Fribourg (Switzerland).
1923 he returned to the Netherlands as professor at the university
roningen. He stayed there until 1946, and then moved to the munici-
university of Amsterdam (1946–1953). From then on he had various
tions in the United States, like Berkeley (California) from 1954 to
3, Stanford (California) and Madison (Wisconsin). As professor emeri-
he returned to the Netherlands and died at Amsterdam on September
ı, 1975, at the age of 85.

Van der Corput's dissertation ("Over roosterpunten in het platte
: — de betekenis van de methoden van Voronoï en Pfeiffer", Noord-
, Groningen, 1919) was in analytic number theory. It contained
ell-organized survey of the various methods for estimating the number
attice points in a big circle and in the region between an orthogonal
erbola and its asymptotes (the Dirichlet problem). All these methods
to the estimate $O(x^{1/3})$ for the error term, also for the more general
ons he investigated. It was a very fine piece of work. Landau, the
ing expert in the field at the time, was very much impressed.

It seemed a kind of harmony that suggested $O(x^{1/3})$ to be the right
r term in all these problems, but in 1922 Van der Corput shocked the
d by showing that the exponent 1/3 could be decreased (Math. Ann.

87 (1922), pp. 39–65). This work was based on his refined techniques for treating sums of exponentials and was certainly one of the deepest pieces of analytic number theory ever made. Some further improvements led him a few years later to the exponent 27/82. The method could be applied to the circle problem by his student L.W. Nieland, with the same error estimate $O(x^{27/82})$. Another problem attacked by the method was the order of growth of the Riemann zeta function, on which Van der Corput collaborated with his student J. F. Koksma.

Until about 1940 Van der Corput was extremely active in analytic number theory, with series of papers on Diophantine approximation, Vinogradov's method, Goldbach's conjecture, and Geometry of numbers. After 1940 Van der Corput added to his work on number theory an active interest in many other branches of mathematics. His earlier work on the asymptotic method of stationary phase initialized extensive work in his later theory of neutrices. He wrote on a wide variety of subjects, like the study of functional equations for the elementary functions, and a new proof for the fundamental theorem of algebra. Nevertheless he kept working in number theory too. Special attention might be given to what was the first complete account on the Erdös–Selberg elementary proof of the prime number theorem (Math. Centrum Amsterdam, Scriptum no. 1 (1948)).

Van der Corput was very stimulating as a teacher, and made his students collaborate with him on his best ideas. In particular four of his best students might be mentioned who all died before him: L. W. Nieland, J. F. Koksma, J. Popken, C. S. Meijer.

A very remarkable episode in Van der Corput's life was his initiative in 1946 to start a national institution for the promotion of both pure and applied mathematics, in order to give the mathematical background for the post-war industrial development in the Netherlands. This Mathematical Centre did indeed provide such a background in various areas, especially in statistics and computer science. Van der Corput was its first director (1946–1953).

The Royal Netherlands Academy of Sciences and Letters made Van der Corput a member in 1929. Furthermore he was honoured by doctorates honoris causa from the University of Bordeaux and from the Technological University at Delft, and by the membership of the Royal Academy for Sciences and Letters of Belgium.

Van der Corput was an editor of Acta Arithmetica from its start in 1936, and he had an article in its first volume.

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Superprimes and a generalized Frobenius symbol

by

WOLFRAM JEHNE and NORBERT KLINGEN (Köln)

Dedicated to Karl Dörge on occasion of his 75th birthday

This paper can be considered from three points of view: 1. ultrafilter invariants, 2. J. Ax's theory of ultraproducts of finite fields [2], and 3. Jarden's "translation principle" ([10], [12]) which connects the Dirichlet density with the Haar measure in the Galois group. We shall give an extension of ordinary arithmetic of global fields k, which — in a certain sense — can be interpreted as arithmetic of special non-standard models of k. The results will be applied to the first two cases mentioned above.

According to the general philosophy, non-principal ultrafilters are so highly unconstructive that they cannot be distinguished from one another (at least in the case of a countable index set, Bell–Slomson [5]). However, we shall define number theoretic invariants, related to class field theory, which allow us to divide all non-principal ultrafilters on the set of all primes into $2^{\aleph_0}$ different classes (Section 2).

More generally, we consider the space Ω_k of all non-principal ultrafilters U on the set P_k of all prime divisors of a global field k. Since they are related to prime divisors of a non-standard model $^*k$ of k we call the ultrafilters $U \in \Omega_k$ the "superprimes" of k. As is well known ([4] or [7]) Ω_k is a compact subspace of the Stone–Čech compactification $\hat{P}_k$ of the discrete countable set P_k. For those superprimes we can define the usual notions, such as "lying over", ramification, residue class fields, etc., and obtain analogous elementary properties, e.g. $n = \sum e_i f_i$ (Section 1). It turns out that superprimes are always unramified.

To each superprime of k we can attach a generalized Frobenius- and Artin symbol. Our main result states that for a (not necessary finite) Galois extension $K|k$ of a global field k the generalized Artin symbol defines a continuous surjective mapping

$$\left(\frac{K|k}{}\right): \Omega_k \to \varkappa G(K|k)$$

of the compact superprime space Ω_k onto the compact space $\varkappa G(K|k)$ of all conjugacy classes of the Galois group $G(K|k)$. This mapping extends